Parallel Game Tree Search

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Abstract

- Use multiprocessor shared-memory or distributed memory machines to search the game tree in parallel.
- Questions:
  - Is it possible to search multiple branches of the game tree at the same time while also gets benefits from the searching window introduced in alpha-beta search?
  - What can be done to parallelize Monte-Carlo based game tree search?
- Tradeoff between overheads and benefits.
  - Communication
  - Computation
  - Synchronization
- Techniques
  - For alpha-beta based search algorithms.
  - Lockless transposition table.
  - For Monte-Carlo based search algorithms.
- Can achieve reasonable speed-up using a moderate number of processors on a shared-memory multiprocessor machine.

Comments on parallelization

- **Parallelization can add more computation power, but synchronization introduces overhead and may be difficult to implement.**
- **Synchronization methods**
  - **Message passing, such as MPI**
  - **Shared memory cells**
    - *Avoid a record becoming inconsistent because one is reading the first item, but the last item is being written.*
    - *Memory locked before using.*
  - **It may be efficient to broadcast a message.**
- **Locking the whole transposition table is definitely too costly.**
  - **The ability to lock each record.**
  - **Lockless transposition table technique.**
- **A global transposition table v.s. distributed transposition tables.**

Speed-up (1/2)

- **Speed-up**: the amount of performance improvement gotten in comparison to the the amount of hardware you used.
    - Assume the amount of resources, e.g., time, consumed is T_n when you use n when you use n processors.
    - Speed-up $= \frac{T_1}{T_n}$ using n processors.
- Speed-up is a function of n and can be expressed as $sp(n)$.
    - **Scalability**: whether you can obtain "reasonable" performance gain when n gets larger.
- Choose the "resources" where comparisons are made.
    - The elapsed time.
    - The total number of nodes visited.
    - The scores.
    - $\cdots$
- Choose the game trees where experiments are performed.
    - Artificial constructed trees with a pre-specified average branching factor and depth.
    - Real game trees.

Speed-up (2/2)

- **Three different setups for experiments.**
  - **Use the a sequential algorithm P_{seq} for the baseline of comparison.**
  - **Use the the best sequential algorithm P_{best} for the baseline of comparison.**
  - **Use a 1-processor version of your parallel program $P_{1,par}$ as the baseline of comparison.**
    - ▷ ***It is usually the case that $P_{1,par}$ is much slower than P_{best}.***
    - ▷ ***It is often the case that $P_{1,par}$ is slower than P_{seq}.***
  - **Use an optimized sequential version of your parallel program $P_{1,opt}$ as the baseline of comparison.**
    - ▷ ***It is also usually the case that $P_{1,opt}$ is slower than P_{best}.***
- **Choose the game trees where experiments are performed.**
  - **Artificial constructed trees with a pre-specified average branching factor and depth.**
  - **Real game trees.**

Amdahl's law

- The best you can do about parallelization [G. Amdahl 1967].
- Assume a program needs to execute T instructions and and x of them can be parallelized.
  - Assume you have n processors and an instruction takes a unit of time.
  - Parallel processing time is

    $$\geq T - x + \frac{x}{n} + O_n \geq T - x.$$

    where O_n is the overhead cost in doing parallelization with n processors.
  - Speed-up is

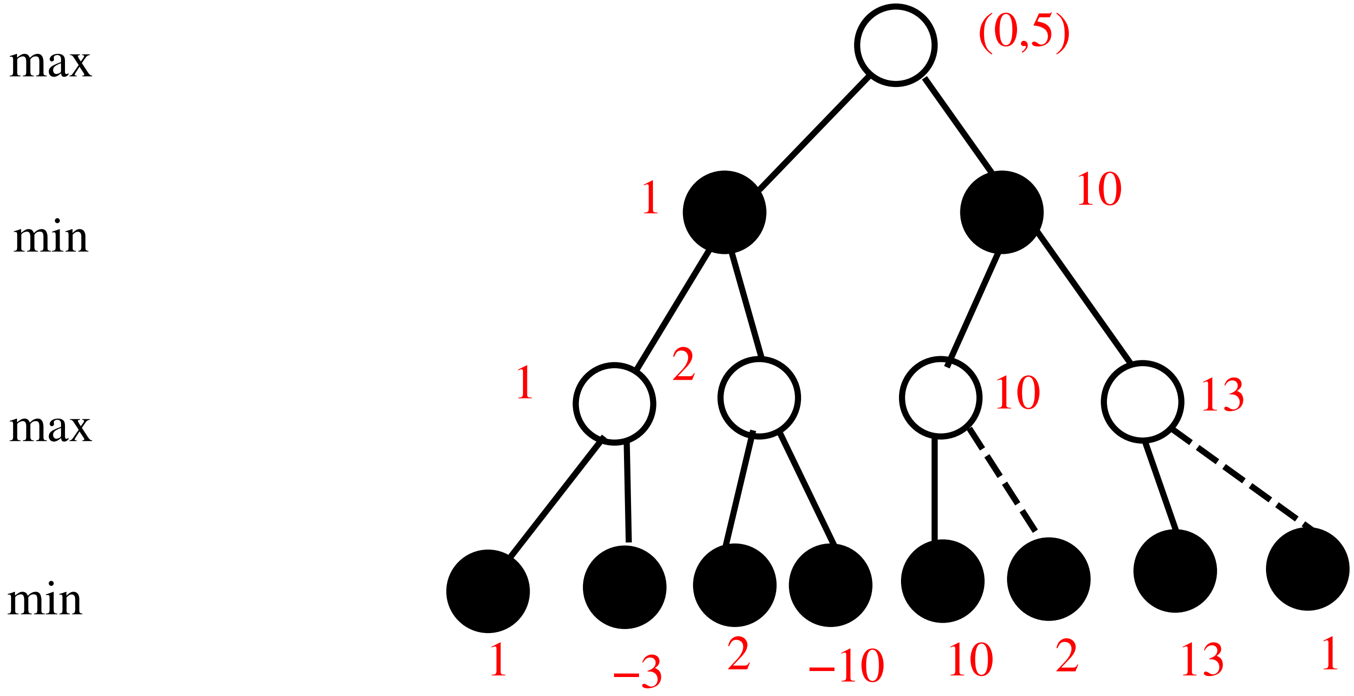
    $$\leq \frac{T}{T - x}.$$
- If 20% of the code cannot be parallelized, then your parallel program can be at most 5 times faster no matter how many processors you use.
- Depending on O_n, it may not be wise to use too many processors.

