Games solved: Now and in the future

by H. J. van den Herik, J. W. H. M. Uiterwijk, and J. van Rijswijck

Tsan-sheng Hsu

徐讚昇

tshsu@iis.sinica.edu.tw

http://www.iis.sinica.edu.tw/~tshsu

Abstract

- **Which game characters are predominant when the solution of a game is the main target?**
  - **It is concluded that decision complexity is more important than state-space complexity.**
  - **There is a trade-off between knowledge-based methods and brute-force methods.**
  - **There is a clear correlation between the first-player's initiative and the necessary effort to solve a game.**

Definitions (1/4)

- **Domain: two-person zero-sum games with perfect information.**
  - **Zero-sum means one player's loss is exactly the other player's gain, and vice versa.**
    - ▷ *There is no way for both players to win at the same time.*
- **Game-theoretic value of a game: the outcome, i.e., win, loss or draw, when all participants play optimally.**
  - **Classification of games' solutions according to L.V. Allis [Ph.D. thesis 1994] if they are considered solved.**
    - ▷ ***Ultra-weakly solved:*** *the game-theoretic value of the initial position has been determined.*
    - ▷ ***Weakly solved:*** *for the initial position a strategy has been determined to achieve the game-theoretic value against any opponent.*
    - ▷ ***Strongly solved:*** *a strategy has been determined for all legal positions.*
  - **The game-theoretical values of many games are unknown or are only known for some legal positions.**

Definitions (2/4)

- **State-space complexity of a game: the number of the legal positions in a game.**
- **Game-tree (or decision) complexity of a game: the number of the leaf nodes in a *solution search tree*.**
  - ▷ *A solution search tree is a tree where the game-theoretic value of the root position can be decided.*
- **A fair game: the game-theoretic value is draw and both players have roughly an equal probability on making a mistake.**
  - ▷ *Paper-scissor-stone*
  - ▷ *Roll a dice and compare who gets a larger number*
- **Initiative: the right to move first.**

Definitions (3/4)

- **A convergent game: the size of the state space decreases as the game progresses.**
  - **Start with many pieces on the board and pieces are gradually removed during the course of the game.**
    - ▷ *Example: Checkers.*
  - **It means the number of possible configurations decreases as the game progresses.**
- **A divergent game: the size of the state space increases as the game progresses.**
  - **May start with an empty board, and pieces are gradually added during the course of the game.**
    - ▷ *Example: Connect-5 before the board is almost filled.*
  - **It means the number of possible configurations increases as the game progresses.**

Definitions (4/4)

- A game may be convergent at one stage and then divergent at other stage.
  - Most games are dynamic.
  - For the game of Tic-Tac-Toe, assume you have played x plys with x being even.
    - *Then you have a possible of*

$$\binom{9}{x/2}\binom{9-x/2}{x/2}$$

    different configurations.
  - This number is not monotone increasing or decreasing.

Predictions made in 1990

- **Predictions were made in 1990 [Allis et al 1991] for the year 2000 concerning the expected playing strength of computer programs.**

solved	over champion	world champion	grand master	amateur
Connect-four	Checkers $(8 * 8)$	Chess	Go $(9 * 9)$	Go $(19 * 19)$
Qubic	Renju	Draughts $(10 * 10)$	Chinese chess	
Nine Men's Morris	Othello		Bridge	
Go-Moku	Scrabble			
Awari	Backgammon			

  - ▷ *Over champion means definitely over the best human player.*
  - ▷ *World champion means equaling to the best human player.*
  - ▷ *Grand master means beating most human players.*

A double dichotomy of the game space

$\log\log(\text{state-space complexity}) \rightarrow$

category 3 if solvable at all, then by knowledge-based methods	**category 4** currently unsolvable by any method
category 1 solvable by any method	**category 2** if solvable at all, then by brute-force methods

$\log\log(\text{game-tree complexity}) \rightarrow$

Questions to be researched

- **Can perfect knowledge obtained from solved games be translated into rules and strategies which human beings can assimilate?**
- **Are such rules generic, or do they constitute a multitude of ad hoc recipes?**
- **Can methods be transferred between games?**
  - **More specifically, are there generic methods for all category-n games, or is each game in a specific category a law unto itself?**

