

# Theory of Computer Games: Concluding Remarks

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# Abstract

- **Practical issues.**
  - The open book.
  - Smart usage of resources.
    - ▷ *time during searching*
    - ▷ *memory*
    - ▷ *coding efforts*
    - ▷ *debugging efforts*
  - Putting everything together.
- **Some advanced research issues.**
  - The graph history interaction (GHI) problem.
  - Opponent models.
  - Searching chance nodes.
- **How to test your program?**

# The open book (1/2)

- During the open game, it is frequently the case
  - branching factor is huge;
  - it is difficult to write a good evaluating function;
  - the number of possible distinct positions up to a limited length is small as compared to the number of possible positions encountered during middle game search.
- Acquire game logs from
  - books;
  - games between masters;
  - games between computers;
    - ▷ *Use off-line computation to find out the value of a position for a given depth that cannot be computed online during a game due to resource constraints.*
  - ...

# The open book (2/2)

- Assume you have collected  $r$  games.
  - For each position in the  $r$  games, compute the following 3 values:
    - ▷ *win*: the number of games reaching this position and then wins.
    - ▷ *loss*: the number of games reaching this position and then loss.
    - ▷ *draw*: the number of games reaching this position and then draw.
- When  $r$  is large and the games are **trustful**, then use the 3 values to compute an **estimated goodness** for this position.
- Comments:
  - Pure statistically.
  - Can build a static open book.
  - Your program may not be able to **take over** when the open book is over.
  - It is difficult to acquire large amount of “trustful” game logs.
  - Automatically analysis of game logs written by human experts. [Chen et. al. 2006]
  - Using high-level meta-knowledge to guide the way in searching:
    - ▷ *Dark chess: adjacent attack of the opponent’s Cannon.* [Chen and Hsu 2013]

# Example: Chinese chess open book (1/3)

- A total of 28,591 (Red win)+21,072 (Red lose)+55,930 (draw) games.



# Example: Chinese chess open book (2/3)

- Can be sorted using different criteria.
  - Win-lose
  - winning rates
  - ...



# Example: Chinese chess open book (3/3)

- A tree-like structure.

The screenshot displays the 'Chinese Chess (V4.2)' software interface. The main window shows a chessboard with pieces in their starting positions. The interface includes a menu bar at the top with options like '檔案', '顯示', '標記', '電腦審局', '專家標記', '擺盤模式', '走譜模式', '棋局模式', and '開始'. Below the menu bar, there are buttons for '紅方' (Red) and '黑方' (Black), along with a timer and step counter. The chessboard is labeled with columns 1-9 and rows 九-一. The pieces are arranged as follows: Red (bottom): 帥, 仕, 相, 馬, 車; Black (top): 將, 士, 象, 馬, 車. Pawns (卒) and cannons (炮) are also present. On the right side, there is a '分數選擇' (Score Selection) section with a dropdown menu set to '2.依照盤面分數<版本4> (單純勝率) (W/N)'. Below this, there are '使用者自訂參數' (User Custom Parameters) for Red and Black, and '篩選條件' (Filter Conditions) for '盤數' (Number of Games), '盤數比例' (Percentage of Games), '分數' (Score), and '分數比例' (Percentage of Score). The '歷史走步' (History Moves) section shows a list of moves: '<初始盤面>', '[1]<子> 炮二平五 <0.28> 審局:-1 [紅: 1]', and '[2]<子> 象7進5 <0.24> 審局:65 [紅: 2]'. The '父節點' (Parent Node) section shows '<返回上一步>'. The '子節點' (Child Node) section shows a list of moves with their scores: '[1]卒1進1 <1.00> [紅: 0 黑: 1 和: 0]', '[2]包2進2 <0.67> 審局:145 [紅: 1 黑: 1]', '[3]卒7進1 <0.33> 審局:151 [紅: 2 黑: 1]', '[4]象7進5 <0.24> 審局:65 [紅: 25 黑: 1]', '[5]馬8進7 <0.23> 審局:25 [紅: 1283 黑: 1]', and '[6]包2平5 <0.21> 審局:78 [紅: 61 黑: 1]'. The bottom left corner of the window displays the text '目前沒有備忘錄' (No memo currently).

