

Scout and NegaScout

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Abstract

- It looks like alpha-beta pruning is the best we can do for an **exact generic searching procedure**.
 - What else can be done generically?
 - Alpha-beta pruning follows basically the “intelligent” searching behaviors used by human when domain knowledge is not involved.
 - Can we find some other “intelligent” behaviors used by human during searching?
- Intuition: **MAX node**.
 - Suppose we know currently we have a way to gain at least 300 points at the first branch.
 - If there is an efficient way to know the second branch is at most gaining 300 points, then there is no need to search the second branch in detail.
 - ▷ *Alpha-beta cut algorithm is one way to make sure of this by returning an exact value.*
 - ▷ *Is there a way to search a tree by only returning a bound?*
 - ▷ *Is searching with a bound faster than searching exactly?*
- Similar intuition holds for a **MIN node**.

SCOUT procedure

- It may be possible to verify whether the value of a branch is greater than a value v or not in a way that is faster than knowing its exact value [Judea Pearl 1980].
- High level idea:
 - While searching a branch T_i of a MAX node, if we have already obtained a lower bound v_ℓ .
 - ▷ First TEST whether it is possible for T_i to return something greater than v_ℓ .
 - ▷ If FALSE, then there is no need to search T_i .
⇒ This is called **fails the test**.
 - ▷ If TRUE, then search T_i .
⇒ This is called **passes the test**.
 - While searching a branch T_j of a MIN node, if we have already obtained an upper bound v_u .
 - ▷ First TEST whether it is possible for T_j to return something smaller than v_u .
 - ▷ If FALSE, then there is no need to search T_j .
⇒ This is called **fails the test**.
 - ▷ If TRUE, then search T_j .
⇒ This is called **passes the test**.

How to TEST $> v$

procedure TEST $>$ (position p , value v)

// test whether the value of the branch at p is $> v$

- determine the successor positions p_1, \dots, p_b of p
- if $b = 0$, then *// terminal*
 - ▷ if $f(p) > v$ then *// f(): evaluating function*
 - ▷ return TRUE
 - ▷ else return FALSE
- if p is a MAX node, then
 - for $i := 1$ to b do
 - ▷ if TEST $>$ (p_i, v) is TRUE, then
return TRUE *// succeed if a branch is $> v$*
 - return FALSE *// fail only if all branches $\leq v$*
- if p is a MIN node, then
 - for $i := 1$ to b do
 - ▷ if TEST $>$ (p_i, v) is FALSE, then
return FALSE *// fail if a branch is $\leq v$*
 - return TRUE *// succeed only if all branches are $> v$*

