Theory of Computer Games: Selected Advanced Topics

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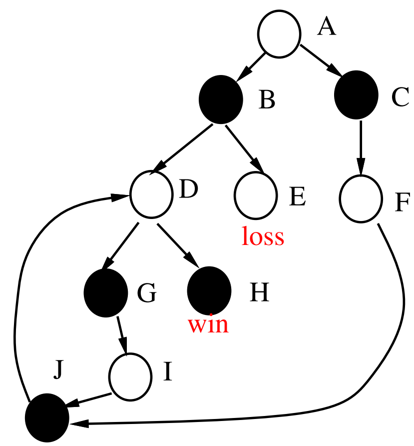
Abstract

- **Some advanced research issues.**
  - **The graph history interaction (GHI) problem.**
  - **Opponent models.**
  - **Multi-player game tree search.**
  - **Bit board speedup.**
  - **Proof-number search.**
- **More research topics.**
  - **The influence of rules on games.**
    - *Allowing long cycles in Go.*
    - *The scoring of a suicide ply in chess.*
  - **Why a position is difficult to human?**
  - **Unique features in games.**

Graph history interaction problem

- **The graph history interaction (GHI) problem [Campbell 1985]:**
  - **In a game graph, a position can be visited by more than one paths from a starting position.**
  - **The value of the position depends on the path visiting it.**
    - *It can be win, loss or draw for Chinese chess.*
    - *It can only be draw for Western chess and Chinese dark chess.*
    - *It can only be loss for Go.*
- **In the transposition table, you record the value of a position, but not the path leading to it.**
  - **Values computed from rules on repetition cannot be used later on.**
  - **It takes a huge amount of storage to store all the paths visiting it.**
- **This is a very difficult problem to be solved in real time [Wu et al '05] [Kishimoto and Müller '04].**

GHI: when loop draws



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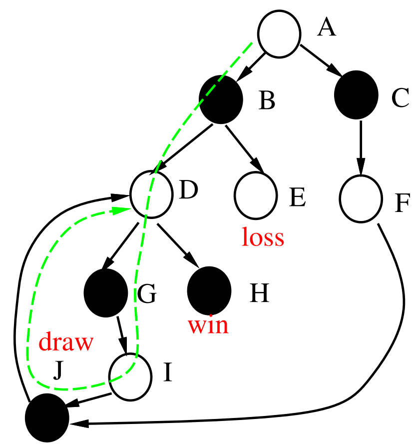

- **Assume if the game falls into a loop, then it is a draw.**
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  - ▷ ***Memorized J as a draw position.***

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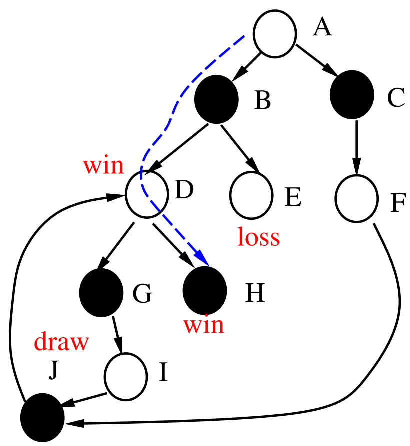

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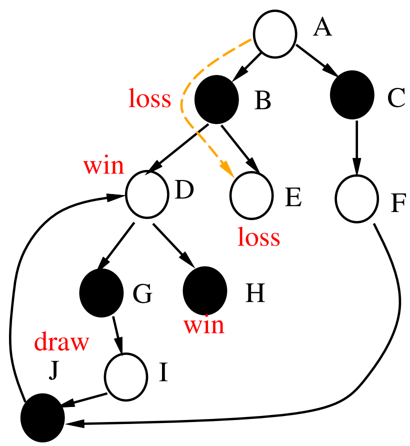

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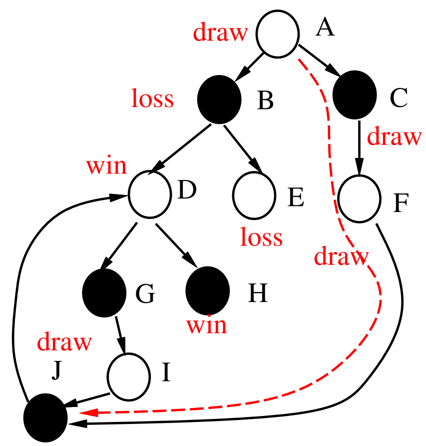

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- A **is draw because one child is loss and the other chile is draw.**

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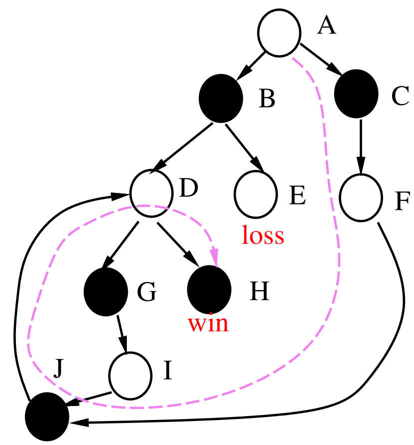

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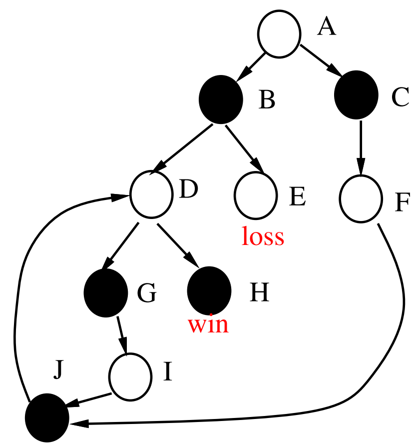

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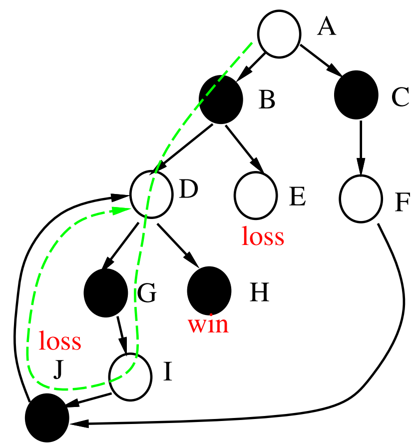

- **Assume the one causes loops wins the game.**
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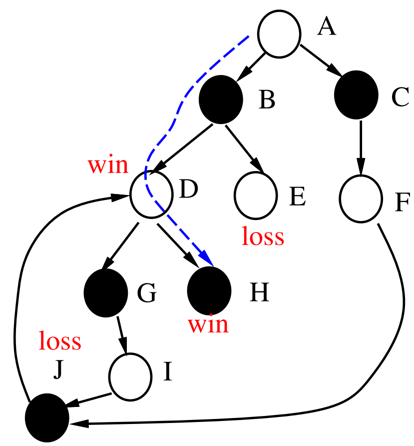

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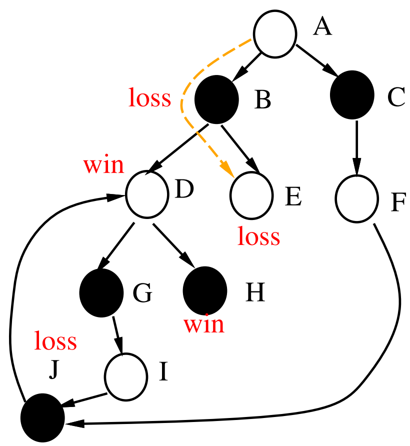