Convergent games

- **Since most games are dynamic, here we consider games whose ending phases are convergent.**
  - **Can be solved by the method of endgame databases if we can enumerate and store all possible positions at a certain stage.**
- **Problems solved:**
  - **Nine Men's Morris: in the year 1995, a total of 7,673,759,269 states.**
    - ▷ *The game theoretic value is draw.*
  - **Mancala games**
    - ▷ *Awari: in the year 2002.*
    - ▷ *Kalah: in the year 2000 upto, but not equal, Kalah(6,6)*
  - **Checkers**
    - ▷ *By combining endgame databases, middle-game databases and verification of opening-game analysis.*
    - ▷ *Solved the so called 100-year position in 1994.*
    - ▷ *The game is proved to be a draw in 2007.*
  - **Chess endgames**
  - **Chinese chess endgames**

Divergent games

- **Since most games are dynamic, here we consider games whose INITIAL phases are divergent.**
- **Connection games**
  - **Connect-four ($6 * 7$)**
  - **Qubic ($4 * 4 * 4$)**
  - **Go-Moku ($15 * 15$)**
  - **Renju**
  - **k-in-a-row games**
  - **Hex ($10 * 10$ or $11 * 11$)**
- **Polynmino games**
  - **Pentominoes**
  - **Domineering**
- **Othello**
- **Chess**
- **Chinese chess**
- **Shogi**
- **Go**

Connection games (1/2)

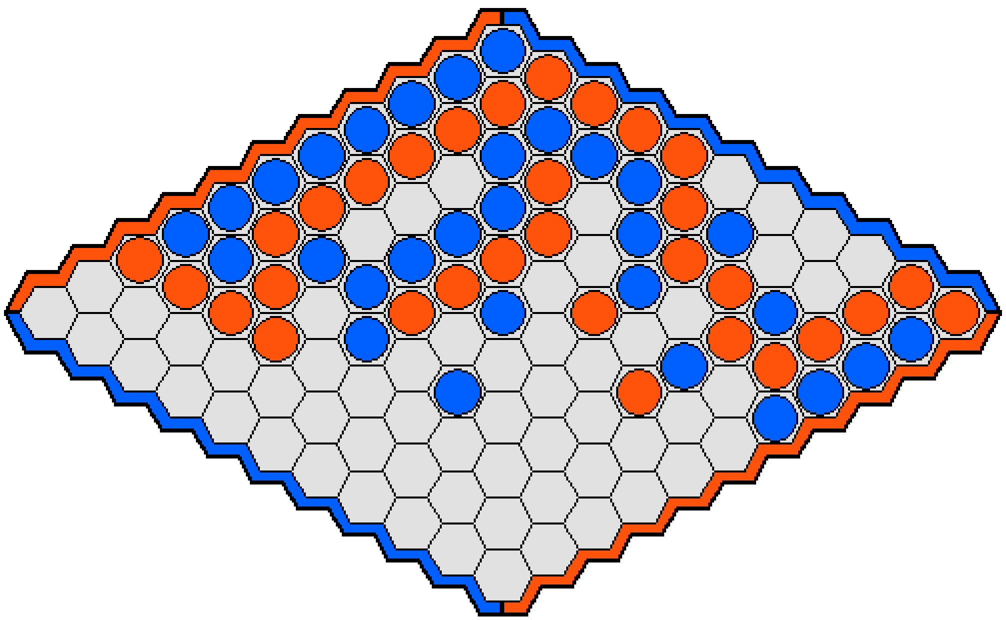
- **Connect-four** ($6 * 7$)
  - Solved by J. Allen in 1989 using a brute-force depth first search with alpha-beta pruning, a transposition table, and killer-move heuristics.
  - Also solved by L.V. Allis in 1988 using a knowledge-based approach by combining 9 strategic rules that identify potential threats of the opponent.
    - ▷ *Threats are something like forced moved or moves you have little choices.*
    - ▷ *Threats are moves with predictable counter-moves.*
  - It is first-player win.
  - Weakly solved on a SUN-4 workstation using 300+ hours.
- **Qubic** ($4 * 4 * 4$)
  - A three-dimensional version of Tic-Tac-Toe.
  - Connect-four played on a $4 * 4 * 4$ game board.
  - Solved in 1980 by O. Patashnik by combining the usual depth-first search with expert knowledge for ordering the moves.
    - ▷ *It is first-player win for the 2-player version.*