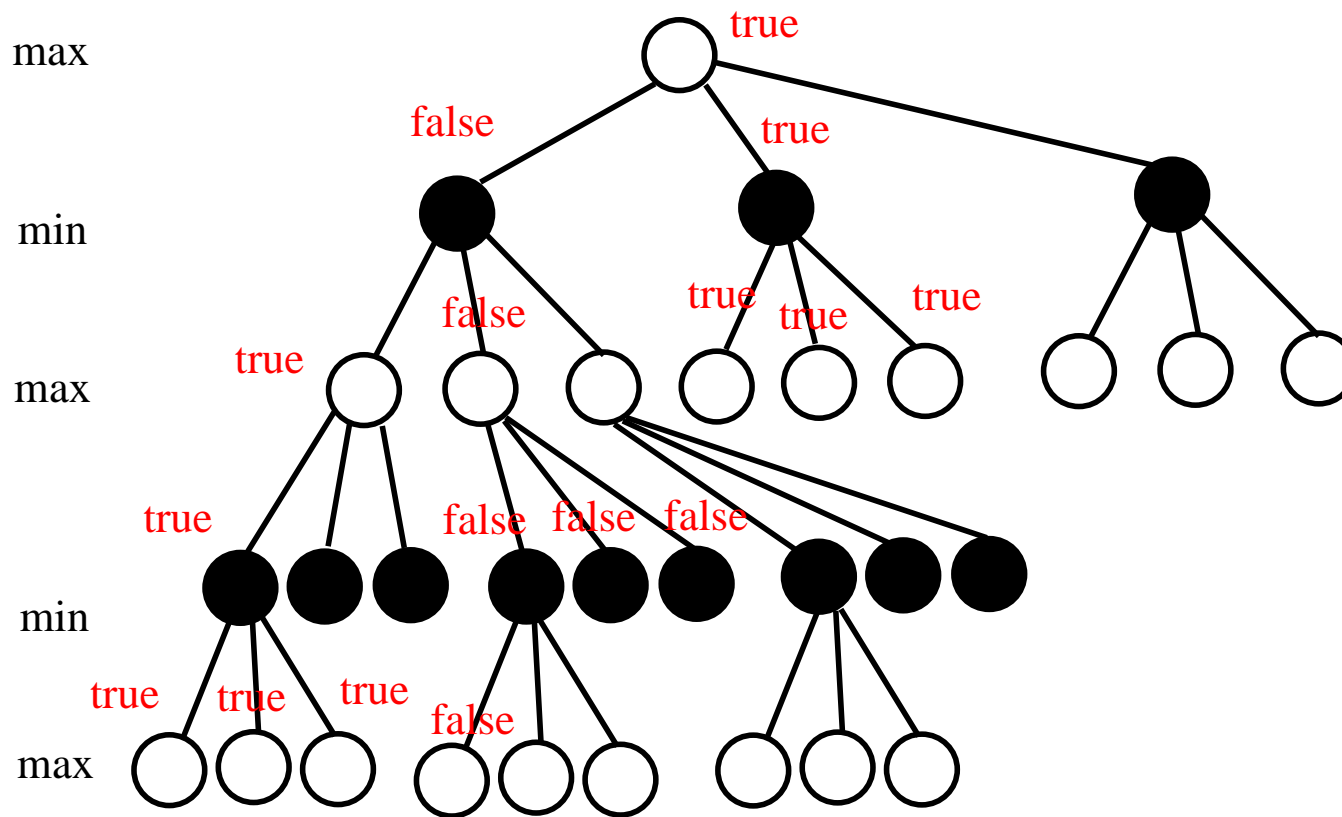
How to TEST $< v$

procedure TEST $<$ (position p , value v)

// test whether the value of the branch at p is $< v$

- determine the successor positions p_1, \dots, p_b of p
- if $b = 0$, then *// terminal*
 - ▷ if $f(p) < v$ then *// f(): evaluating function*
 - ▷ return TRUE
 - ▷ else return FALSE
- if p is a MAX node, then
 - for $i := 1$ to b do
 - ▷ if TEST $<$ (p_i, v) is FALSE, then
return FALSE *// fail if a branch is $\geq v$*
 - return TRUE *// succeed only if all branches $< v$*
- if p is a MIN node, then
 - for $i := 1$ to b do
 - ▷ if TEST $<$ (p_i, v) is TRUE, then
return TRUE *// succeed if a branch is $< v$*
 - return FALSE *// fail only if all branches are $\geq v$*

Illustration of TEST >



How to TEST — Discussions

- Sometimes it may be needed to test for “ $\geq v$ ”, or “ $\leq v$ ”.

- $\text{TEST}_{>}(p,v)$ is TRUE \equiv $\text{TEST}_{\leq}(p,v)$ is FALSE
- $\text{TEST}_{>}(p,v)$ is FALSE \equiv $\text{TEST}_{\leq}(p,v)$ is TRUE
- $\text{TEST}_{<}(p,v)$ is TRUE \equiv $\text{TEST}_{\geq}(p,v)$ is FALSE
- $\text{TEST}_{<}(p,v)$ is FALSE \equiv $\text{TEST}_{\geq}(p,v)$ is TRUE

- **Practical consideration:**

- Set a depth limit and evaluate the position's value when the limit is reached.

Main SCOUT procedure

Algorithm SCOUT(position p)

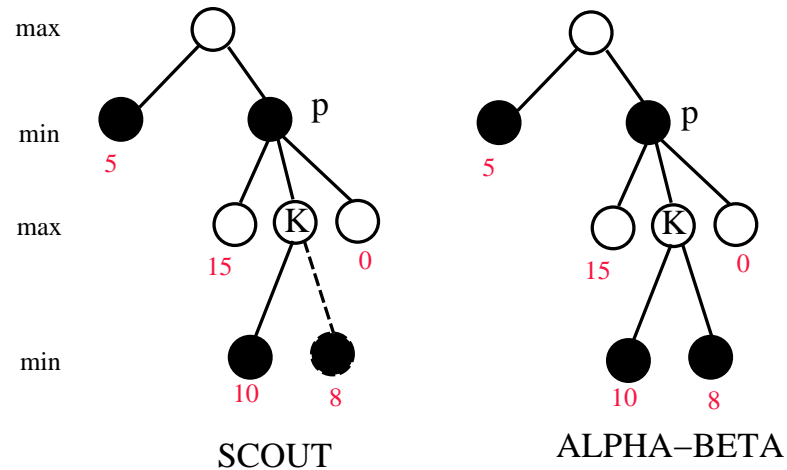
- determine the successor positions p_1, \dots, p_b
- if $b = 0$, then return $f(p)$
- else $v = SCOUT(p_1)$ // **SCOUT the first branch**
- if p is a **MAX** node
 - for $i := 2$ to b do
 - ▷ if $TEST_{>}(p_i, v)$ is **TRUE**, // **TEST first for the rest of the branches**
then $v = SCOUT(p_i)$ // **find the value of this branch if it can be $> v$**
- if p is a **MIN** node
 - for $i := 2$ to b do
 - ▷ if $TEST_{<}(p_i, v)$ is **TRUE**, // **TEST first for the rest of the branches**
then $v = SCOUT(p_i)$ // **find the value of this branch if it can be $< v$**
- return v

Discussions for SCOUT (1/3)

- Note that v is the current best value at any moment.
- MAX node:
 - For any $i > 1$, if $\text{TEST}_{>}(p_i, v)$ is TRUE,
 - ▷ then the value returned by $\text{SCOUT}(p_i)$ must be greater than v .
 - We say the p_i **passes the test** if $\text{TEST}_{>}(p_i, v)$ is TRUE.
- MIN node:
 - For any $i > 1$, if $\text{TEST}_{<}(p_i, v)$ is TRUE,
 - ▷ then the value returned by $\text{SCOUT}(p_i)$ must be smaller than v .
 - We say the p_i **passes the test** if $\text{TEST}_{<}(p_i, v)$ is TRUE.