# Using resources: time and others

- Time is the most critical resource [Hyatt 1984] [Šolak and Vučković 2009].
- Watch out different timing rules.
  - An upper bound on the total amount of time can be used.
    - ▷ *It is hard to predict the total number of moves in a game in advance. However, you can have some rough ideas.*
  - Fixed amount of time per ply.
  - An upper bound  $T_1$  on the total amount of time is given, and then you need to play  $X$  plys every  $T_2$  amount of time.
- Thinking style of human players.
  - Using almost no time while you are in the open book.
  - More time is spent in the beginning immediately after the program is out of the book.
  - Stop searching a path further when you think the position is **stable** in the middle game.
  - In the endgame phase, use more time in critical positions or when you try to initiate an attack.
  - Do not think at all if you have only one possible logical move left.



# Pondering

- **Pondering:**
  - Use the time when your opponent is thinking.
  - Guessing and then pondering.
- **How pondering is done.**
  - In your turn, keep the first 2 plys  $m_1$  and  $m_2$  in the PV you obtained.
  - You choose to play  $m_1$ , and then it's the opponent's turn to think.
  - In pondering, you can assume the opponent plays  $m_2$ .
  - Then you continue to think at the same time your opponent thinks.
  - If the opponent plays  $m_2$ , then you can continue the pondering search in your turn.
  - If the opponent plays other moves, then you restart a new search.

# Using other resources

## ■ Memory

- Using a large transposition table occupies a large space and thus slows down the program.
  - ▷ *A large number of positions are not visited too often.*
- Using no transposition table makes you to search a position more than once.

## ■ CPU

- Do not fork a process to search branches that have little hope of finding the PV even you have more than enough hardware.

## ■ Other resources.

# Putting everything together

## ■ Game playing system

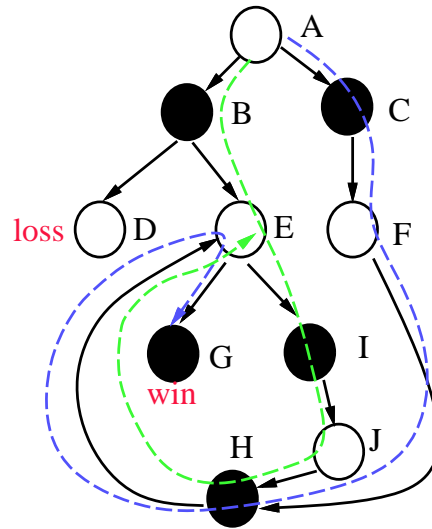
- GUI
- Use some sorts of open books.
- Middle-game searching: usage of a search engine.
  - ▷ *Evaluating function: knowledge.*
  - ▷ *Main search algorithm.*
  - ▷ *Enhancements: transposition tables, Quiescent search and possible others.*
- Use some sorts of endgame databases.

## ■ Debugging and testing

# Graph history interaction problem

- The graph history interaction (**GHI**) problem [Campbell 1985]:
  - In a game graph, a position can be visited by more than one paths.
  - The value of the position depends on **the path** visiting it.
    - ▷ *It can be win, loss or draw for Chinese chess.*
    - ▷ *It can only be draw for Western chess.*
    - ▷ *It can only be loss for Go.*
- In the transposition table, you record the value of a position, but not the path leading to it.
  - Values computed from rules on repetition cannot be used later on.
  - It takes a huge amount of storage to store all the paths visiting it.
- This is a very difficult problem to be solved in real time [Wu et al. '05].