Other practical concerns

- **Load balancing**
  - The ratio between the amount of the largest work on a PE and the amount of the lightest work on another PE.
  - Good load balancing is a key to have a good speed-up factor.
- **Granuality**
  - A factor affecting how well can you match Amdahl's law.
  - How fine can you parallelize your code?
    - ▷ *instruction level*
    - ▷ *procedure level*
    - ▷ *task level*
- **Speed-up factor**: ratio between the parallel version with a given number of processors and the baseline version.
- Is it possible to achieve **super linear** speed-up?
  - Super linear speed-up means you can make the code to run N times faster using less than N times about of hardware.
    - ▷ *Yes, on badly ordered game trees.*
    - ▷ *Not in real game trees with a reasonable good algorithm.*

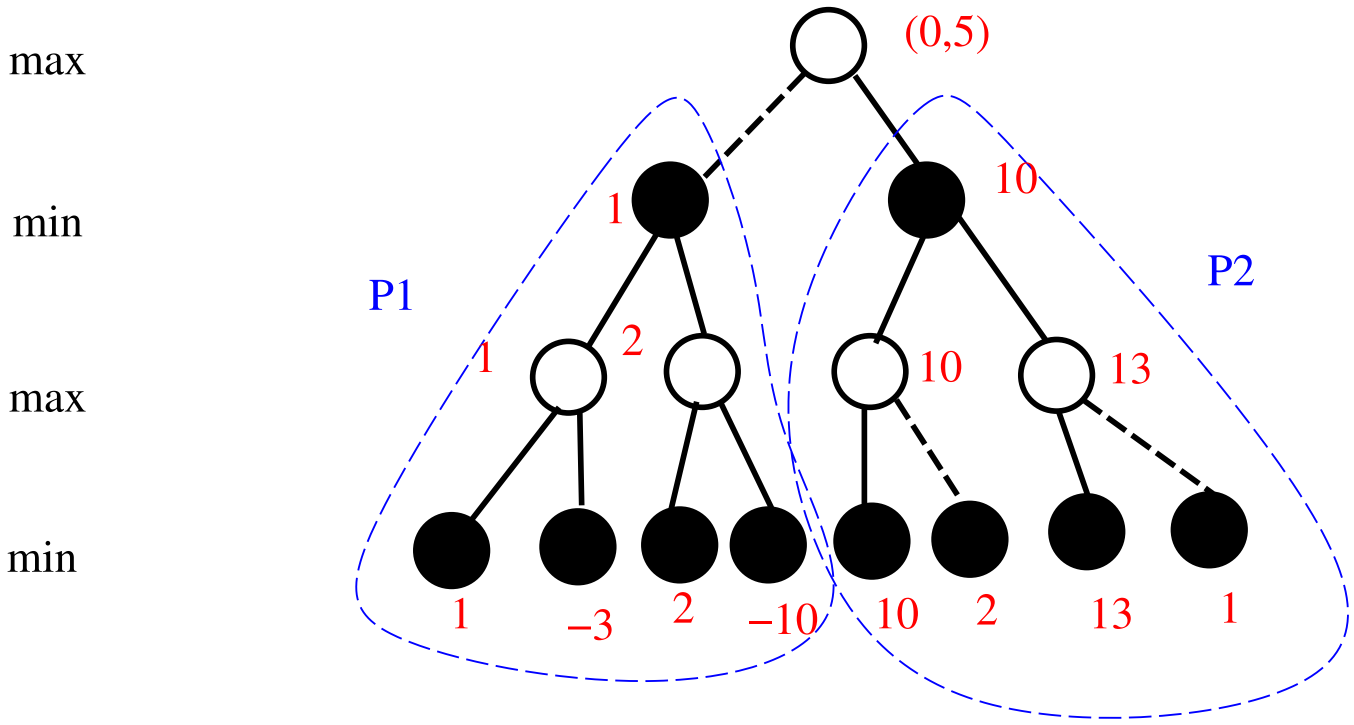
Super-linear speed-up (1/3)