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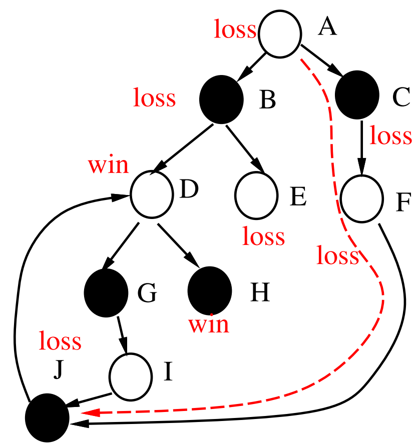

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- A **is loss because both branches lead to loss.**

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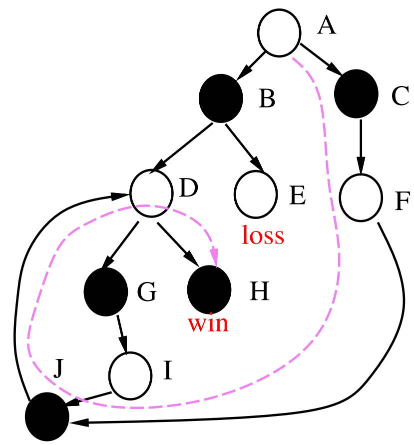

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Comments

- **Using DFS to search the above game graph from left first or from right first produces two different results.**
- **Position A is actually a win position.**
  - **Problem: memorize J being draw is only valid when the path leading to it causes a loop.**
- **Storing the path leading to a position in a transposition table requires too much memory.**
  - **Maybe we can store some forms of hash code to verify it.**
- **Finding a better data structure for solving this problem remains to be a challenging research issue.**
- **Remark: It real settings, it is usually the case that the rule of loops is enforced after 3 repetitions. However, GHI problem holds for any times of repetition.**

Opponent models

- In a normal alpha-beta search, it is assumed that you and the opponent use the same strategy.
  - What is good to you is bad to the opponent and vice versa!
  - Hence we can reduce a minimax search to a NegaMax search.
  - This is normally true when the game ends, but may not be true in the middle of the game.
- What will happen when there are two strategies or evaluation functions f_1 and f_2 so that
  - for some positions p, $f_1(p)$ is better than $f_2(p)$
    - ▷ *"better" means closer to the real value $f(p)$*
  - for some positions q, $f_2(q)$ is better than $f_1(q)$
- If you are using f_1 and you know your opponent is using f_2, what can be done to take advantage of this information.
  - This is called OM (opponent model) search [Carmel and Markovitch 1996].
    - ▷ *In a MAX node, use f_1.*
    - ▷ *In a MIN node, use f_2.*

Other usage of the opponent model

- **Depend on strength of your opponent, decide whether to force an easy draw or not.**
  - **This is called the contempt factor.**
- **Example in CDC:**
  - **It is easy to chase the king of your opponent using your pawn.**
  - **Drawing a weaker opponent is a waste.**
  - **Drawing a stronger opponent is a gain.**
- **It is feasible to use a learning model to “guess” the level of your opponent as the game goes and then adapt to its model in CDC [Chang et al 2021].**

Opponent models – comments

- **Comments:**
  - **Need to know your opponent's model precisely or to have some knowledge about your opponent.**
  - **How to learn the opponent model on-line or off-line?**
  - **When there are more than 2 possible opponent strategies, use a probability model (PrOM search) to form a strategy.**
- **Remark: A common misconception is that if your opponent uses a worse strategy f_3 than the one, namely f_2, used in your model, then he may get advantage.**
  - **This is impossible if f_2 is truly better than f_3.**
  - **If f_1 can beat f_2, then f_1 can sure beat f_3.**

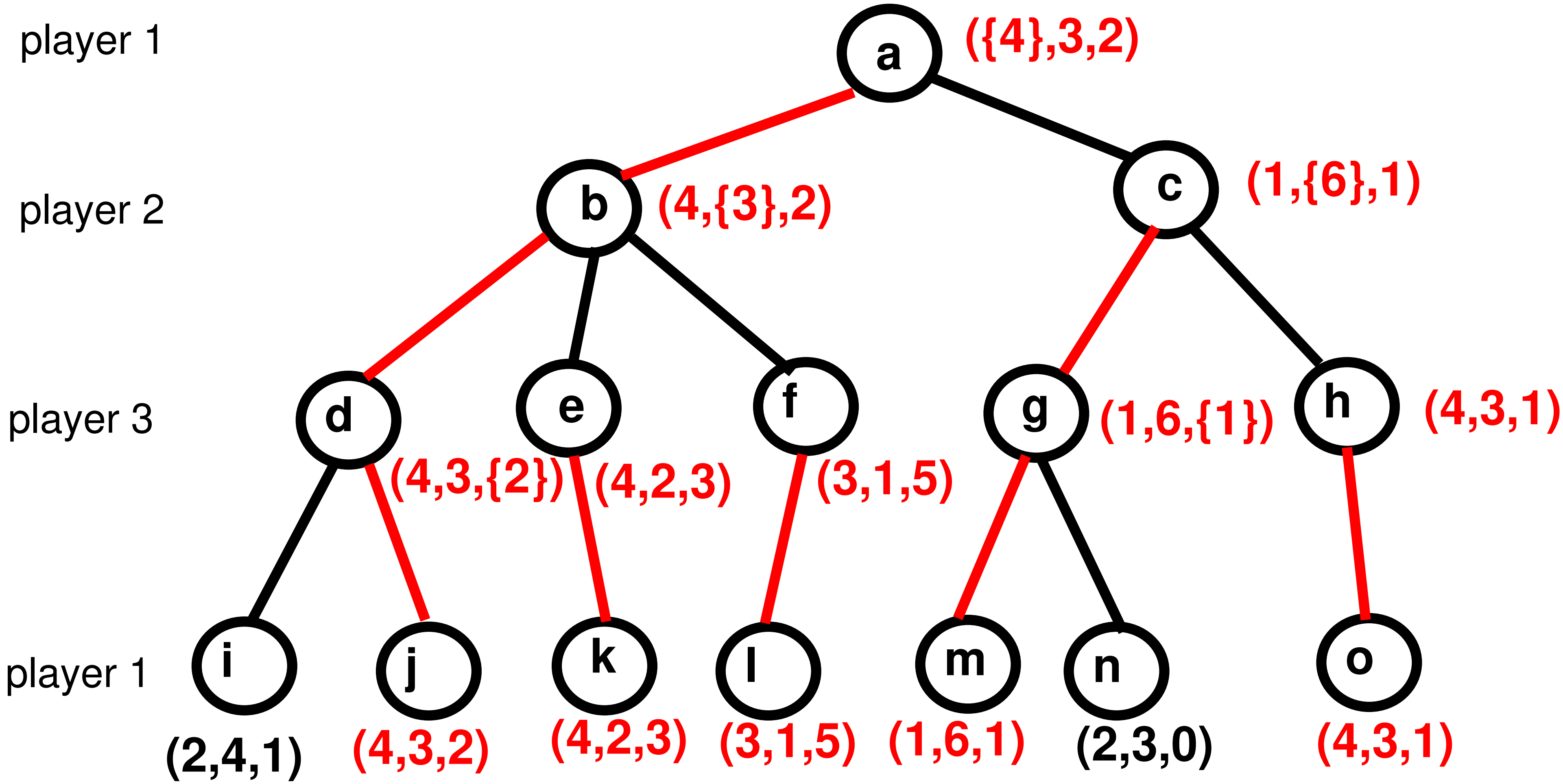
Multi-player game tree search

- **Games with more than 2 players.**
  - **Mahjong: 4 players**
  - **Contract bridge or bridge: 4 players**
  - **Monopoly: 2 to many players**
  - **Scrabble: 2 to 4 players**
  - **Risk: 2 to 6 players**
- **Assume we have n players, $y_1, \ldots, y_n$ in a game.**
  - **We have n evaluating functions, $score_i$, one for each player.**
  - **Given a position p with the children $p_1, \ldots, p_m$, let $score_i(p)$ be the score of y_i for p.**
    - ▷ ***If p is a terminal position for y_i, then $m = 0$ and $score_i(p)$ is the "true" score of y_i in p.***
    - ▷ ***Otherwise, $score_i(p) = \max_{j=1}^{m} score_i(p_j)$.***
  - **The above algorithm is called MAX^n where stands for during each turn, each player maximizes his own score without considering scores of others.**