# GHI problem – example



- Assume the one causes loops loses the game.
- $A \rightarrow B \rightarrow E \rightarrow I \rightarrow J \rightarrow H \rightarrow E$  is loss because of **rules of repetition**.
  - ▷ *Memorized H as a loss position.*
- $A \rightarrow B \rightarrow D$  is a loss.
- $A \rightarrow C \rightarrow F \rightarrow H$  is loss because  $H$  is recorded as loss.
- $A$  is loss because both branches lead to loss.
- **However,  $A \rightarrow C \rightarrow F \rightarrow H \rightarrow E \rightarrow G$  is a win.**

# Opponent models

- In a normal alpha-beta search, it is assumed that you and the opponent use the same strategy.
  - What is good to you is bad to the opponent and vice versa!
  - Hence we can reduce a minimax search to a NegaMax search.
  - This is normally true when the game ends, but may not be true in the middle of the game.
- What will happen when there are two strategies or evaluating functions  $f_1$  and  $f_2$  so that
  - for some positions  $p$ ,  $f_1(p)$  is **better** than  $f_2(p)$ 
    - ▷ “better” means closer to the real value  $f(p)$
  - for some positions  $q$ ,  $f_2(q)$  is **better** than  $f_1(q)$
- If you are using  $f_1$  and you know your opponent is using  $f_2$ , what can be done to take advantage of this information.
  - This is called OM (**opponent model**) search [Carmel and Markovitch 1996].
    - ▷ In a MAX node, use  $f_1$ .
    - ▷ In a MIN node, use  $f_2$ .

# Opponent models – comments

## ■ Comments:

- Need to know your opponent model precisely.
- How to learn the opponent on-line or off-line?
- When there are more than 2 possible opponent strategies, use a probability model (PrOM search) to form a strategy.

# Search with chance nodes

## ■ Chinese dark chess

- Two player, zero sum, complete information
- **Perfect information**
- **Stochastic**
- There is a **chance** node during searching [Ballard 1983].
  - ▷ *The value of a node is a distribution, not a fixed value.*

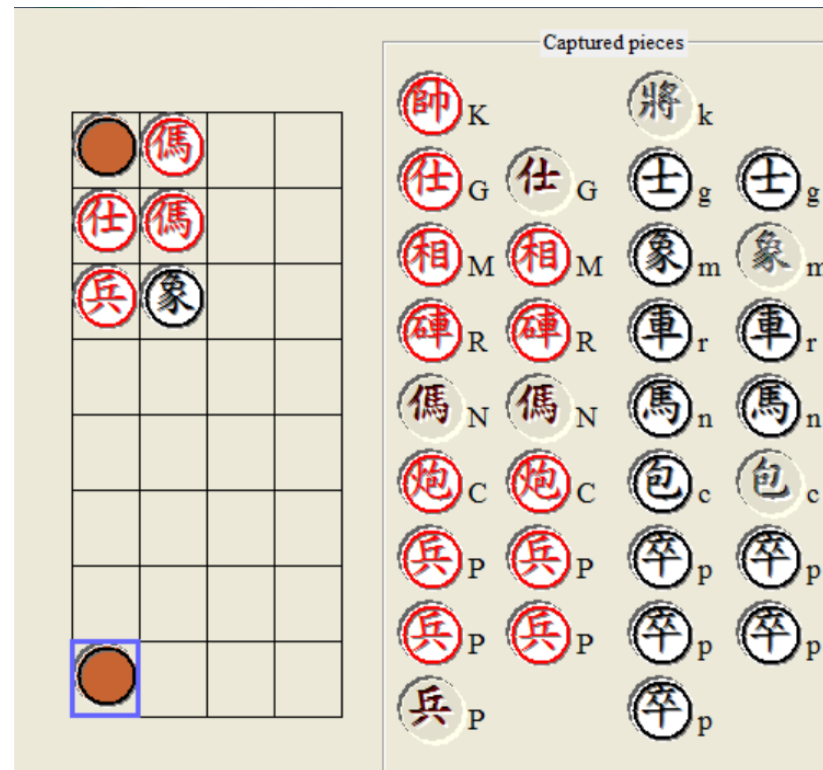
## ■ Previous work

- Alpha-beta based [Ballard 1983]
- Monte-Carlo based [Lancoto et al 2013]



# Example

- Black to flip a1.
  - If a1 is black cannon, then black can win.
  - If a1 is black king, then it is difficult for black to win.