Discussions for SCOUT (2/3)

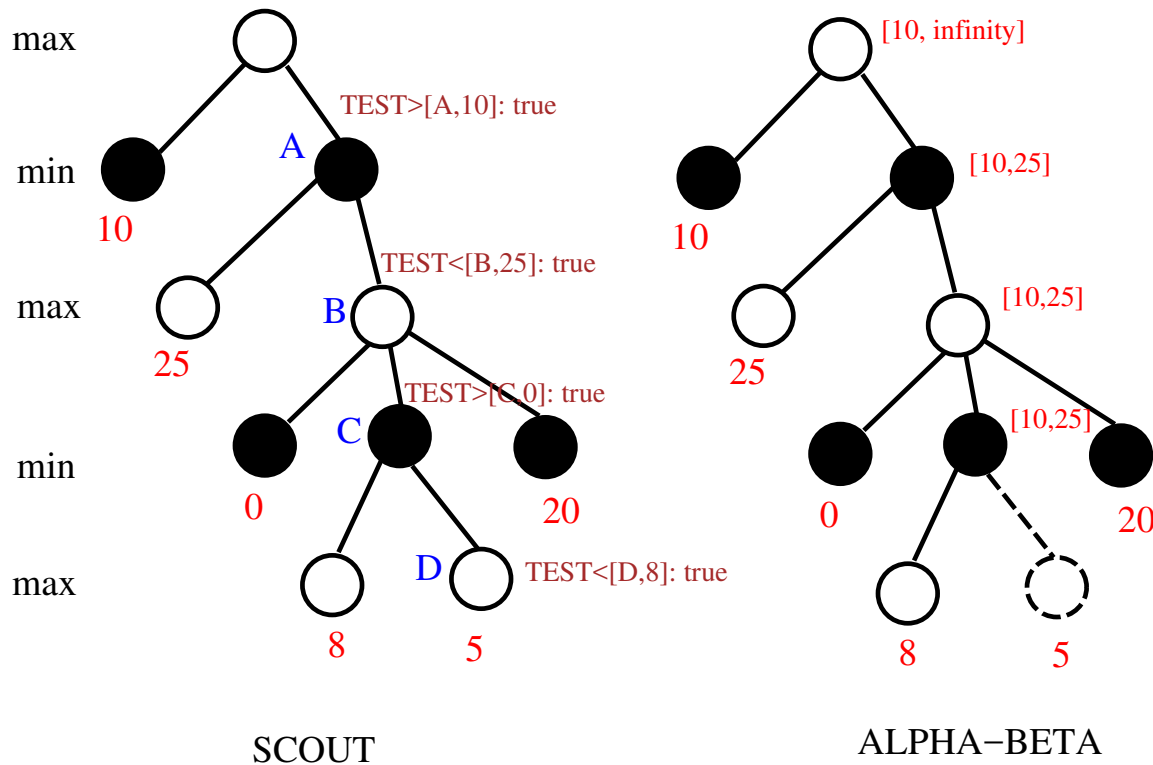
- TEST which is called by SCOUT may visit less nodes than that of alpha-beta.



- Assume $TEST_{>}(p,5)$ is called by the root after the first branch of the root is evaluated.
 - ▷ It calls $TEST_{>}(K,5)$ which skips K 's second branch.
 - ▷ $TEST_{>}(p,5)$ is FALSE, i.e., fails the test, after returning from the 3rd branch.
 - ▷ No need to do SCOUT for the branch rooted p .
- Alpha-beta needs to visit K 's second branch.

Discussions for SCOUT (3/3)

- SCOUT may pay many visits to a node that is cut off by alpha-beta.



Number of nodes visited (1/4)

- **For TEST to return TRUE for a subtree T , it needs to evaluate at least**
 - ▷ *one child for a MAX node in T , and*
 - ▷ *and all of the children for a MIN node in T .*
 - ▷ *If T has a fixed branching factor b and uniform depth b , the number of nodes evaluated is $\Omega(b^{\ell/2})$ where ℓ is the depth of the tree.*
- **For TEST to return FALSE for a subtree T , it needs to evaluate at least**
 - ▷ *one child for a MIN node in T , and*
 - ▷ *and all of the children for a MAX node in T .*
 - ▷ *If T has a fixed branching factor b and uniform depth b , the number of nodes evaluated is $\Omega(b^{\ell/2})$.*

Number of nodes visited (2/4)

■ Assumptions:

- Assume a full complete b -ary tree with depth ℓ where ℓ is even.
- The depth of the root, which is a MAX node, is 0.

■ The total number of nodes in the tree is $\frac{b^{\ell+1}-1}{b-1}$.

■ H_1 : the minimum number of nodes visited by TEST when it returns TRUE.

$$\begin{aligned} H_1 &= 1 + 1 + b + b + b^2 + b^2 + b^3 + b^3 + \dots + b^{\ell/2-1} + b^{\ell/2-1} + b^{\ell/2} \\ &= 2 \cdot (b^0 + b^1 + \dots + b^{\ell/2}) - b^{\ell/2} \\ &= 2 \cdot \frac{b^{\ell/2+1}-1}{b-1} - b^{\ell/2} \end{aligned}$$

Number of nodes visited (3/4)