Connection games (2/2)

- **Go-Moku ($15 * 15$)**
  - **First-player win.**
  - **Weakly solved by L.V. Allis in 1995 using a combination of threat-space search and database construction.**
- **Renju**
  - **Does not allow the first player to play certain moves.**
  - **An *asymmetric* game.**
  - **Weakly solved by Wágner and Viráag in 2000 by combining search and knowledge.**
    - ▷ ***Took advantage of an iterative-deepening search based on threat sequences up to 17 plies.***
    - ▷ ***It is still first-player win.***
- **k-in-a-row games**
  - **mnk-Game: a game playing on a board of m rows and n columns with the goal of obtaining a straight line of length k.**
  - **Variations: first ply picks only one stone, the rest picks two stones in a ply.**
    - ▷ ***Connect 6.***
    - ▷ ***Try to balance the advantage of the initiative!***

Hex ($10 * 10$ or $11 * 11$)

- **Properties:**
  - **It is a finite game.**
  - **It is not possible for both players to win at the same.**
  - **Exactly one of the players can win.**



Red won

Courtesy of ICGA web site

Proof on exactly one player win (1/2)

- A topological argument.
  - A vertical chain can only be cut by a horizontal chain and vice versa because each cell is connected with 6 adjacent cells.
    - ▷ *Note if a cell has 4 neighbors as in the case of Go, then it is possible to cut off a vertical chain by cells that are not horizontally connected and vice versa.*
- Other arguments such as one using graph theory exist.

Proof on exactly one player win (2/2)

- **W.l.o.g. let B be the set of cells that can be reached by chains originated from the rightmost column.**
- **B does not contain a cell of the leftmost column; otherwise we have a contradiction.**
  - **Let $N(B)$ be the red cells that can be reached by chains originated from the rightmost column.**
  - **$N(B)$ must be connected**
    - ▷ ***Otherwise, B can advance further.***
  - **$N(B)$ must contain a cell in the top row.**
    - ▷ ***Otherwise, B contains all cells in the first row, which is a contradiction.***
  - **$N(B)$ must contain a cell in the bottom row.**
    - ▷ ***Otherwise, B contains all cells in the bottom row, which is a contradiction.***
  - **Hence $N(B)$ is a red winning chain.**

Illustration of the ideas (1/2)