# Basic ideas for searching chance nodes

- Assume a chance node  $x$  has a score probability distribution function  $Pr(*)$  with the range of possible outcomes from 1 to  $N$  where  $N$  is a positive integer.
  - For each possible outcome  $i$ , there is a  $score(i)$  to be computed.
  - The expected value  $E = \sum_{i=1}^N score(i) * Pr(x = i)$ .
  - The minimum value is  $m = \min_{i=1}^N \{score(i) \mid Pr(x = i) > 0\}$ .
  - The maximum value is  $M = \max_{i=1}^N \{score(i) \mid Pr(x = i) > 0\}$ .
- Example: in Chinese dark chess.
  - For the first ply,  $N = 14 * 32$ .
    - ▷ *Using symmetry, we can reduce it to 7\*8.*
  - We now consider the chance node of flipping the piece at the cell a1.
    - ▷  $N = 14$ .
    - ▷ *Assume  $x = 1$  means a black King is revealed and  $x = 8$  means a red King is revealed.*
    - ▷ *Then  $score(1) = score(8)$ .*
    - ▷  *$Pr(x = 1) = Pr(x = 8) = 1/14$ .*

# Bounds in a chance node

- Assume the various possibilities of a chance node is evaluated one by one in the order that at the end of phase  $i$ ,  $i = N$  is evaluated.
  - Assume  $v_{min} \leq score(i) \leq v_{max}$ .
- How do the lower and upper bounds, namely  $m_i$  and  $M_i$ , of the chance node change at the end of phase  $i$ ?
  - $i = 0$ .
    - ▷  $m_0 = v_{min}$
    - ▷  $M_0 = v_{max}$
  - $i = 1$ , we first compute  $score(1)$ , and then know
    - ▷  $m_1 \geq score(1) * Pr(x = 1) + v_{min} * (1 - Pr(x = 1))$ , and
    - ▷  $M_1 \leq score(1) * Pr(x = 1) + v_{max} * (1 - Pr(x = 1))$ .
  - ...
  - $i = i^*$ , we have computed  $score(1), \dots, score(i^*)$ , and then know
    - ▷  $m_{i^*} \geq \sum_{i=1}^{i^*} score(i) * Pr(x = i) + v_{min} * (1 - \sum_{i=1}^{i^*} Pr(x = i))$ , and
    - ▷  $M_{i^*} \leq \sum_{i=1}^{i^*} score(i) * Pr(x = i) + v_{max} * (1 - \sum_{i=1}^{i^*} Pr(x = i))$ .

# Algorithm: Chance\_Search

- **Algorithm  $F4.8'$ (position  $p$ , value  $alpha$ , value  $beta$ , integer  $depth$ )**
  - **determine the successor positions  $p_1, \dots, p_b$**
  - **...**
  - **for  $h = 1$  to  $b$  do**
  - **if  $p_h$  is not a chance node, then search normally**
  - **else we searching a chance node  $p_h$  with  $N$  choices such that with a probability  $Pr_i$  it will be  $k_i$**
  - $m_0 = alpha;$
  - $M_0 = beta;$
  - **for each possible choice  $k_i$  from 1 to  $N$  do**
    - ▷  $t := G4.8'(k_i, m_{i-1}, M_{i-1}, depth - 1);$
    - ▷  $m_i = m_{i-1} + (t - alpha) * Pr_i;$
    - ▷  $M_i = M_{i-1} + (t - beta) * Pr_i;$
  - **...**

# Example: Chinese dark chess

## ■ Assumption:

- The range of the scores of Chinese dark chess is  $[-10,10]$  inclusive.
- $N = 7$ .
- $Pr(x = i) = 1/N = 1/7$ .

## ■ Calculation:

- $i = 0$ ,
  - ▷  $m_0 = -10$ .
  - ▷  $M_0 = 10$ .
- $i = 1$  and **if**  $score(1) = -2$ , then
  - ▷  $m_1 = -2 * 1/7 + -10 * 6/7 = -62/7 \simeq -8.86$ .
  - ▷  $M_1 = -2 * 1/7 + 10 * 6/7 = 58/7 \simeq 8.26$ .
- $i = 1$  and **if**  $score(1) = 3$ , then
  - ▷  $m_1 = 3 * 1/7 + -10 * 6/7 = -57/7 \simeq -8.14$ .
  - ▷  $M_1 = 3 * 1/7 + 10 * 6/7 = 63/7 = 9$ .