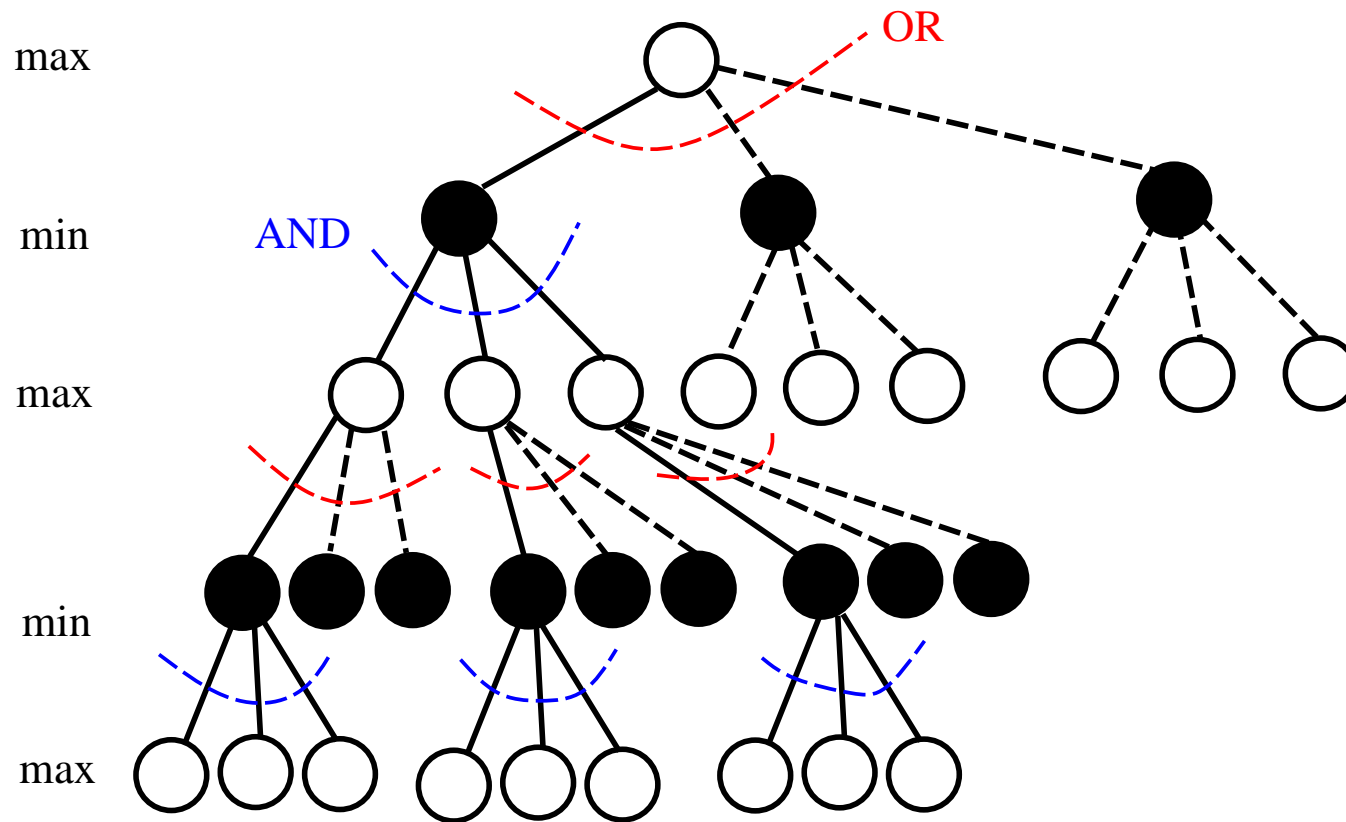
■ Assumptions:

- Assume a full complete b -ary tree with depth ℓ where ℓ is even.
- The depth of the root, which is a MAX node, is 0.

■ H_2 : the minimum number of nodes visited by alpha-beta.

$$\begin{aligned} H_2 &= \sum_{i=0}^{\ell} (b^{\lceil i/2 \rceil} + b^{\lfloor i/2 \rfloor} - 1) \\ &= \sum_{i=0}^{\ell} b^{\lceil i/2 \rceil} + \sum_{i=0}^{\ell} b^{\lfloor i/2 \rfloor} - (\ell + 1) \\ &= \sum_{i=0}^{\ell} b^{\lceil i/2 \rceil} + H_1 - (\ell + 1) \\ &= (1 + b + b + \dots + b^{\ell/2-1} + b^{\ell/2} + b^{\ell/2}) + H_1 - (\ell + 1) \\ &= (H_1 - 1 + b^{\ell/2} - b^{\ell/2-1}) + H_1 - (\ell + 1) \\ &= 2 \cdot H_1 + b^{\ell/2} - b^{\ell/2-1} - (\ell + 2) \\ &\sim (2.x) \cdot H_1 \end{aligned}$$

Number of nodes visited (4/4)



Comparisons

- When the first branch of a node has **the best** value, then TEST scans the tree fast.
 - The best value of the first $i - 1$ branches is used to test whether the i th branch needs to be searched exactly.
 - If the value of the first $i - 1$ branches of the root is better than the value of i th branch, then we do not have to evaluate exactly for the i th branch.
- Compared to alpha-beta pruning whose cut off comes from bounds of search windows.
 - It is possible to have some cut-off for alpha-beta as long as there are some relative move orderings are “good.”
 - ▷ *The moving orders of your children and the children of your ancestor who is odd level up decide a cut-off.*
 - The search bound is updated during the searching.
 - ▷ *Sometimes, a deep alpha-beta cut-off occurs because a bound found from your ancestor a distance away.*

Performance of SCOUT (1/2)