- **Sequential alpha-beta search with a pre-assigned window $(0, 5)$:**
  - **Visited 13 nodes.**



Super-linear speed-up (2/3)

- **Parallel alpha-beta search with a pre-assigned window $(0, 5)$ on two processors:**
  - **P2: visited 5 nodes, and then the root performs a beta cut.**
  - **P1: being terminated by the root after 5 nodes are visited.**



Super-linear speed-up (3/3)

- **Total sequential time: visited 13 nodes.**
- **Total parallel time for 2 processors: visited 6 nodes.**
- **We have achieved a super-linear speed-up.**

Comments on super-linear speed-up (1/2)

- Parallelization can achieve super-linear speed-up only if the solution can be found without enumerating all possibilities.
  - For example: finding an entry of 1 in an array.
- If the solution needs to be found by exhaustively examining all possibilities, then there is no chance of getting a super-linear speed-up.
  - For example: counting the total number of 1's in an array.
- Overhead in parallelization comes from how much work should each processor "talks" to each other in order to decide the solution.
  - Trivially parallelizable: almost no need to talk to each other.

Comments on super-linear speed-up (2/2)

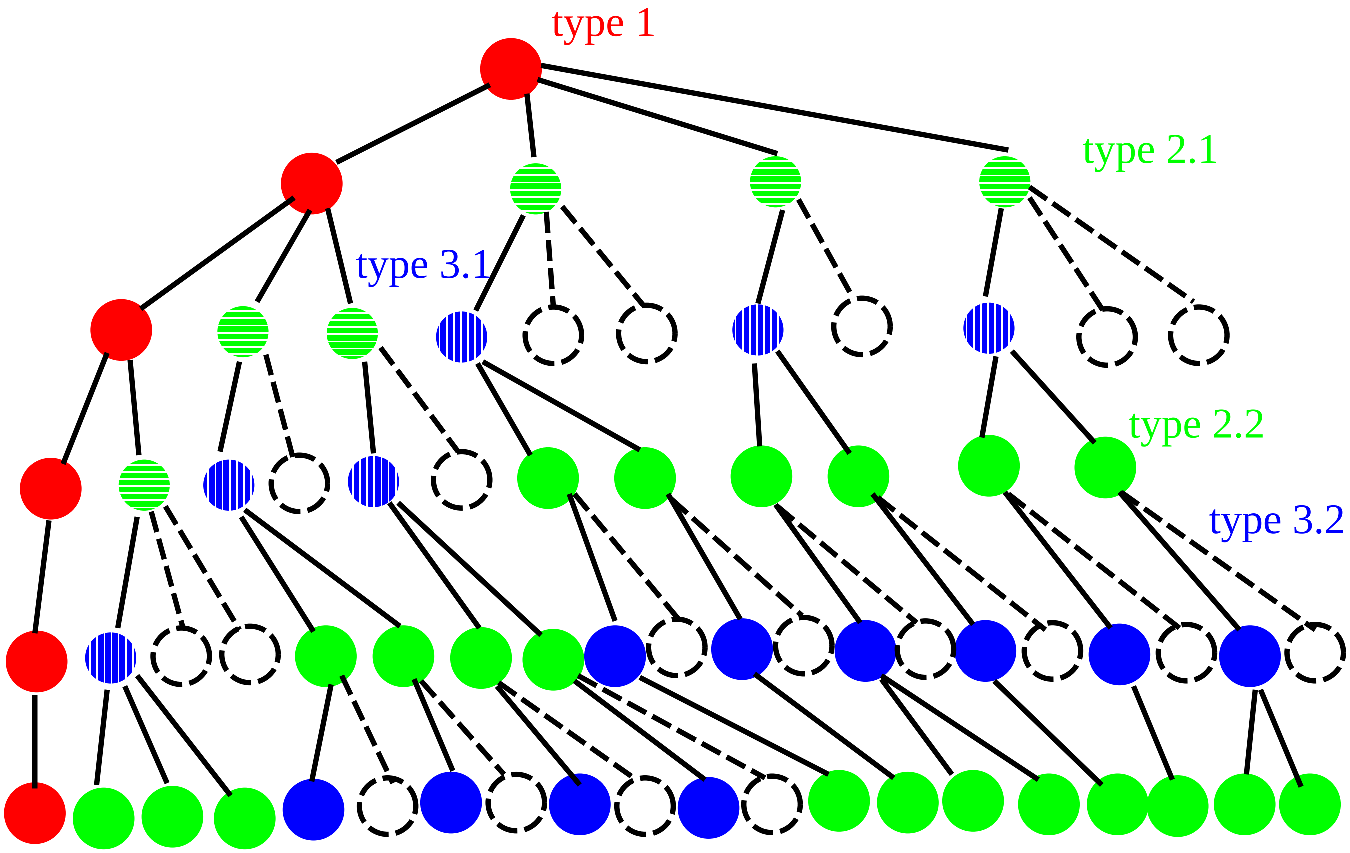
- Why is it possible to obtain a super-linear speed-up in searching a game tree using alpha-beta based algorithms?
  - Assume some cut-off happens during the execution.
  - Parallel algorithms offer a chance of getting a different “move ordering”.
  - It is possible to find a solution faster.
- It is also possible to get poor speed-up if the “move ordering” of the parallel version is bad.
  - You may perform unnecessary work, e.g., searching a branch that will be cut in the future.
- For Monte-Carlo based search algorithm, super-linear speed-up may be obtained by trying out different PV branches at the same time.
  - Increase the chance of finding the right branch.