# How to use these bounds

- The lower and upper bounds of the expected score can be used to do alpha-beta pruning.
  - Nicely fit into the alpha-beta search algorithm.
- Can do better by not searching the DFS order.
  - It is not necessary to search completely the subtree of  $x = 1$  first, and then start to look at the subtree of  $x = 2$ .
  - Assume it is a MAX chance node, e.g., the opponent takes a flip.
    - ▷ *Knowing some value  $v'_1$  of a subtree for  $x = 1$  gives an upper bound, i.e.,  $score(1) \geq v'_1$ .*
    - ▷ *Knowing some value  $v'_2$  of a subtree for  $x = 2$  gives another upper bound, i.e.,  $score(2) \geq v'_2$ .*
    - ▷ *These bounds can be used to make the search window further narrower.*
- For Monte-Carlo based algorithm, we need to use a sparse sampling algorithm to efficiently estimate the expected value of a chance node [Kearn et al 2002].

# Testing

- You have two versions  $P_1$  and  $P_2$ .
- You make the 2 programs play against each other using the same resource constraints.
- To make it fair, during a round of testing, the numbers of a program plays first and second are equal.
- After a few rounds of testing, how do you know  $P_1$  is better or worse than  $P_2$ ?

# How to know you are successful

- Assume during a **self-play** experiment, two copies of the same program are playing against each other.
  - Since two copies of the same program are playing against each other, the outcome of each game is an independent random trial and can be modeled as a trinomial random variable.
  - Assume for a copy playing first,

$$Pr(game_{first}) = \begin{cases} p & \text{if win} \\ q & \text{if draw} \\ 1 - p - q & \text{if lose} \end{cases}$$

- Hence for a copy playing second,

$$Pr(game_{last}) = \begin{cases} 1 - p - q & \text{if win} \\ q & \text{if draw} \\ p & \text{if lose} \end{cases}$$



# Outcome of self-play games

- Assume  $2n$  games,  $g_1, g_2, \dots, g_{2n}$  are played.
  - In order to offset the initiative, namely first player's advantage, each copy plays first for  $n$  games.
  - We also assume each copy alternatives in playing first.
  - Let  $g_{2i-1}$  and  $g_{2i}$  be the  $i$ th pair of games.
- Let the outcome of the  $i$ th pair of games be a random variable  $X_i$  from the prospective of the copy who plays  $g_{2i-1}$ .
  - Assume we assign a score of  $x$  for a game won, a score of  $0$  for a game drawn and a score of  $-x$  for a game lost.
- The outcome of  $X_i$  and its occurrence probability is thus

$$Pr(X_i) = \begin{cases} p(1 - p - q) & \text{if } X_i = 2x \\ pq + (1 - p - q)q & \text{if } X_i = x \\ p^2 + (1 - p - q)^2 + q^2 & \text{if } X_i = 0 \\ pq + (1 - p - q)q & \text{if } X_i = -x \\ (1 - p - q)p & \text{if } X_i = -2x \end{cases}$$

# How good we are against the baseline?

- **Properties of  $X_i$ .**
  - The mean  $E(X_i) = 0$ .
  - The standard deviation of  $X_i$  is

$$\sqrt{E(X_i^2)} = x\sqrt{2pq + (2q + 8p)(1 - p - q)},$$

and it is a multi-nominally distributed random variable.

- **When you have played  $n$  pairs of games, what is the probability of getting a score of  $s$ ,  $s > 0$ ?**
  - Let  $X[n] = \sum_{i=1}^n X_i$ .
    - ▷ *The mean of  $X[n]$ ,  $E(X[n])$ , is 0.*
    - ▷ *The standard deviation of  $X[n]$ ,  $\sigma_n$ , is  $x\sqrt{n}\sqrt{2pq + (2q + 8p)(1 - p - q)}$ ,*
  - If  $s > 0$ , we can calculate the probability of  $Pr(|X[n]| \leq s)$  using well known techniques from calculating multi-nominal distributions.