- **A node may be visited more than once.**
 - First visit is to TEST.
 - Second visit is to SCOUT.
 - ▷ *During SCOUT, it may be TESTed with a different value.*
 - Q: Can information obtained in the first search be used in the second search?
- **SCOUT is a recursive procedure.**
 - A node in a branch that is not the first child of a node with a depth of ℓ .
 - ▷ *Note that the depth of the root is defined to be 0.*
 - ▷ *Every ancestor of you may initiate a TEST to visit you.*
 - ▷ *It can be visited ℓ times by TEST.*

Performance of SCOUT (2/2)

- Show great improvements on $depth > 3$ for games with small branching factors.
 - It traverses most of the nodes without evaluating them precisely.
 - Few subtrees remained to be revisited to compute their exact mini-max values.
- Experimental data on the game of Kalah show [UCLA Tech Rep UCLA-ENG-80-17, Noe 1980]:
 - SCOUT favors “skinny” game trees, that are game trees with high depth-to-width ratios.
 - On depth = 5, it saves over 40% of time.
 - Maybe bad for games with a large branching factor.
 - **Move ordering is very important.**
 - ▷ *The first branch, if is good, offers a great chance of pruning further branches.*

Alpha-beta revisited

- In an alpha-beta search with a window $[alpha, beta]$:
 - **Failed-high** means it returns a value that is larger than or equal to its upper bound $beta$.
 - **Failed-low** means it returns a value that is smaller than or equal to its lower bound $alpha$.
- **Null or Zero window search:**
 - Using alpha-beta search with the window $[m, m + 1]$.
 - The result can be either failed-high or failed-low.
 - Failed-high means the return value is at least $m + 1$.
 - ▷ *Equivalent to $TEST_{>}(p, m)$ is TRUE.*
 - Failed-low means the return value is at most m .
 - ▷ *Equivalent to $TEST_{>}(p, m)$ is FALSE.*
- The above argument works for the original, fail hard and fail soft versions of the alpha-beta algorithm.

Behaviors of Null window search

- When $F3'(p, m, m + 1, \infty)$ returns $m + 1$:
 - for the MAX node p , returns immediately after the first child p_i , namely the smallest index i , the value $m + 1$.
 - for the MIN node p_i , every child $p_{i,j}$ returns $m + 1$
 - for each MAX node $p_{i,j}$, returns immediately after the first child $r_{i,j,k}$, namely the smallest index k , the value $m + 1$.
 - ...
- Exactly like the OR-AND tree shown in $TEST_{>}$ when the TEST is passed.
- We can observe similar behaviors when $F3'(p, m, m + 1, \infty)$ returns m as if the TEST is failed.

Alpha-Beta + Scout

■ Intuition:

- Try to incooperate SCOUT and alpha-beta together.
- The searching window of alpha-beta if properly set can be used as TEST in SCOUT.
- Using a searching window is better than using a single bound as in SCOUT.
- Can also apply alpha-beta cut if it applies.

■ Modifications to the SCOUT algorithm:

- Traverse the tree with two bounds as the alpha-beta procedure does.
 - ▷ *A searching window.*
 - ▷ *Use the current best bound to guide the value used in TEST.*
- Use a fail soft version to get a better result when the returned value is out of the window.

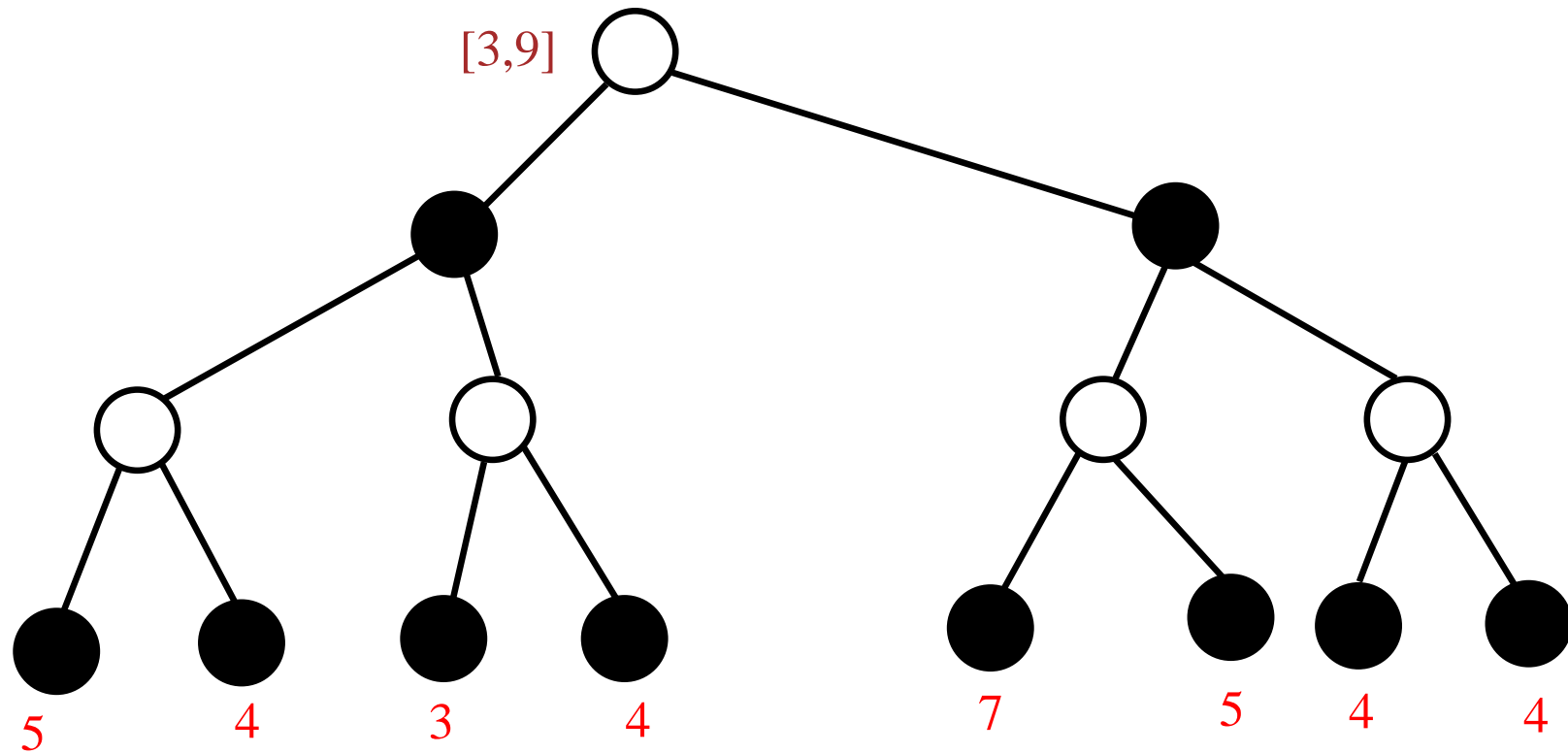
The NegaScout Algorithm – MiniMax (1/2)