MAX^n: algorithm

- $next\_player(idx)$: the player who is next to player idx.
- Brute force algorithm for multi-player games.
- Algorithm MAXN(position p, player idx)
  - output: $best$ which is an array with $best[i]$ being the best value for player i so far.
  - If p is terminal, then return $best[i] = score_i(p), \forall i$;
  - initialize $best$ to be $best[i] = -\infty, \forall i$;
  - Let p_i be the ith child of p;
  - for $i = 1$ to last child of p do
    - ▷ $current$ = MAXN(p_i, next_player(idx));
    - ▷ if $current[idx] > best[idx]$, $best = current$; // maximized player idx
  - return $best$;

MAX^n: example ($n = 3$)



Opportunities for pruning (1/2)

- **Let p be a position in a multi-player game.**
- **Alpha-beta pruning is a special case for $n = 2$ and cannot be generalized for $n > 2$.**
  - **Property used in alpha-beta pruning:**
    - ▷ ***What is good for y_1 is definitely bad for y_2 by using the zero sum principle which is for a position p, $score_1(p) + score_2(p) = 0$.***
  - **The above may not be true for $n > 2$.**
    - ▷ ***When $n = 3$, what is good for y_1 may be also good for y_2, but very bad for y_3.***

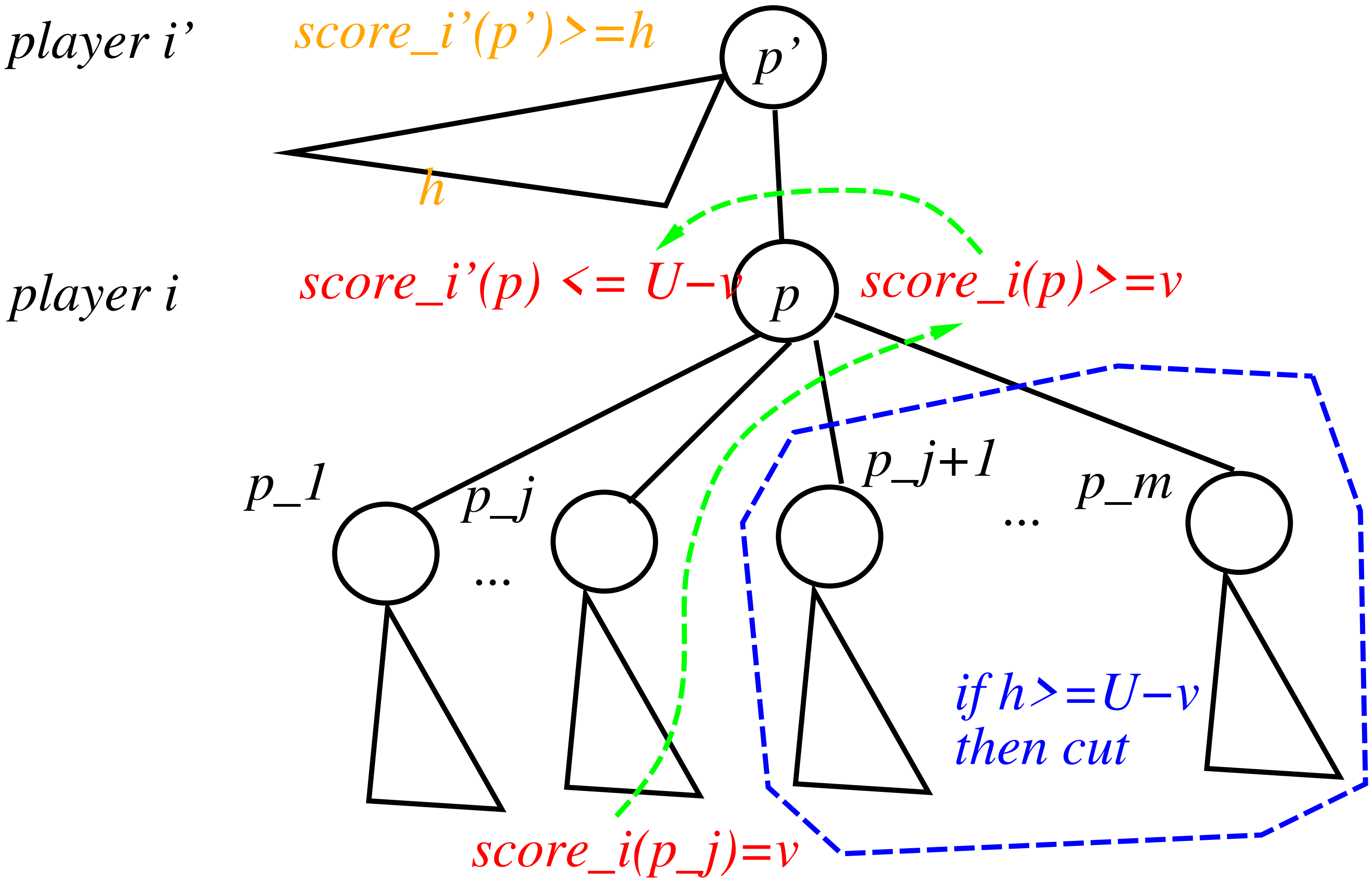
Opportunities for pruning (2/2)

- **For a position p, if there is no constraints on the n scores of p, then it is impossible to have any cut offs for MAX^n.**
  - **In applications we often have the following properties.**
    - ▷ ***Zero sum.***
    - ▷ ***The sum of all n scores for p has an upper bound U.***
    - ▷ ***The score of p for any player has a lower bound L.***
  - **Examples:**
    - ▷ ***Go for n players: each player owns pieces of a distinct color.***
      $\rightarrow$ ***the sum of all points $\leq$ the board size, and the score cannot be negative.***
    - ▷ ***Othello for n players: each player owns pieces of a distinct color and flips all pieces of different colors.***
      $\rightarrow$ ***the sum of all points $\leq$ the plys played so far and the score cannot be negative.***