Parallel α-β search

- **Three major approaches: depend on what tasks can be parallelized and the model of parallelism.**
    - Principle variation splitting (PV split)
        - ▷ *Central control or global synchronization model of parallelism.*
    - Young Brothers Wait Concept (YBWC)
        - ▷ *Client-server model of parallelism.*
    - Dynamic Tree Splitting (DTS)
        - ▷ *Peer-to-peer model of parallelism.*

Classification of nodes (1/2)

- **Classify nodes in a game tree according to [Knuth & Moore 1975].**

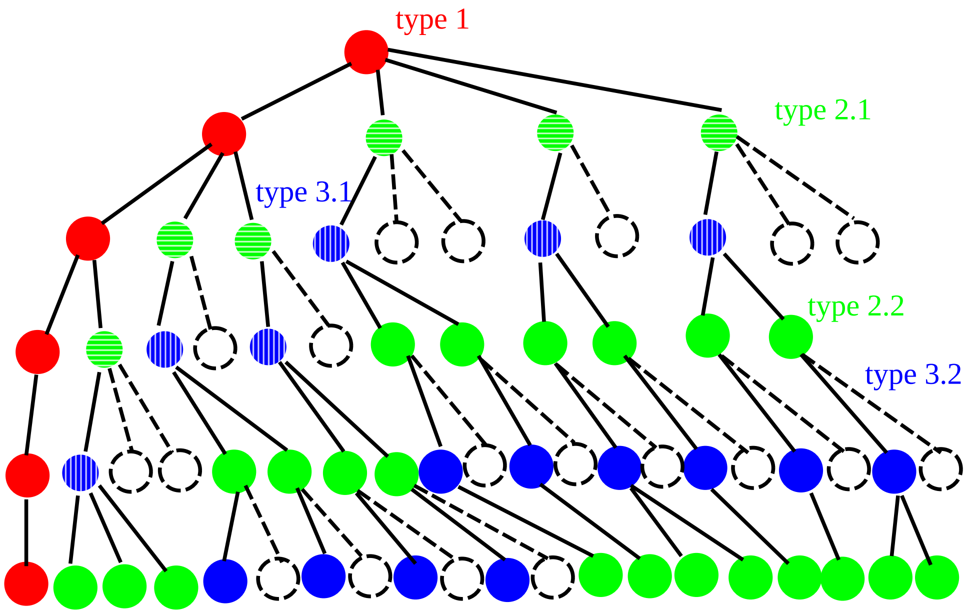


Classification of nodes (2/2)

- **Type 1 (PV): principle variation.**
  - *Nodes in the leftmost branch.*
  - *PV nodes needs to be searched first to established a good search bound.*
  - *After the first child is searched, the rest of its children can be searched in parallel.*
- **Type 2 (CUT): cut nodes.**
  - *Children of type-1 and type-3 nodes.*
  - *Because children of a cut node may be cut, it is not wise to perform searches in parallel for children of a cut node.*
- **Type 3 (ALL): all nodes.**
  - *The first branch of a cut node.*
  - *All children of an all node need to be explored.*
  - *It is better to search these children in parallel.*

Principle variation splitting

- **Algorithm PVS(game tree G)**
  - **Execute the first branch to get a PV branch $n_1, n_2, n_3, \ldots, n_d$ where n_d is a leaf node.**
  - **for $i = d - 1$ down to 1 do**
    - ▷ ***Update the bound information using information backed-up from n_{i+1}***
    - ▷ ***for each non-PV branch of n_i do in parallel***
    - ▷ ***A processor gets a branch and searches***
    - ▷ ***Update the bounds when a branch is done***