- Algorithm $F4'$ (position p , value $alpha$, value $beta$, integer $depth$)
 - determine the successor positions p_1, \dots, p_b
 - if $b = 0$ // a terminal node
or $depth = 0$ // $depth$ is the remaining depth to search
or time is running up // from timing control
or some other constraints are met // apply heuristic here
 - then return $f(p)$ else
begin
 - ▷ $m := -\infty$ // m is the current best lower bound; fail soft
 $m := \max\{m, G4'(p_1, alpha, beta, depth - 1)\}$ // the first branch
if $m \geq beta$ then return(m) // beta cut off
 - ▷ for $i := 2$ to b do
 - ▷ 9: $t := G4'(p_i, m, m + 1, depth - 1)$ // null window search
 - ▷ 10: if $t > m$ then // failed-high
 - 11: if ($depth < 3$ or $t \geq beta$)
 - 12: then $m := t$
 - 13: else $m := G4'(p_i, t, beta, depth - 1)$ // re-search
 - ▷ 14: if $m \geq beta$ then return(m) // beta cut off
 - end
 - return m

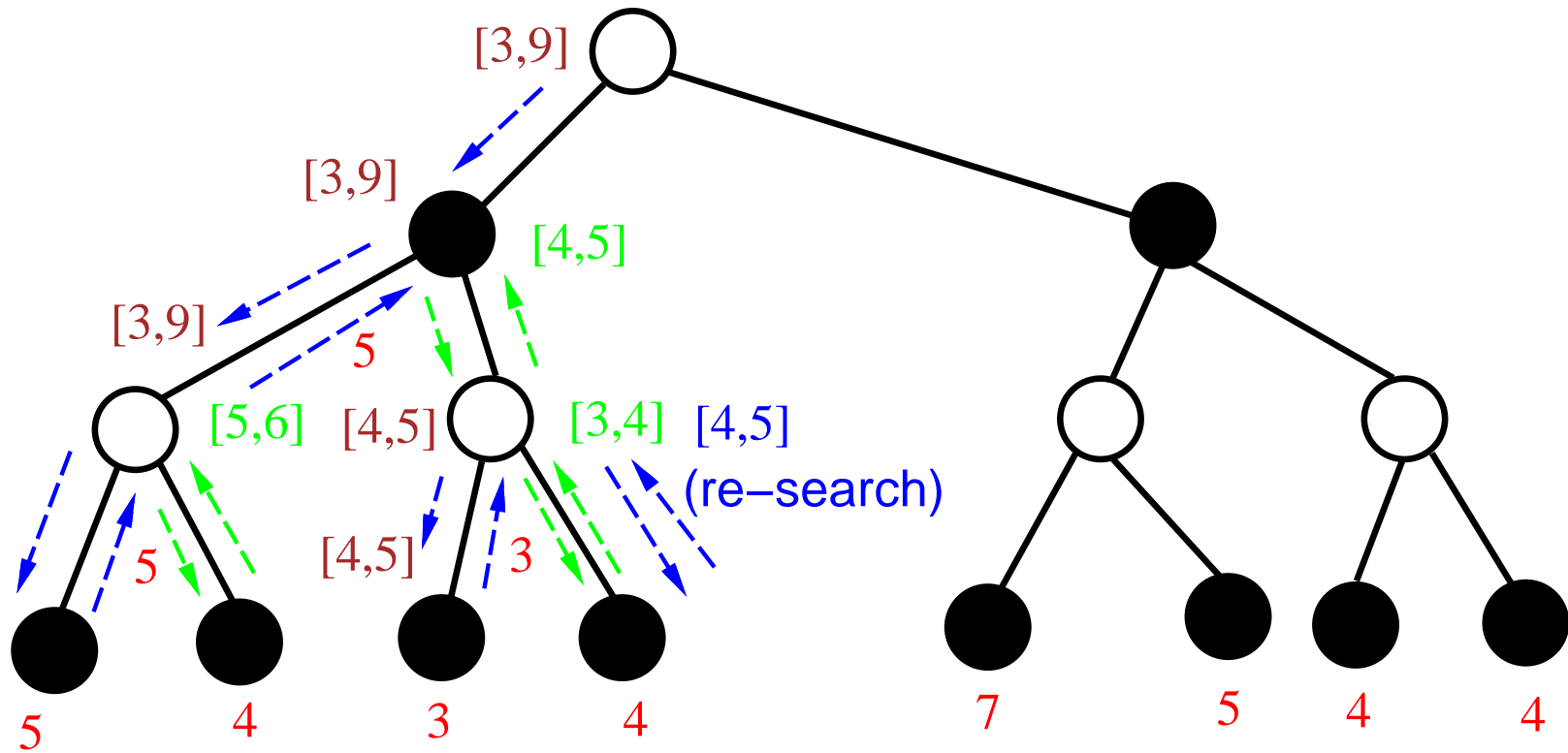
The NegaScout Algorithm – MiniMax (2/2)