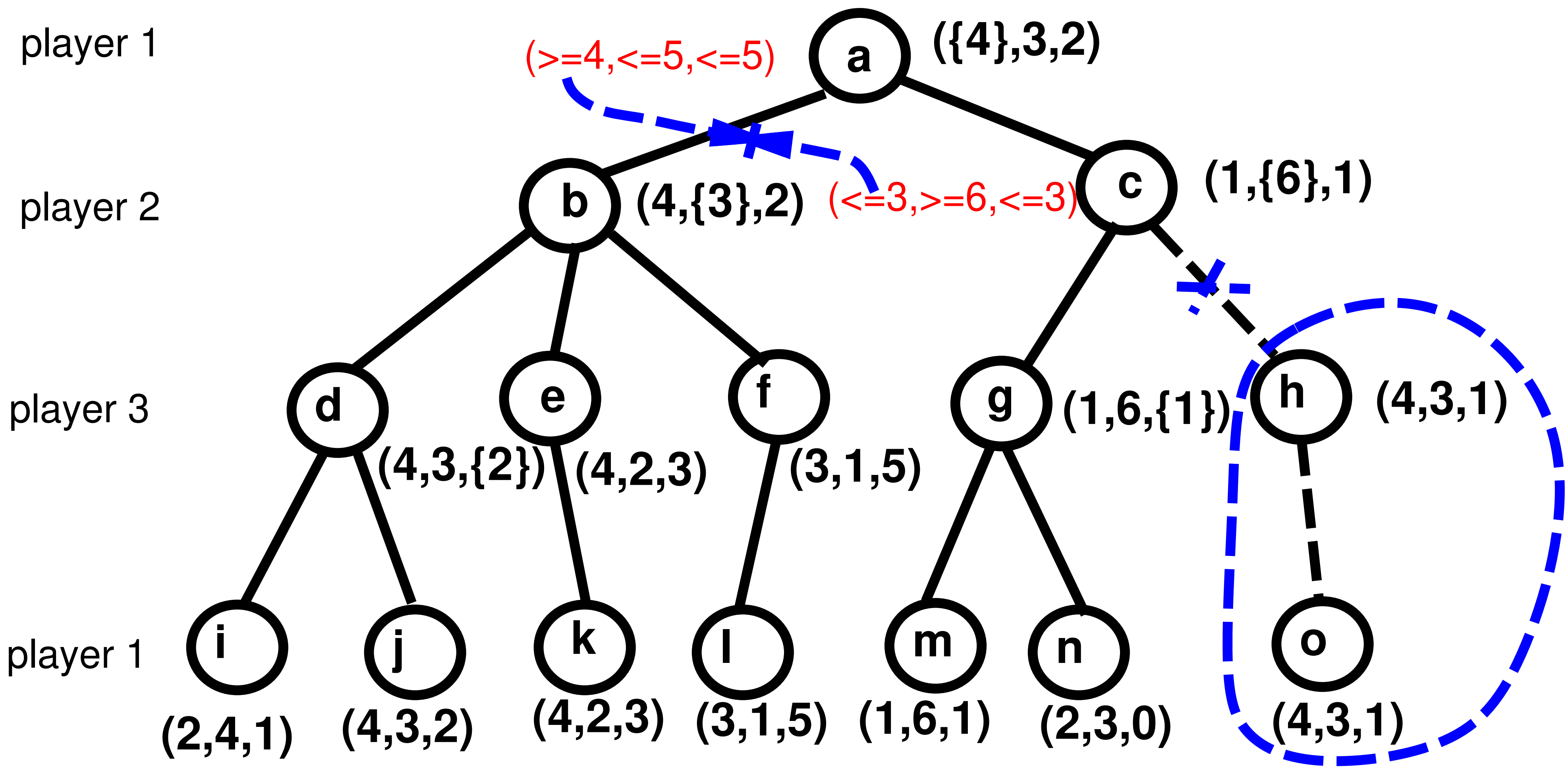
Pruning

- **Recall: a position p with the children $p_1, \ldots, p_m$ and the parent p', and $score_i(p)$ is the score of player i for p.**
- **Direct pruning:**
  - **During the turn of the ith player, if $score_i(p_j) = U$, then no more search is needed.**
- **Shallow pruning:**
  - **Without loss of generality, assume $L = 0$.**
  - **During the turn of the ith player, if $score_i(p_j) = v$ so far, then $score_i(p) \geq v$ since each player is a max player.**
  - **This implies $score_j(p) \leq U - v$ if $j \neq i$.**
  - **Let i' be the index of the immediate previous player.**
  - **We know $score_{i'}(p') \geq h$ if he has done some searching.**
  - **If $h \geq U - v$, then we have a cut off.**

MAX^n: ideas for cutoff



MAX^n: cutoff example $(n = 3, U = 9)$



Remarks about pruning in MAX^n

- **Direct pruning is a degenerated case of the shallow pruning by the following settings.**
  - **If $v = U$, then the scores of all other players are all zero.**
  - **Using the lower bound L, you can get a cut off.**
- **Compared to two-player alpha-beta pruning, both direct and shallow pruning can be used in $n \geq 2$.**
- **Deep pruning does not work when $n > 2$.**
  - **Assume you are searching the node w, v is your parent and u is an ancestor that is not v.**
  - **Assume node x is the turn of player $player(x)$.**
  - **Any value of $score_{player(u)}(u)$ cannot produce any cutoff on searching the tree T_w because $player(v)$ makes the decision first in propagating the values up.**
  - **Any value of $score_{player(u)}(w)$ can be propagated up and be used by u.**

Algorithm for shallow cut off

- **Functions and data structures**
  - $next\_player(idx)$: the player who is next to player idx.
  - $score_i(p)$: the score of player i for the position p.
  - U: the upper bound of sum of all scores among all players on a position.
  - Assume L is 0.
  - $best$ and $current$ are both arrays of size n.
- **Algorithm shallow(position p, player idx, value $bound$)**
  - return value: $best$ which is an array with $best[i]$ being the best value for player i so far.
  - If p is terminal, then return $best[i] = score_i(p), \forall i$;
  - Let p_i be the ith child of p;
  - $best = shallow(p_1, next(idx), U)$; // recursive call on the first child
  - for $i = 2$ to last child of p do

    4.1: if $best[idx] = U$, then return $best$ // immediate cut off
    4.2: if $best[idx] \geq bound$, then return $best$ // shallow cut off
    4.3: $current = shallow(p_i, next\_player(idx), U - best[idx])$;
    4.4: if $current[idx] > best[idx]$, $best = current$; // maximize player idx

  - return $best$;

Comments

- **A generalization of alpha-beta cutoff on adjacent depths.**
- **Does not work on deep alpha-beta cutoff [Korf 1991].**
- **In the best case, the effective branching factor is $\frac{1+\sqrt{4b-3}}{2}$ where b is the average branching factor.**
  - **Comparing to alpha-beta cut off, the best effective branching factor is $\sqrt{b}$.**
- **In the average case, the effective branching factor is approaching $O(b)$.**
  - **Comparing to alpha-beta cut off, the the average effective branching factor is $b^{0.75}$ [Fuller et al 1975].**
  - **This implies most of the cut off come from deep pruning in the average case.**
- **More research are needed to get more cutoff by observing additional constraints on the values from the application domain.**
- **MCTS can be easily extended to work on any number of players, but need to work on better properties of convergence.**

Hardware Speedup

- **Using hardware to speed up searching is not new.**
  - **Parallel computing.**
    - ▷ *The Northwestern University CHESS program series on the 1970's makes full usage of hardware advantages from supercomputers [Atkin & Slate 1977].*
  - **Special hardware acceleration:**
    - ▷ *Belle: a chess machine with special micro instructions for move generation, alpha-beta pruning and transposition table operations [Condon & Thompson 1982].*
    - ▷ *Deep Blue: custom VLSI FPGA chips for operating chess playing expert systems [Hsu et al 1995].*
- **The above's are very costed.**