Comments for PV splitting

- **Comments:**
  - **Parallelism is done on type-2 branches of a type-1 node.**
  - **May not be able to use a large number of processors efficiently.**
  - **Load balancing is not good.**
    - ▷ *The ratio between the amount of the largest work on a PE and the amount of the lightest work on another PE.*
  - **Synchronization overhead is large.**
  - **When the first branch is usually not the best branch, then the overhead is huge.**
  - **Achieve a speed-up of 4.1 for 8 processors and 4.6 for 16 processors [Manohararjah '01].**
    - ▷ *Poor scalability.*
    - ▷ *Limited speed-up: within 5.*
  - **Improvements:**
    - ▷ *When a processor is idle, it helps out a busy processor by sharing its tasks.*
    - ▷ *Observe some improvements, but not much.*

Young brothers wait concept (1/2)

- **Concept: at each node, when the first branch is explored and a bound is obtained, then all the other branches can be executed in parallel.**
  - **Split point: a node whose value of the first branch is known.**
  - **Highest split point of a tree: a split point whose depth is the least.**
  - **A processor is assigned and owns a subtree rooted at a node.**
    - ▷ *This processor is the server of this subtree.*
  - **An idle processor asks a server for a subtree to search.**
    - ▷ *This processor is a client of this server.*

Young brothers wait concept (2/2)

- **Algorithm $YBWC$(game tree G)**
  - **Let P_1 own the root of the game tree G and begin to search using alpha-beta pruning until the tree is completely searched.**
    - ▷ ***During searching, maintain the split point information.***
  - **While the game tree is not searched completely, do**
    In parallel for each processor P_i do
    - ▷ ***If P_i is idle, it looks for server processors with split points.***
    - ▷ ***P_i gets a branch from a highest split point and owns this subtree.***
    - ▷ ***P_i begins to search using alpha-beta pruning and maintain the split point information.***
    - ▷ ***When a subtree owned by P_i has been searched, returns the information to the server processor where it gets the job from.***
    - ▷ ***P_i is idle again.***

Comments for YBWC

- **Comments:**
  - **Can utilize many processors.**
  - **Parallelism is done on almost all nodes.**
  - **It is possible to use non-shared-memory architectures.**
    - ▷ ***For example: distributed memory machines.***
    - ▷ ***Speed-up: 137 using 256 processors [Manohararjah '01].***
    - ▷ ***Scalability is moderate.***
    - ▷ ***Load balancing is not always good.***
  - **The cost of splitting a node needs to be calculated to avoid splitting small trees.**

Dynamic tree splitting (DTS)

- **Concepts:**
  - **Peer-to-peer approach so that no one owns any subtree.**
  - **The processor who finished last on a split point reports the value to the parent of the split point.**
  - **More criteria for the selection of split points.**

DTS: Classification of nodes

- **D-PV: a node that has the same alpha and beta values with the root.**
- **D-CUT**
  - **On a MAX node with the same alpha value with the root,**
    - ▷ *if some branches are searched, then the returned values from the branches may update the lower bound.*
    - ▷ *If the lower bound is raised (updated), then it is possible to visit less nodes.*
    - ▷ *Hence it may not be cost effective to parallelize.*
    - ▷ *Note: It takes time to initialize a new job.*
  - **On a MIN node with the same beta value with the root,**
    - ▷ *if some branches are searched, then the returned values from the branches may update the upper bound.*
    - ▷ *If the upper bound is lowered (updated), then it is possible to visit less nodes.*
    - ▷ *Hence it may not be cost effective to parallelize.*
    - ▷ *Note: It takes time to initialize a new job.*
- **D-ALL: any node that is neither D-PV nor D-CUT.**
  - ▷ *Nothing much is known here.*

Split point: confidence

- A confidence factor is associated with each D-CUT and D-ALL node.
  - That is, the chance of a node being the specified type.
- If many moves (up to a limit, say 3) have been searched at a D-CUT node, then the confidence that it is a D-CUT node decreases.
- If several moves have been searched at a D-ALL node, then the confidence that it is a D-ALL node increases.

DTS: Split point

- **Criteria for a split point:**
  - **The node must be of type D-PV, D-ALL with a high confidence or or D-CUT with a low confidence.**
  - **If it is a D-PV node, its first branch must have been searched.**
  - **Set thresholds for confidence factors.**
    - ▷ ***A D-ALL node with a high confidence factor remains to be a candidate for a split point.***
    - ▷ ***Can also fork a D-ALL node with the highest confidence factor first.***
    - ▷ ***A D-CUT node with a low confidence factor may be a split point.***
- **Note:**
  - ▷ ***Nodes that are higher up in the tree (closer to the root) represent more work.***
  - ▷ ***You want to fork a branch that are higher up and with a larger confidence factor for D-ALL, or with a smaller confidence factor for D-CUT.***
  - ▷ ***Use the above information to compute a global priority.***

DTS: Algorithm

- Algorithm DTS(game tree G)
  - Initialize a global job list with the root as the only available job.
  - while the job list is not empty do
    - ▷ *Idle processors look for jobs with the highest priority in the global job list.*
    - ▷ *A working processor maintains its own split point information at the global job list.*
    - ▷ *A working processor updates bounds when a job is finished and then becomes idle.*
- Comments:
  - Used by several state-of-the-art chess programs.
  - Spend a bit more time to decide whether a node is a split point or not.