- Algorithm $G4'$ (position p , value $alpha$, value $beta$, integer $depth$)
 - determine the successor positions p_1, \dots, p_b
 - if $b = 0$ // a terminal node
or $depth = 0$ // $depth$ is the remaining depth to search
or time is running up // from timing control
or some other constraints are met // apply heuristic here
 - then return $f(p)$ else
begin
 - ▷ $m = \infty$ // m is the current best upper bound; fail soft
 $m := \min\{m, F4'(p_1, alpha, beta, depth - 1)\}$ // the first branch
if $m \leq alpha$ then return(m) // alpha cut off
 - ▷ for $i := 2$ to b do
 - ▷ 9: $t := F4'(p_i, m - 1, m, depth - 1)$ // null window search
 - ▷ 10: if $t < m$ then // failed-low
 - 11: if ($depth < 3$ or $t \leq alpha$)
 - 12: then $m := t$
 - 13: else $m := F4'(p_i, alpha, t, depth - 1)$ // re-search
 - ▷ 14: if $m \leq alpha$ then return(m) // alpha cut off
 - end
 - return m

NegaScout – MiniMax version (1/2)



NegaScout – MiniMax version (2/2)



The NegaScout Algorithm

- Use Nega-MAX format.
- Algorithm $F4(\text{position } p, \text{value } \alpha, \text{value } \beta, \text{integer } \textit{depth})$
 - determine the successor positions p_1, \dots, p_b
 - if $b = 0$ // a terminal node
or $\textit{depth} = 0$ // \textit{depth} is the remaining depth to search
or time is running up // from timing control
or some other constraints are met // apply heuristic here
 - then return $h(p)$ else
 - ▷ $m := -\infty$ // the current lower bound; fail soft
 - ▷ $n := \beta$ // the current upper bound
 - ▷ for $i := 1$ to b do
 - ▷ 9: $t := -F4(p_i, -n, -\max\{\alpha, m\}, \textit{depth} - 1)$
 - ▷ 10: if $t > m$ then
 - ▷ 11: if ($n = \beta$ or $\textit{depth} < 3$ or $t \geq \beta$)
 - ▷ 12: then $m := t$
 - ▷ 13: else $m := -F4(p_i, -\beta, -t, \textit{depth} - 1)$ // re-search
 - ▷ 14: if $m \geq \beta$ then return(m) // cut off
 - ▷ 15: $n := \max\{\alpha, m\} + 1$ // set up a null window
 - return m

Search behaviors (1/3)

- If the depth is enough or it is a terminal position, then stop searching further.

- Return $h(p)$ as the value computed by an evaluation function.
- Note:

$$h(p) = \begin{cases} f(p) & \text{if depth of } p \text{ is 0 or even} \\ -f(p) & \text{if depth of } p \text{ is odd} \end{cases}$$

- Fail soft version.

- For the first child p_1 , search using the normal alpha beta window.

- line 9: normal window for the first child
- the initial value of m is $-\infty$, hence $-\max\{\alpha, m\} = -\alpha$
 - ▷ m is the current best value
- that is, searching with the normal window $[\alpha, \beta]$