Bit board techniques

- **Everyone can make use of the benefits of hardware acceleration now by smart usage of fast parallel bitwise operations provided by modern day CPU's.**
  - **Intel CPU's: MMX and SSE [Intel 2021]**
  - **AMD: 3D Now! [AMD 2000]**
- **Main technique**
  - **Using bits to represent the board and pieces on the board.**
    - ▷ *Transfer a board into an $n \times m$ picture*
    - ▷ *Transfer pieces into patterns of pixel rectangles*
  - **These instructions are usually in the form of SIMD (single instruction multiple data).**
  - **Many are for image related operations.**
  - **May also make use of GPU.**

Special instruction sets (1/2)

- **Make use of fast parallel bitwise operations provided by modern day CPU's.**
- **Many different types**
  - **Find aggregated information**
  - **Parallel bit deposit and extract**
  - **...**
- **Most of the instructions can be done using AND, OR, NOT operations, but can be done much faster using special CPU instructions.**

Special instruction sets (2/2)

- **Find aggregated information:**
  - **population count (POPCNT): the number of 1-bits in a "word".**
  - **leading/trailing zero count: LZCNT, TZCNT**
- **Parallel bit deposit and extract**
  - **Pack in sequence selected bits (PEXT): extract something out**
    - ▷ $PEXT(W, Mask)$ ***returns a word by packing to the right those bits in the word*** W ***whose corresponding bits in the word*** $Mask$ ***are equal to*** 1.
    - ▷ ***Example:*** $PEXT(010110010, 010101010)$ ***extracts the four even numbered bit and then pack it to the right. Thus it returns*** 01100.
  - **Distribute bits in sequence to selected locations (PDEP): deposit something into.**
    - ▷ $PDEP(W, Mask)$ ***returns a word by sending the*** i***th bit in the word*** W ***to the location addressed by the*** i***th*** 1.
    - ▷ ***Example:*** $PEXT(01100, 010101010)$ ***deposits the four bits to the even numbered location. Thus it returns*** 010100000.

Example I

- **In Go, how to find the number of empty intersections on the board?**
  - **Assume you have a long hardware word W of 19*2=38 bits.**
    - ▷ *Use 19 words $W_1, \ldots, W_{19}$ to represent the rows.*
  - **Encoding: bits i and $i+1$ in W_j represents the status of the intersection at the ith column and jth row.**
    - ▷ 00 *means empty.*
    - ▷ 10 *means a black stone.*
    - ▷ 01 *means a white stone.*
  - **POPCOUNT(W_j) gives the number of stones in the jth row.**
  - **$19-$POPCOUNT(W_j) gives the number of empty intersections in the jth row.**

Example II

- **In Chinese Dark Chess (CDC), how to find all revealed pieces of a color on the board?**
  - **Assume you have a long hardware word W of 32*3=96 bits.**
  - **Encoding: bits $3i$, $3i+1$, and $3i+2$ in W_b represents the status of the ith cell on the board with regard to the black side. Similarly, we have W_r for the red side.**
    - ▷ 000 *means empty, or pieces of other color or dark.*
    - ▷ xyz *means the xyzth kind of piece where there are up to only 7 different kinds of pieces of a color. Thus the encodings used are from 1 to 7.*
- **Algorithm Find_PCES(color c)**
  - **// find all pieces of color c and put them in m[]**
  - $i = 0$
  - **while** $W_c\ != \ 0$ **do**
    - ▷ $a = TZCNT(W_c)$ *// count the number of tailing zeros*
    - ▷ $W_c >>= a$ *// right shift a bits, find next piece*
    - ▷ $m[i++] = W_c\ \&\ 07$ *// gives a piece of color c*
    - ▷ $W_c\ \&= \sim(07)$ *// mask off the lowest 3 bits*
  - **return** m

Example III

- **In Othello, how to pack information in a column in a continuous sequence of cells?**
  - **Problem:**
    - ▷ *The board of Othello is a 8 by 8 rectangle. Assume we use a word to represent the board and use the row-major ordering, then cells in a column are non-adjacent.*
    - ▷ *Example: The first (leftmost) column are numbered 0, 8, 16, 24, 32, 40, 48, and 56 in a row-major ordering.*
  - **Encoding:**
    - ▷ *Assume you have a hardware word W of 64 bits.*
    - ▷ W_b *and* W_w *are words for black and white stones respectively.*
    - ▷ 0 *means empty or other color.*
    - ▷ $\overline{(W_b|W_w)}$ *gives the word for empty spaces.*
- **Algorithm Find_Column(color c, int idx)**
  - **// pack information in column idx into adjacent bits**
  - **//** Loc **is an array which gives the masks of bits in column idx**
  - $Mask = Loc[idx]$
  - $W = PEXT(W_c, Mask)$
  - **return** W

Comments

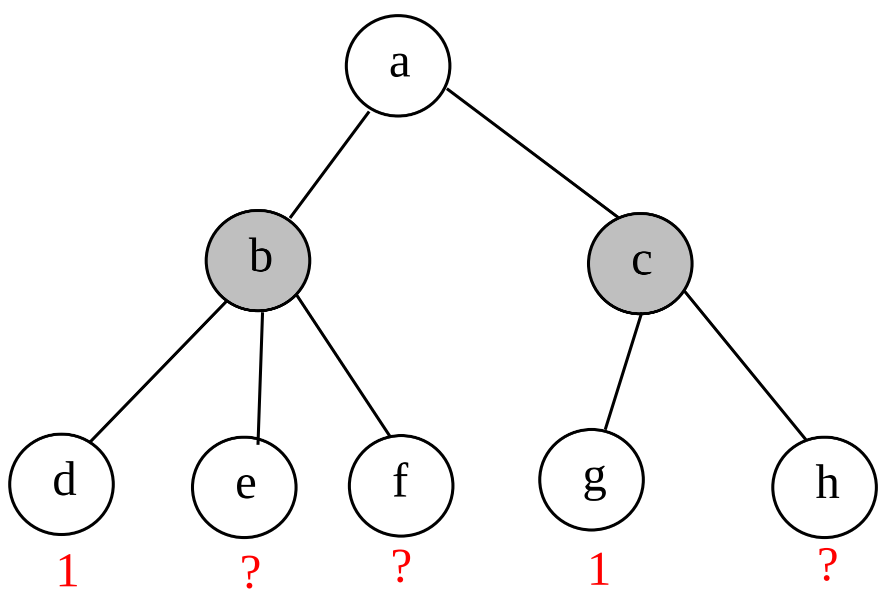
- **Read carefully the instruction set of the CPU used to find out any special SIMD operations that are or aren't provided.**
- **The speedup is a lot, sometimes more than 50 times, if the encoding used is good [Browne 2014].**