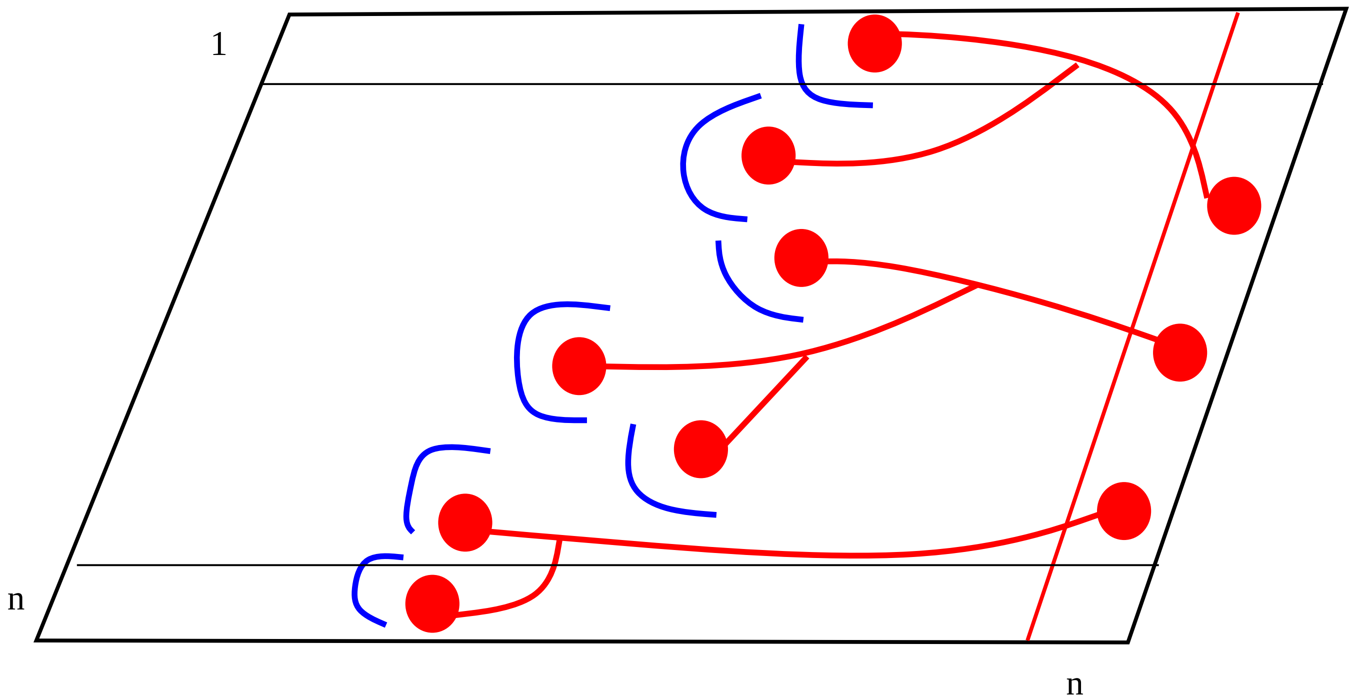
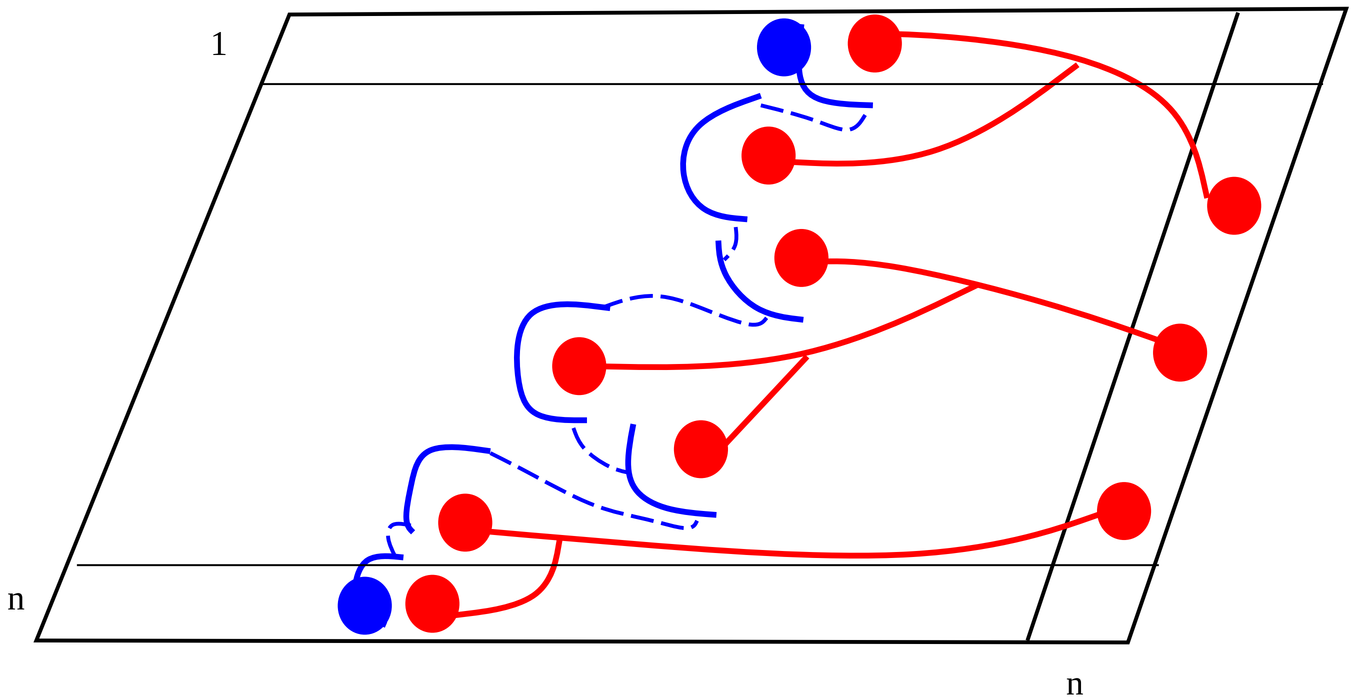