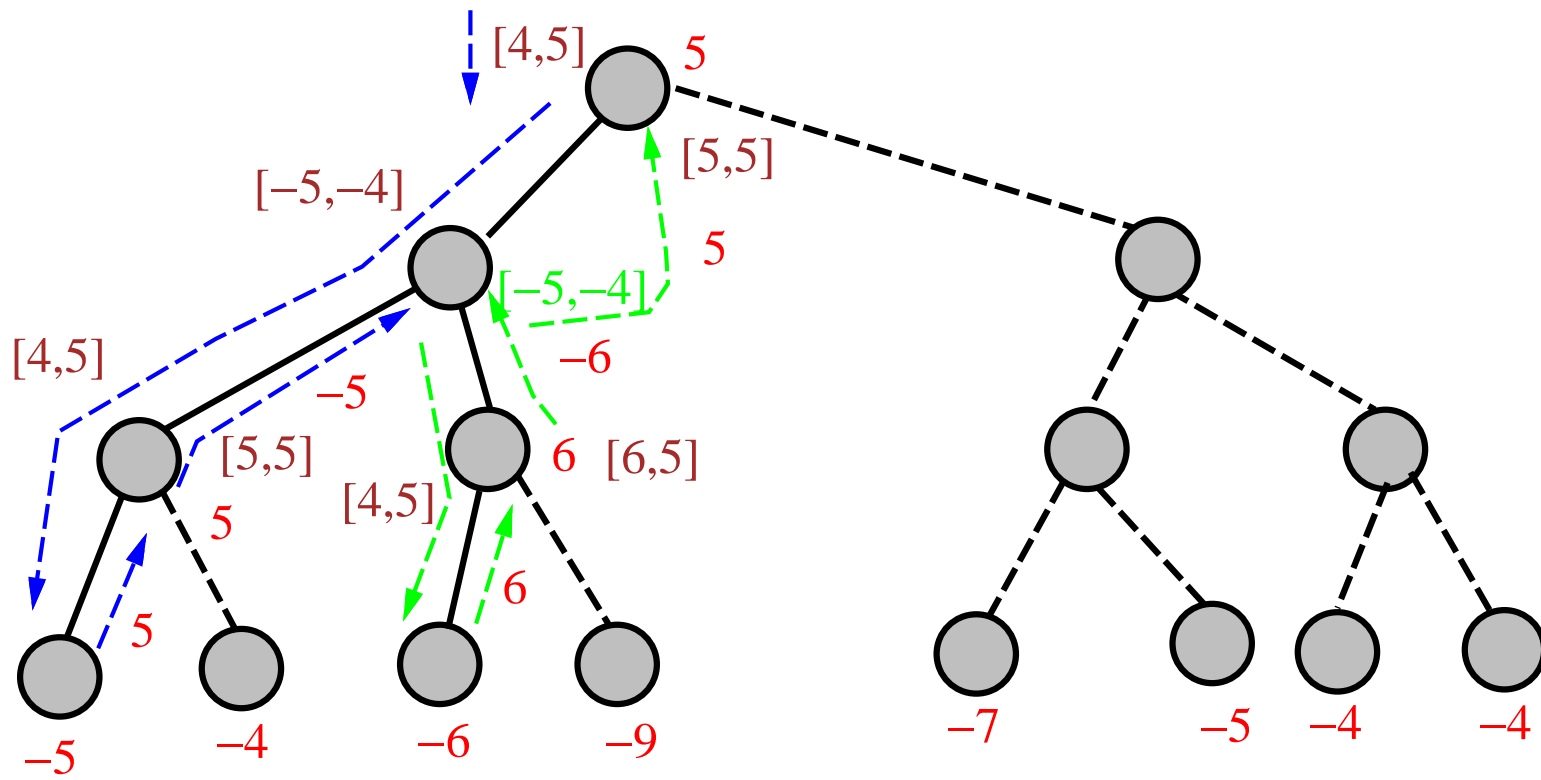
Search behaviors (2/3)

- For the second child and beyond p_i , $i > 1$, first perform a null window search for testing whether m is the answer.
 - line 9: a null-window of $[n - 1, n]$ searches for the second child and beyond where $n = \max\{\alpha, m\} + 1$.
 - ▷ m is best value obtained so far
 - ▷ α is the previous lower bound
 - ▷ m 's value will be first set at line 12 because $n = \beta$
 - ▷ The value of n is set at line 15.
 - line 11:
 - ▷ $n = \beta$: we are at the first iteration.
 - ▷ $\text{depth} < 3$: on a smaller depth subtree, i.e., depth at most 2, NegaScout always returns the best value.
 - ▷ $t \geq \beta$: we have obtained a good enough value from the failed-soft version to guarantee a beta cut.

Search behaviors (3/3)

- For the second child and beyond p_i , $i > 1$, first perform a null window search for testing whether m is the answer.
 - line 11: on a smaller depth subtree, i.e., depth at most 2, NegaScout always returns the best value.
 - ▷ *Normally, no need to do alpha-beta or any enhancement on very small subtrees.*
 - ▷ *The overhead is too large on small subtrees.*
 - line 13: **re-search** when the null window search fails high.
 - ▷ *The value of this subtree is at least t .*
 - ▷ *This means the best value in this subtree is more than m , the current best value.*
 - ▷ *This subtree must be re-searched with the the window $[t, beta]$.*
 - line 14: the normal pruning from alpha-beta.

Example for NegaScout



Refinements

- When a subtree is re-searched, it is best to use information on the previous search to speed up the current search.
 - Restart from the position that the value t is returned.
- Maybe want to re-search using the normal alpha-beta procedure.
- $F4$ runs much better with a good move ordering and some form of transposition table.
 - Order the moves in a priority list.
 - Reduce the number of re-searches.

Performances

- Experiments done on a uniform random game tree [Reinefeld 1983].
 - Normally superior to alpha-beta when searching game trees with branching factors from 20 to 60.
 - Shows about 10 to 20% of improvement.

Comments

- Incooperating both SCOUT and alpha-beta.
- Used in state-of-the-art game search engines.
- The first search, though maybe unsuccessful, can provide useful information in the second search.
 - Information can be stored and then reused.

References and further readings

- * J. Pearl. Asymptotic properties of minimax trees and game-searching procedures. *Artificial Intelligence*, 14(2):113–138, 1980.
- * A. Reinefeld. An improvement of the scout tree search algorithm. *ICCA Journal*, 6(4):4–14, 1983.