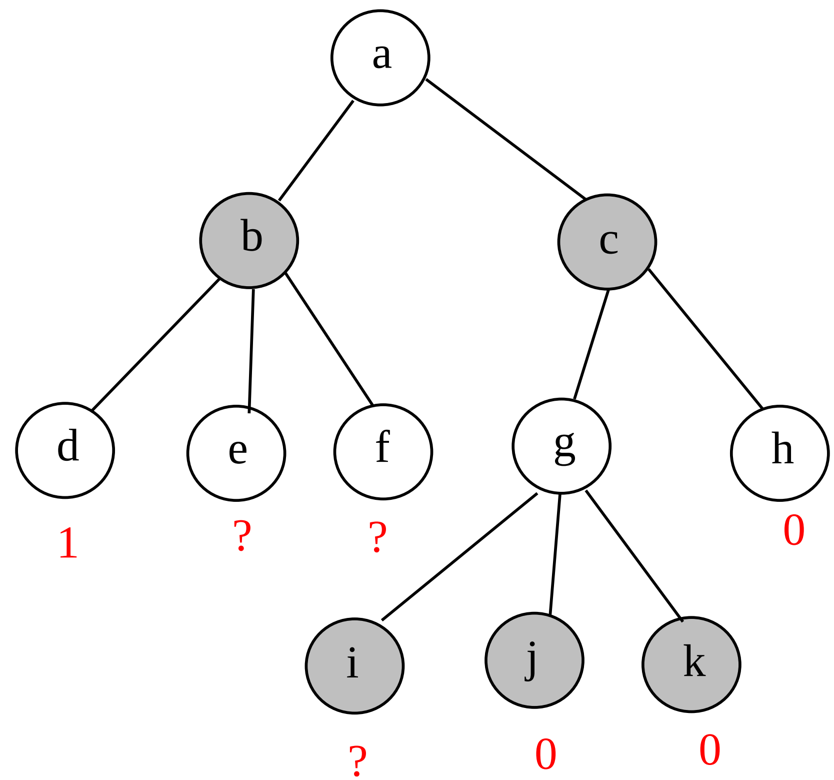
Proof number search

- Consider the case of a 2-player game tree with either 0 or 1 on the leaves.
  - win, or not win which is lose or draw;
  - lose, or not lose which is win or draw;
  - Call this a binary valued game tree.
- If the game tree is known as well as the values of some leaves are known, can you make use of this information to search this game tree faster?
  - The value of the root is either 0 or 1.
  - If a branch of the root returns 1, then we know for sure the value of the root is 1.
  - The value of the root is 0 only when all branches of the root returns 0.
  - An AND-OR game tree search.

Which node to search next?

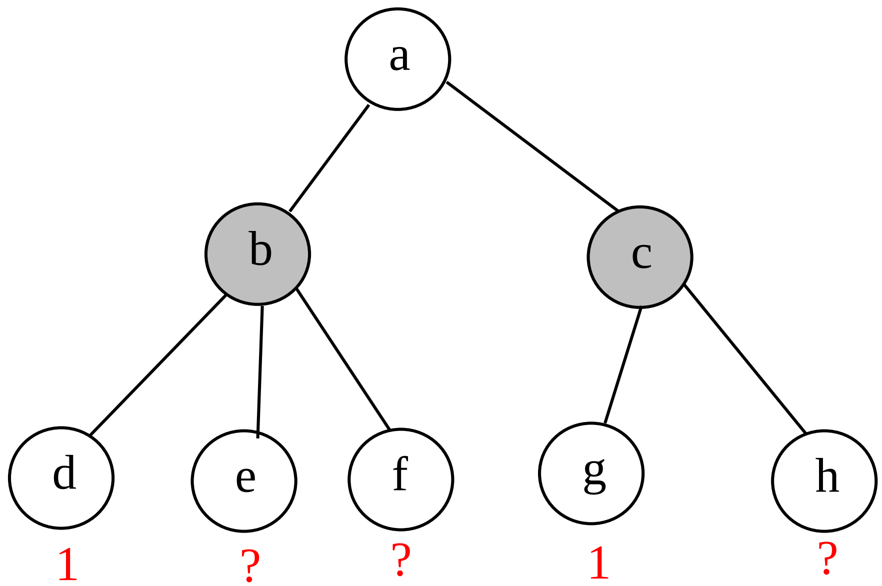
- A **most proving node** for a node u: a descendent node if its value is 1, then the value of u is 1.
- A **most disproving node** for a node u: a descendent node if its value is 0, then the value of u is 0.





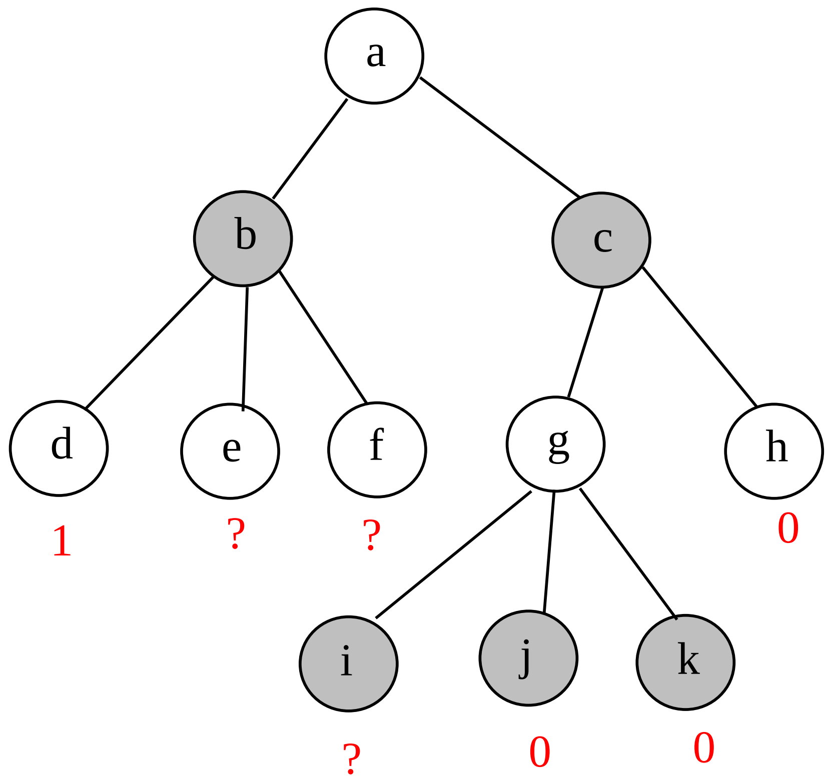
Most proving node

- **Node h is a most proving node for a.**



Most disproving node

- **Node e or f is a most disproving node for a.**

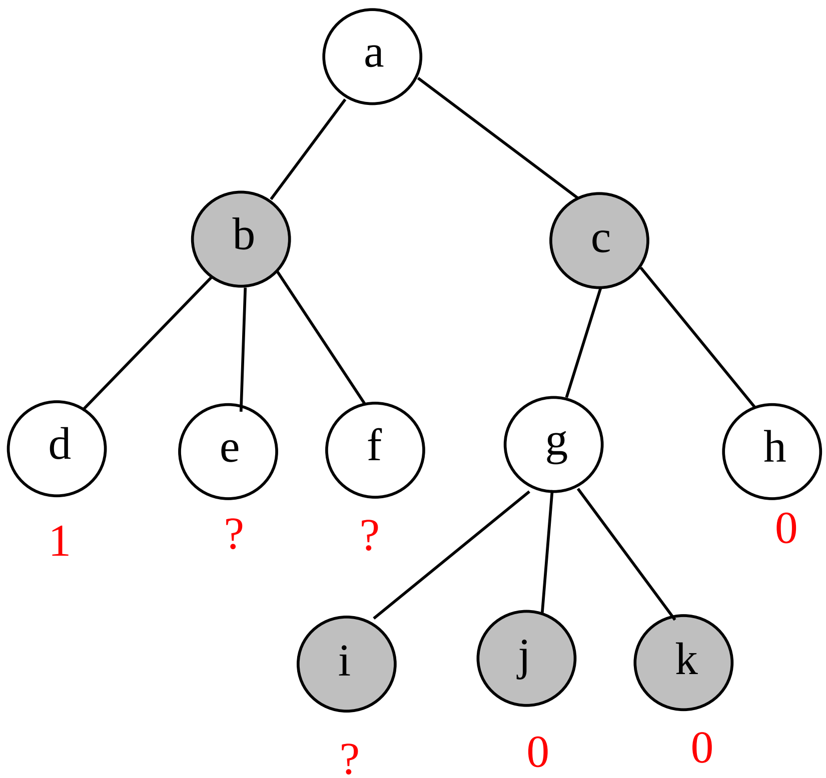


Proof or Disproof Number

- **Assign a proof number and a disproof number to each node u in a binary valued game tree.**
  - $proof(u)$**: the minimum number of leaves needed to visited in order for the value of u to be 1.**
  - $disproof(u)$**: the minimum number of leaves needed to visited in order for the value of u to be 0.**
- **The definition implies a bottom-up ordering.**