    - ▷ *Takes some time to tune for the best parameters.*
  - Speed-up factor is very good: 3.7 for 4 processors, 6.6 for 8 processors and 11.1 for 16 processors [Manohararjah '01].
  - Load balancing is good.
  - Scalability is reasonable.

Comments: parallel α-β search

- DTS is currently being used by most Chess-like programs.
- It also takes time to tune the system parameters for DTS to work well.
    - The threshold for confidence factors.
    - Dynamically adjusting of the confidence factors.

Memory issues (1/2)

- **During searching, each process needs to maintain the following information.**
    - **Local data: such as the current depth, current best move.**
    - **Data that can be used later: such as the hash information.**
- **Distributed memory model.**
    - **Maintain each own data in a private memory area.**
    - **Exchange information when needed.**
        - ▷ ***Using message passing to probe a hash entry.***
        - ▷ ***Using message passing to return the value of a probe.***
- **Shared memory model.**
    - **Maintain each local data in a private memory area.**
    - **Maintain the re-used information in a global area.**
        - ▷ ***Concurrent read is often allowed in the model.***
        - ▷ ***Concurrent write is usually not allowed in the model.***
        - ▷ ***Lock the cell when it needs to be written.***

Memory issues (2/2)

- **Advantage and disadvantage**
  - **Distributed memory model.**
    - *Coding is easy.*
    - *Slow response time.*
  - **Shared memory model.**
    - *Overhead in locking.*
    - *Fast response time when there is no extensive memory contention.*
- **Often used techniques: Lockless transposition tables.**
  - **Allow concurrent read, but not concurrent write.**
  - **Assume writing of an entry is atomic.**
    - *Records in data structures used consist of multiple entries.*

Lockless transposition table

- **Scenario**
  - **Assume each entry of the transposition table H contains two parts where reading/writing each part is atomic.**
    - ▷ *$Position\_hash\_key$: $H_1[]$ array, say each entry is 64-bit which is called the hash key value.*
    - ▷ *$Data$: $H_2[]$ array, say each entry is 64-bit long.*
  - **Assume the hash table index $hash\_Tindex$ is the rightmost h, say $h = 32$, bits of $Position\_hash\_key$.**
- **To read or write an hash entry given a position P, you do the followings.**
  - **Compute $Position\_hash\_key(P)$ and $Data(P)$.**
  - **Let $hash\_Tindex(P)$ be the rightmost h bits of $Position\_hash\_key(P)$.**
  - **Read or write $H_1[hash\_Tindex(P)]$.**
  - **Read or write $H_2[hash\_Tindex(P)]$.**
- **Problem: The hash entry is corrupted if**
  - **P is being visited at the same time by two processes C_1 and C_2 so that**
    - ▷ *C_1 writes $H_1[hash\_Tindex(P)]$.*
    - ▷ *C_2 writes $H_2[hash\_Tindex(P)]$.*

Solution

- **Algorithm for writing an entry**
  - **Compute** $Position\_hash\_key(P)$ **and** $Data(P)$.
  - **Let** $hash\_Tindex(P)$ **be the rightmost** h **bits of** $Position\_hash\_key(P)$.
  - **write:** $H_1[hash\_Tindex(P)] \leftarrow Position\_hash\_key(P) \oplus Data(P)$.
  - **write:** $H_2[hash\_Tindex(P)] \leftarrow Data(P)$.
- **Algorithm for reading an entry**
  - **Compute** $Position\_hash\_key(P)$.
  - **Let** $hash\_Tindex(P)$ **be the rightmost** h **bits of** $Position\_hash\_key(P)$.
  - **read:** $W_1 \leftarrow H_1[hash\_Tindex(P)]$
  - **read:** $W_2 \leftarrow H_2[hash\_Tindex(P)]$
  - **reconstruct:** $W_1 \leftarrow W_1 \oplus W_2$
  - **verify: check whether** $W_1 = Position\_hash\_key(P)$
    - ▷ ***if they equal, then use this entry.***
    - ▷ ***if they do not equal, then the entry is corrupted.***

Why this works

- $H_1[hash\_Tindex(P)] = Position\_hash\_key(P) \oplus Data(P)$.
- $H_2[hash\_Tindex(P)] = Data(P)$.
- $H_1[hash\_Tindex(P)] \oplus H_2[hash\_Tindex(P)] = Position\_hash\_key(P)$.
- **If $H_1[i]$ and $H_2[i]$ are written by two different processes with $Data(P_1)$ and $Data(P_2)$, then it will probably not produce the right position hash key value.**
- **Comments:**
  - **May have errors because of hash collisions.**
  - **It is not too difficult to extend this method to an hash table with more than 2 entries.**