# Practical setup

## ■ Parameters that are usually used.

- $x = 1$ .
- **For Chinese chess,  $q$  is about 0.3161,  $p = 0.3918$  and  $1 - p - q$  is 0.2920.**
  - ▷ *Data source: 63,548 games played among masters recorded at [www.dpxq.com](http://www.dpxq.com).*
  - ▷ *This means the first player has a better chance of winning.*
- **The mean of  $X[n]$ ,  $E(X[n])$ , is 0.**
- **The standard deviation of  $X[n]$ ,  $\sigma_n$ , is**

$$x\sqrt{n}\sqrt{2pq + (2q + 8p)(1 - p - q)} = \sqrt{1.16n}.$$

# Results (1/3)

$Pr( X[n]  \leq s)$	$s = 0$	$s = 1$	$s = 2$	$s = 3$	$s = 4$	$s = 5$	$s = 6$
$n = 10, \sigma_{10} = 3.67$	0.108	0.315	0.502	0.658	0.779	0.866	0.924
$n = 20, \sigma_{20} = 5.19$	0.076	0.227	0.369	0.499	0.613	0.710	0.789
$n = 30, \sigma_{30} = 6.36$	0.063	0.186	0.305	0.417	0.520	0.612	0.693
$n = 40, \sigma_{40} = 7.34$	0.054	0.162	0.266	0.366	0.460	0.546	0.624
$n = 50, \sigma_{50} = 8.21$	0.049	0.145	0.239	0.330	0.416	0.497	0.571

# Results (2/3)

$Pr( X[n]  \leq s)$	$s = 7$	$s = 8$	$s = 9$	$s = 10$	$s = 11$	$s = 12$	$s = 13$
$n = 10, \sigma_{10} = 3.67$	0.960	0.981	0.991	0.997	0.999	1.000	1.000
$n = 20, \sigma_{20} = 5.19$	0.851	0.899	0.933	0.958	0.974	0.985	0.991
$n = 30, \sigma_{30} = 6.36$	0.761	0.819	0.865	0.902	0.930	0.951	0.967
$n = 40, \sigma_{40} = 7.34$	0.693	0.753	0.804	0.847	0.883	0.912	0.934
$n = 50, \sigma_{50} = 8.21$	0.639	0.699	0.753	0.799	0.839	0.872	0.900

# Results (3/3)

$Pr( X[n]  \leq s)$	$s = 14$	$s = 15$	$s = 16$	$s = 17$	$s = 18$	$s = 19$	$s = 20$
$n = 10, \sigma_{10} = 3.67$	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$n = 20, \sigma_{20} = 5.19$	0.995	0.997	0.999	0.999	1.000	1.000	1.000
$n = 30, \sigma_{30} = 6.36$	0.978	0.986	0.991	0.994	0.997	0.998	0.999
$n = 40, \sigma_{40} = 7.34$	0.952	0.966	0.976	0.983	0.989	0.992	0.995
$n = 50, \sigma_{50} = 8.21$	0.923	0.941	0.956	0.967	0.976	0.983	0.988

# Statistical behaviors

- Hence assume you have two programs that are playing against each other and have obtained a score of  $s + 1$ ,  $s > 0$ , after trying  $n$  pairs of games.
  - Assume  $Pr(|X[n]| \leq s)$  is say 0.95.
    - ▷ *Then this result is meaningful, that is a program is better than the other, because it only happens with a low probability of 0.05.*
  - Assume  $Pr(|X[n]| \leq s)$  is say 0.05.
    - ▷ *Then this result is not very meaningful, because it happens with a high probability of 0.95.*
- In general, it is a very rare case, e.g., less than 5% of chance that it will happen, that your score is more than  $2\sigma_n$ .
  - For our setting, if you perform  $n$  pairs of games, and your net score is more than  $2 * \sqrt{1.16} * \sqrt{n} \simeq 2.154\sqrt{n}$ , then it means something statistically.
- You can also decide your “definition” of “a rare case”.

# Concluding remarks

- **Consider your purpose of studying a game:**
  - It is good to solve a game completely.
    - ▷ *You can only solve a game once!*
  - It is better to acquire the knowledge about why the game wins, draws or loses.
    - ▷ *You can learn lots of knowledge.*
  - It is even better to discover knowledge in the game and then use it to make the world a better place.
    - ▷ *Fun!*
- **Try to use the techniques learned from this course in other areas!**



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