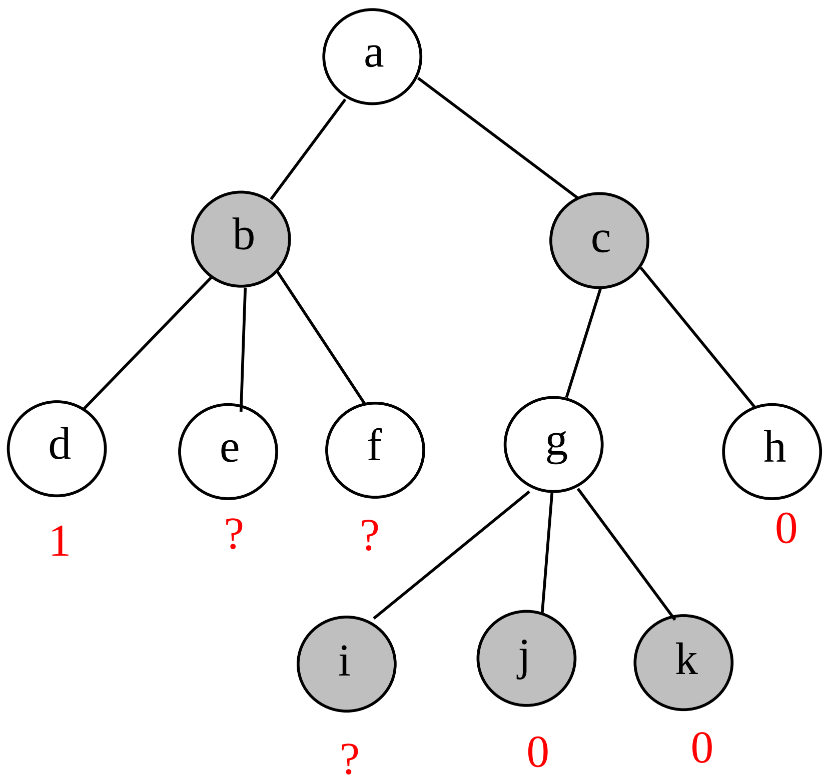
Proof number

- **Proof number for the root a is 2.**
  - ▷ ***Need to at least prove e and f.***



Disproof number

- **Disproof number for the root a is 2.**
  - ▷ ***Need to at least disprove i, and either e or f.***



Proof Number: Definition

- u **is a leaf:**
  - **If** $value(u)$ **is unknown, then** $proof(u)$ **is the cost of evaluating** u**.**
  - **If** $value(u)$ **is 1, then** $proof(u) = 0$**.**
  - **If** $value(u)$ **is 0, then** $proof(u) = \infty$**.**
- u **is an internal node with all of the children** $u_1, \ldots, u_b$**:**
  - **if** u **is a MAX node,**

$$proof(u) = \min_{i=1}^{i=b} proof(u_i);$$

  - **if** u **is a MIN node,**

$$proof(u) = \sum_{i=1}^{i=b} proof(u_i).$$

Disproof Number: Definition

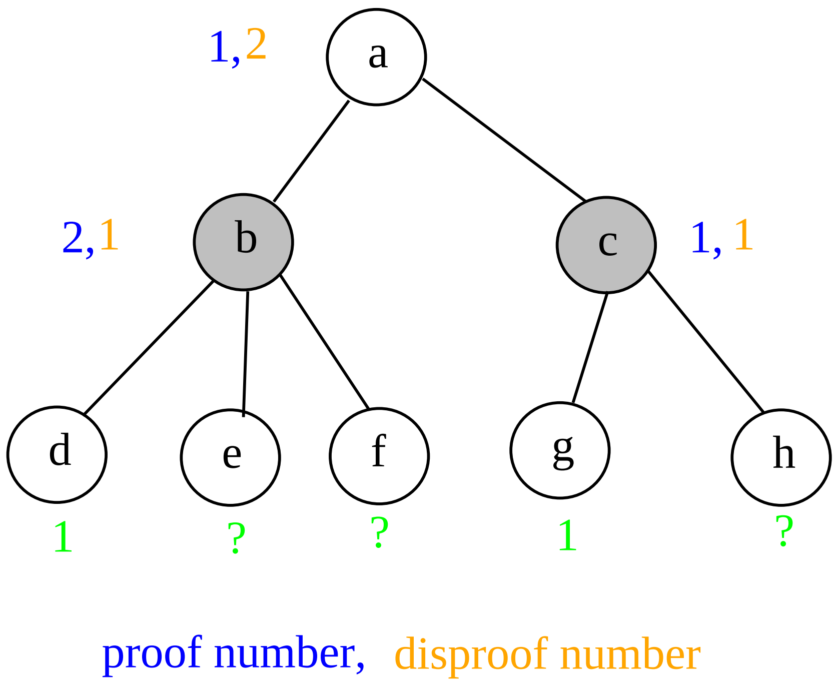
- u **is a leaf:**
  - **If** $value(u)$ **is unknown, then** $disproof(u)$ **is cost of evaluating** u**.**
  - **If** $value(u)$ **is 1, then** $disproof(u) = \infty$**.**
  - **If** $value(u)$ **is 0, then** $disproof(u) = 0$**.**
- u **is an internal node with all of the children** $u_1, \ldots, u_b$**:**
  - **if** u **is a MAX node,**

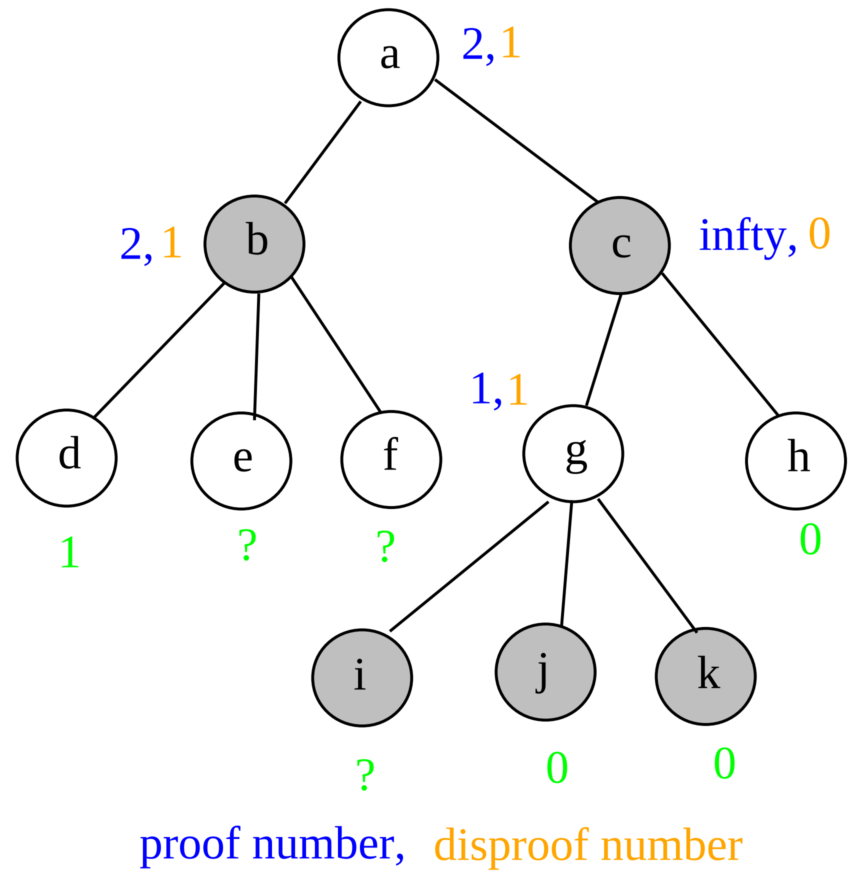
$$disproof(u) = \sum_{i=1}^{i=b} disproof(u_i);$$