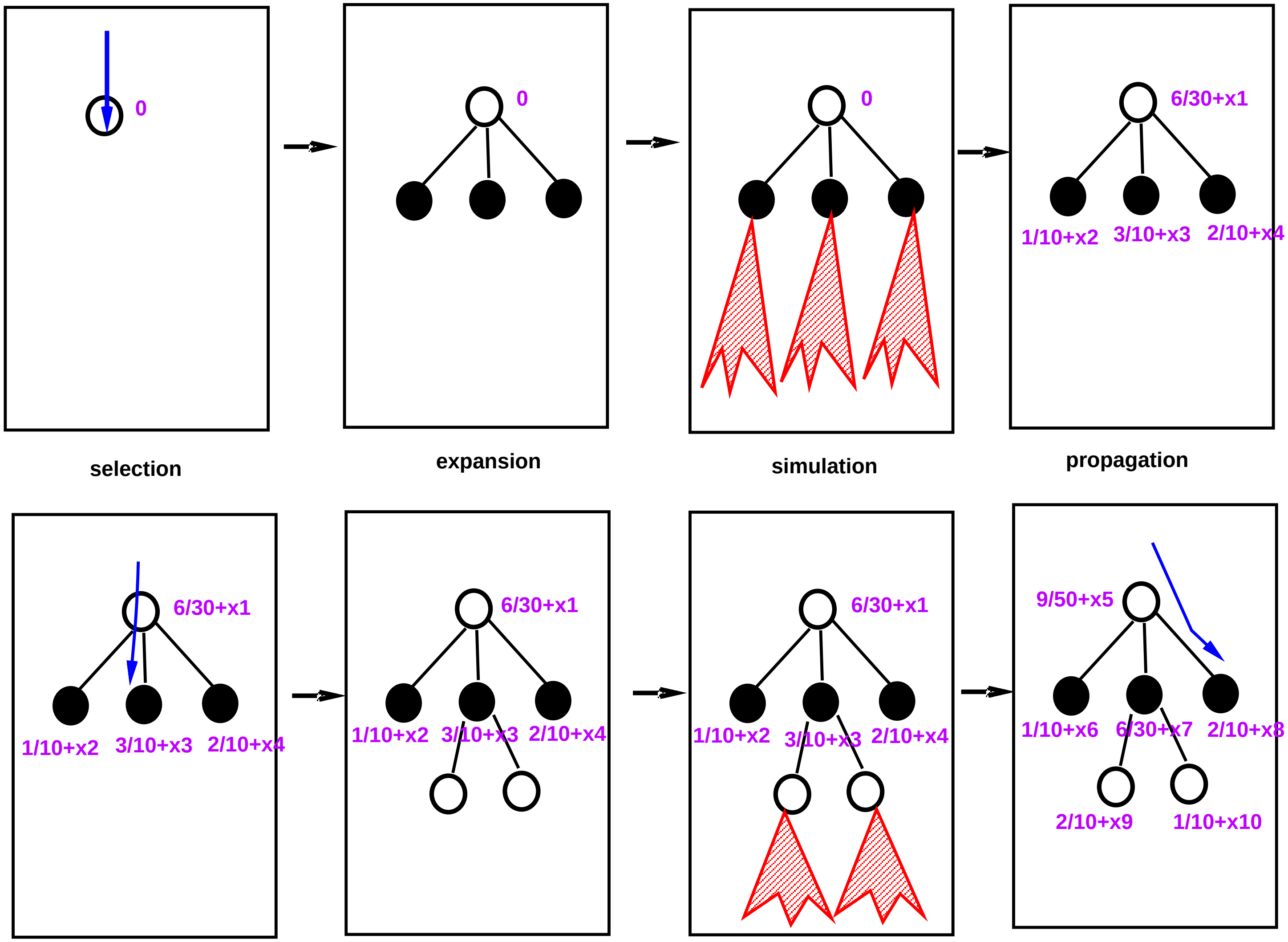
Parallel Monte-Carlo tree search

- **Leaf parallelization.**
- **Root parallelization.**
- **Tree parallelization.**
    - **With global synchronization.**
    - **With local synchronization.**

MCTS with UCT

- **Algorithm MCTS:**
- **1: Obtain an initial game tree**
- **2: Repeat the following sequence N_{total} times**
  - **2.1: Selection**
    - *From the root, pick a PV path to a leaf such that each node has the best UCB "score" among its siblings*
    - *May decide to "trust" the score of a node if it is visited more than a threshold number of times.*
    - *May decide to "prune" a node if its score is too bad now to save time.*
  - **2.2: Expansion**
    - *From a best leaf, expand it by one level.*
    - *Use some node expansion policy to expand.*
  - **2.3: Simulation**
    - *For the expanded leaves, perform some trials (playouts).*
    - *May decide to add knowledge into the trials.*
  - **2.4: Back propagation**
    - *Update the "scores" for nodes using a good back propagation policy.*
- **Pick a child of the root with the best score as your move.**