Illustration of the ideas (2/2)



Strategy-stealing argument (1/2)

- **The** ***unrestricted form of Hex is a first-player win game. using the "strategy-stealing" argument made by John Nash in 1949.***
  - **If there is a winning strategy for the second player, the first player can still win by making an arbitrary first move and using the second-player strategy from then on.**
    - ▷ *The first player ignores the arbitrary first move by assuming that move doe snot exist.*
    - ▷ *Hence the second move made by the second player becomes the first move.*
    - ▷ *The third move made by the first player becomes the second move.*
  - **If using the second-player strategy requires playing the chosen first move or any move played before, then make another arbitrary move.**
    - ▷ *An arbitrary extra move can never be a disadvantage in Hex.*
  - **We have obtained a contradiction, and thus the second player cannot win.**
  - **Since we have proved there is no draw, and there is always a winner, and both players cannot win at the same time, the first player must have a winning strategy.**

Strategy-stealing argument (2/2)

- This is not a constructive proof.
- The strategy-stealing argument cannot be used for every game.
  - An arbitrary extra move can never be a disadvantage in Hex.
  - This may not be true for other games.
- The argument works for any game when
  - it is symmetric,
  - it always has exactly one winner, and
    - ▷ *namely, it cannot have a draw by having no winners or 2 winners,*
  - an arbitrary extra move can never be a disadvantage.
    - ▷ *Note: it requires that a player is always possible to place an arbitrary move which may not be true for some games.*

Properties of Hex

- **Variations of Hex**
  - **The one-move-equalization rule: one player plays an opening move and the other player then has to decide which color to play for the reminder of the game.**
    - ▷ ***The revised version is a second-player win game (ultra-weakly).***
- **Hex exhibits considerable mathematical structure.**
  - **Hex in its general form has been proved to be PSPACE-complete by Even and Tarjan in 1976 by converting it to a Shannon switching game.**
  - **The state-space and decision complexities are comparable to those of Go on an equally-sized board.**
- **Solutions**
  - **(Weakly or strongly) solved on a $6 * 6$ board in 1994.**
  - **Maybe possible to solve the $7 * 7$ case.**
    - ▷ ***The $7 * 7$ case was solved in 2001. [Yang et. al. 2001]***
  - **Not likely to solve the $8 * 8$ version without fundamental breakthroughs.**
    - ▷ ***The $8 * 8$ case was solved in 2009. [Henderson et. al. 2009]***

More divergent games (1/2)

- **Polynmino games: placing 2-D pieces of a connected subset of a square grid to construct a special form.**
  - **Pentominoes**
  - **Domineering**
  - **Games on smaller boards have been solved.**
- **Othello**
  - **M. Buro's LOGISTELLO beat the resigning World Champion by 6-0 in 1997.**
  - **Weakly solved on a $6 * 6$ board by J. Feinstein in 1993.**
- **Chess**
  - **DEEP BLUE beat the human World Champion in 1997.**

More divergent games (2/2)

- **Chinese chess**
  - **Still in progress,**
  - **Professional 7-dan in 2007.**
- **Shogi**
  - **Still in progress,**
  - **Claimed to be professional 2-dan in 2007.**
  - **Defeat a Lady professional player in 2010.**
- **Go**
  - **Still in progress.**
  - **Recent success and breakthrough using Monte Carlo UCT based methods.**
  - **Amateur 1 – 4 kyu in 2008.**
    - ▷ *Beat a professional 8-dan by having an 8-stone advantage.*
    - ▷ *Beaten by a professional 9-dan by giving a 7-stone advantage.*
  - **Amateur 1 dan in 2010.**
  - **Amateur 3 dan in 2011.**