MCTS: example



Leaf parallelization

- **Algorithm PMCTS$_{leaf}$:**
  - **loop:**
    - ▷ ***Select the best leaf and the PV path.***
    - ▷ ***Perform Expansion in sequential.***
    - ▷ ***Perform Simulation, i.e., multiple trials, in parallel on the same leaf.***
    - ▷ ***Perform Back propagation in sequential.***
    - ▷ ***if the terminating condition is not matched, go to loop;***

PMCTS$_{leaf}$: comments

- **Comments:**
  - **Coding is very easy.**
  - **Good parallelization for performing a large number of trials.**
    - ▷ *Speedup is good up to using a reasonable number of PE's.*
  - **Can utilize a large number of PE's.**
    - ▷ *Scalability is reasonable.*
  - **Problem: the best leaf may no longer be the best after only a few more trials.**

Root parallelization

- **Algorithm PMCTS$_{root}$:**
    - **loop:**
    - **Duplicate k copies of the current game tree.**
    - **for each copy do in parallel**
        - ▷ ***Selection: pick the ith largest PV in the ith copy***
        - ▷ ***Expansion: ...***
        - ▷ ***Simulation: ...***
    - **Combine the copies into one copy by merging statistics on nodes and put the information into the current game tree.**
    - **Back propagation: ...**
    - **if the terminating condition is not matched, go to loop;**

PMCTS$_{root}$: comments

- **Comments:**
  - **Coding is easy.**
  - **Can utilize as many PE's as available.**
    - ***Scalability is good.***
  - **Problem: the best leaf may no longer be the best after only**
  - **May need to make sure that each tree does not pick the same best leaf.**
  - **Problem: Need to have a mechanism to properly choose the best leaves among all trees.**
    - ***Avoid duplicated efforts.***

Tree parallelization — global synchronization

- **Algorithm PMCTS$_{Tg}$:**
  - Use only one game tree.
  - Perform Selection, Expansion and Simulation in parallel.
    - *Different threads may work on different nodes in parallel.*
    - *Need a mechanism to ensure threads are not working on the same leaf.*
  - Use a global lock to make sure the game tree is writable by one thread during the Back propagation phase.
- **Comments:**
  - Speed-up is bad.

Tree parallelization — local synchronization

- **Algorithm PMCTS$_{Tl}$:**
  - **Make every node of the game tree as a global variable.**
  - **Perform Selection, Expansion, Simulation and Back propagation in parallel.**
    - ▷ *Different threads may work on different nodes in parallel.*
    - ▷ *Need a mechanism to ensure threads are not working on the same leaf.*
  - **Use a lock to make sure each node is writable by one thread during Back propagation.**
- **Comments:**
  - **Heavy O.S. overhead.**
  - **Unsure about the scalability.**

Problems of parallel Monte-Carlo search

- Each iteration of a Monte-Carlo simulation is a Markov chain process.
    - You need to know the result of the previous trial to decide the current selection.
    - Making trials in parallel has a larger statistical error.
    - May explore the wrong branch if synchronization is done only after a lot of trials.
    - May not have too much parallelism if synchronization is done after only a few trials.
- The cost of synchronization.
    - Shared global variable.
    - Cost of lock and unlock.
    - Memory bandwidth.
    - Network bandwidth.
- The cost of coding.

Parallel Monte-Carlo search: Analysis

- **Amdahl's law: assume a program needs to execute T instructions and and x of them can be parallelized. can be parallelized.**
  - **Assume you have n processors and an instruction take a unit of time.**
  - **Parallel processing time $\geq T - x + x/n + p_n \geq T - x$ where p_n is the cost for overhead in doing parallelization with n processors.**
  - **Speed-up $\leq T/(T - x)$.**
    - ▷ ***If 20% of the code cannot be parallelized, then your parallel program can be at most 5 times faster no matter how many processors you have.***
- **Leaf and root parallelization both have a large portion that is not parallelizable.**
- **Global or local synchronization has a large overhead.**
- **Comments**
  - **Need a better parallel implementation.**
  - **Need a better way to deal with the increasing error in doing more samplings.**

Concluding remarks

- **Need to think about tradeoff between costs in doing parallelism and benefits of saving in searching efforts because of parallelism.**
- **May need to think how to maintain distributed transposition tables.**
- **May need to think about the machine architecture.**
  - **Shared-memory vs. distributed memory.**
  - **Fine grain or coarse grain.**
  - **Whether the parallel version is stable or not?**
    - ▷ ***Ease of debugging.***
    - ▷ ***Ease of coding.***

References and further readings (1/2)

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