Table of complexity

Game	$\log_{10}$(state-space)	$\log_{10}$(game-tree size)
Nine Men's Morris	10	50
Pentominoes	12	18
Awari	12	32
Kalak(6,4)	13	18
Connect-four	14	21
Domineering $(8 * 8)$	15	27
Dakon-6	15	33
Checkers	21	31
Othello	28	58
Qubic	30	34
Draughts	30	54
Chess	46	123
Chinese chess	48	150
Hex $(11 * 11)$	57	98
Shogi	71	226
Renju $(15 * 15)$	105	70
Go-Moku $(15 * 15)$	105	70
Go $(19 * 19)$	172	360

State-space versus game-tree size

- **In 1994, the boundary of solvability by complete enumeration was set at 10^{11}.**
  - **The current estimation is about 10^{13} (since the year 2007).**
- **It is often possible to use heuristics in searching a game tree to cut the number of nodes visited tremendously when the structure of the game is well studied.**
  - **Example: Connect-Four.**

Methods developed for solving games

- **Brute-force methods**
  - **Retrograde analysis**
  - **Enhanced transposition-table methods**
- **Knowledge-based methods**
  - **Threat-space search and λ-search**
  - **Proof-number search**
  - **Depth-first proof-number search**
  - **Pattern search**
    - ▷ *To search for **threat patterns**, which are collections of cells in a position.*
    - ▷ *A threat pattern can be thought of as representing the **relevant area** on the board, an area that human players commonly identify when analyzing a position.*
- **Recent advancements:**
  - **Monte Carlo UCT based game tree simulation.**
    - ▷ *Monte Carlo method has a root from statistic.*
    - ▷ *Biased sampling.*
    - ▷ *Using methods from machine learning.*
    - ▷ *Combining domain knowledge with statistics.*
  - **A majority vote algorithm.**

Brute-force versus knowledge-based methods

- **Games with both a relative low state-space complexity and a low game-tree complexity have been solved by both methods.**
  - Category 1
  - Connect-four and Qubic
- **Games with a relative low state-space complexity have mainly been solved with brute-force methods.**
  - Category 2
  - Namely by constructing endgame databases
  - Nine Men's Morris
- **Games with a relative low game-tree-complexities have mainly been solved with knowledge-based methods.**
  - Category 3
  - Namely, by intelligent (heuristic) searching
  - Sometimes, with the helps of endgame databases
  - Go-Moku, Renju, and k-in-a-row games

Advantage of the initiative

- **Theorem (or argument) made by Singmaster in 1981: The first player has advantages.**
  - **Two kinds of positions**
    - ▷ ***P-positions: the previous player can force a win.***
    - ▷ ***N-positions: the next player can force a win.***
  - **Arguments**
    - ▷ ***For the first player to have a forced win, just one of the moves must lead to a P-position.***
    - ▷ ***For the second player to have a forced win, all of the moves must lead to N-positions.***
    - ▷ ***It is easier to the first player to have a forced win assuming all positions are randomly distributed.***
    - ▷ ***Can be easily extended to games with draws.***
- **Remarks:**
  - **One small boards, the second player is able to draw or even to win for certain games.**
  - **Cannot be applied to the infinite board.**

How to make use of the initiative

- **A potential universal strategy for winning a game:**
  - **Try to obtain a small advantage by using the initiative.**
    - *The opponent must react adequately on the moves played by the other player.*
  - **To reinforce the initiative the player searches for threats, and even a sequence of threats using an evaluation function E.**
  - **Force the opponent to always play the moves you expected.**
- **Threat-space search**
  - **Search for threats only!**

Offsetting the initiative

- **An example of a game with a huge initiative:**
  - **A connection $mn1$-game.**
    - ▷ *一子棋 was mentioned in 張系國著名小說"棋王"(1978年出版).*
  - **A connection $mn2$-game.**
  - **A connection $mn3$-game.**
- **Need to offset the initiative.**
  - **The offsetting rule must be simple.**
  - **The revised game must be as fair as possible.**
    - ▷ *It is difficult to prove a game is fair.*
    - ▷ *Example: Paper-scissor-stone is fair.*
  - **The revised game needs be fun to play with.**
  - **The revised game cannot be too much different from the original game.**
- **Knowing how to properly offsetting the initiative may uncover some fundamental properties of the game such as the level of difficulty.**

Examples (1/2)

- **Enforce rules so that the first player cannot win by selective patterns.**
  - **Renju.**
    - ▷ *Still first-player win.*
  - **Go ($19 * 19$).**
    - ▷ *The first player must win by more than 7 stones.*
    - ▷ *Komi $= 7.5$ in 2011.*
    - ▷ *The value of Komi changes with the time and maybe different because of using different set of rules.*
- **The one-move-equalization rule: one player plays an opening move and the other player then has to decide which color to play for the reminder of the game.**
  - ▷ *Hex.*
  - ▷ *Second-player win.*

Examples (2/2)

- **The first move plays one stone, the rest plays two stones each.**
  - *Connect 6.*
  - *Intuitively, in each turn the initiative goes to different players alternatively.*
  - *Still not able to prove it is a fair game (at 2012).*
- **The first player uses less resource.**
  - **For example: using less time.**
    - *Chinese chess.*
  - **A resource-auctioning scheme.**
- **Unclear how to obtain a fair game.**

Conclusions

- **The knowledge-based methods mostly inform us on the structure of the game, while exhaustive enumeration rarely does.**
- **Many ad-hoc recipes are produced currently.**
  - **The database can be used as a corrector or verifier of strategies formulated by human experts.**
- **It may be hopeful to use data mining techniques to obtain cross-game methods.**
  - **Currently not very successful.**

1990's Predictions — 2000's Status

- Predictions were made in 1990 [Allis et al 1991] for the year 2000 concerning the expected playing strength of computer programs.

solved	over champion	world champion	grand master	amateur
Connect-four	Checkers $(8 * 8)$	Chess	Go $(9 * 9)$	Go $(19 * 19)$
Qubic	Renju	Draughts $(10 * 10)$	Chinese chess	
Nine Men's Morris	Othello		Bridge	
Go-Moku	Scrabble			
Awari	Backgammon			

- color code
  - Green: Performs much better than expected
  - Red: right on the target.
  - Black: have some progress towards the target.
  - Blue: not so.

Predictions for 2010

- **Predictions were made at the year 2000 for the year 2010 concerning the expected playing strength of computer programs.**

solved	over champion	world champion	grand master	amateur
Awari	Chess	Go $(9 * 9)$	Bridge	Go $(19 * 19)$
Othello	Draughts $(10 * 10)$	Chinese chess	Shogi	
Checkers $(8 * 8)$	Scrabble	Hex		
	Backgammon	Amazons		
	Lines of Action			

Predictions for 2010 – Status

- **My personal opinion about the status of Prediction-2010 at October, 2010, right after the Computer Olympiad held in Kanazawa, Japan.**

solved	over champion	world champion	grand master	amateur
Awari	Chess	Go $(9 * 9)$	Bridge	Go $(19 * 19)$
Othello	Draughts $(10 * 10)$	Chinese chess	Shogi	
Checkers $(8 * 8)$	Scrabble	Hex		
	Backgammon	Amazons		
	Lines of Action			

- **color code**
  - **Red: right on the target.**
  - **Black: have some progress towards the target.**
  - **Blue: not so.**

References and further readings (1/2)

- L.V. Allis, H.J. van den Herik, and I.S. Herschberg. Which games will survive? In: D.N.L. Levy, D.F. Beal (Eds.), Heuristic Programming in Artificial Intelligence 2: The Second Computer Olympiad, Ellis Horwood, Chichester, 1991, pp. 232-243.

* H. J. van den Herik, J. W. H. M. Uiterwijk, and J. van Rijswijck. Games solved: Now and in the future. *Artificial Intelligence*, 134:277–311, 2002.

- Jonathan Schaeffer. The games computers (and people) play. *Advances in Computers*, 52:190–268, 2000.
- L. V. Allis, M. van der Meulen, and H. J. van den Herik. Proof-number search. *Artificial Intelligence*, 66(1):91–124, 1994.

References and further readings (2/2)

- J. Yang, S. Liao, and M. Pawlak. A decomposition method for finding solution in game Hex 7x7. In *Proceedings of International Conference on Application nd Development of Computer games in the 21st century*, pages 93–112, November 2001.
- P. Henderson, B. Arneson, and R. B. Hayward. Solving 8x8 Hex. In *Proceedings of IJCAI*, pages 505–510, 2009.