  - **if** u **is a MIN node,**

$$disproof(u) = \min_{i=1}^{i=b} disproof(u_i).$$

Illustrations





How these numbers are used (1/2)

- **Scenario:**
  - **For example, the tree T represents an open game tree or an endgame tree.**
    - *If T is an open game tree, then maybe it is asked to prove or disprove a certain open game is win.*
    - *If T is an endgame tree, then maybe it is asked to prove or disprove a certain endgame is win o loss.*
    - *Each leaf takes a lot of time to evaluate.*
    - *We need to prove or disprove the tree using as few time as possible.*
  - **Depend on the results we have so far, pick a leaf to prove or disprove.**
- **Goal: solve as few leaves as possible so that in the resulting tree, either $proof(root)$ or $disproof(root)$ becomes 0.**
  - **If $proof(root) = 0$, then the tree is proved.**
  - **If $disproof(root) = 0$, then the tree is disproved.**
- **Need to be able to update these numbers on the fly.**

How these numbers are used (2/2)

- **Let $GV = \min\{proof(root), disproof(root)\}$.**
  - **GT is "prove" if $GV = proof(root)$, which means we try to prove it.**
  - **GT is "disprove" if $GV = disproof(root)$, which means we try to disprove it.**
  - **In the case of $proof(root) = disproof(root)$, we set GT to "prove" for convenience.**
- **From the root, we search for a leaf whose value is unknown.**
  - **The leaf found is a most proving node if GT is "prove", or a most disproving node if GT is "disprove".**
  - **To find such a leaf, we start from the root downwards recursively as follows.**
    - ***If we have reached a leaf, then stop.***
    - ***If GT is "prove", then pick***
      a child with the least proof number for a MAX node, and
      any node that has a chance to be proved for a MIN node.
    - ***If GT is "disprove", then pick***
      a child with the least disproof number for a MIN node, and
      any node that has a chance to be disproved for a MAX node.

PN-search: algorithm (1/2)

- $\{*$ **Compute and update proof and disproof numbers of the root in a bottom up fashion until it is proved or disproved.** $*\}$
- *loop:*
  - **If** $proof(root) = 0$ **or** $disproof(root) = 0$**, then we are done, otherwise**
    - ▷ $proof(root) \leq disproof(root)$: ***we try to prove it.***
    - ▷ $proof(root) > disproof(root)$: ***we try to disprove it.***
  - $u \leftarrow root$; $\{*$ **find a leaf to prove or disprove** $*\}$
  - **if we try to prove, then**
    - ▷ ***while*** u ***is not a leaf do***
    - ▷ ***if*** u ***is a MAX node, then***
      $u \leftarrow$ ***leftmost child of*** u ***with the smallest non-zero proof number;***
    - ▷ ***else if*** u ***is a MIN node, then***
      $u \leftarrow$ ***leftmost child of*** u ***with a non-zero proof number;***
  - **else if we try to disprove, then**
    - ▷ ***while*** u ***is not a leaf do***
    - ▷ ***if*** u ***is a MAX node, then***
      $u \leftarrow$ ***leftmost child of*** u ***with a non-zero disproof number;***
    - ▷ ***else if*** u ***is a MIN node, then***
      $u \leftarrow$ ***leftmost child of*** u ***with the smallest non-zero disproof number;***

PN-search: algorithm (2/2)

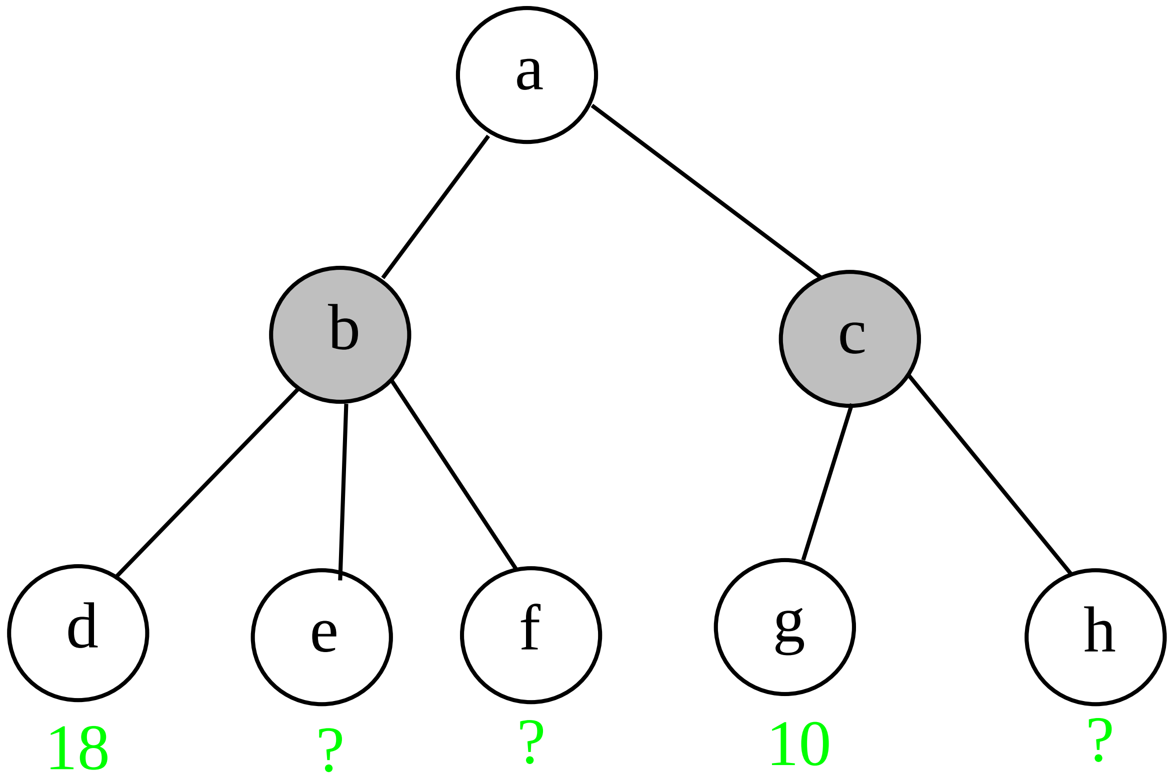
- {$*$ **Continued from the last page** $*$}
  - **solve** u;
  - **repeat** {$*$ **bottom up updating the values** $*$}
    - ▷ ***update*** $proof(u)$ ***and*** $disproof(u)$
    - ▷ $u \leftarrow u'$***s parent***

    until u **is the root**
  - **go to** $loop$;

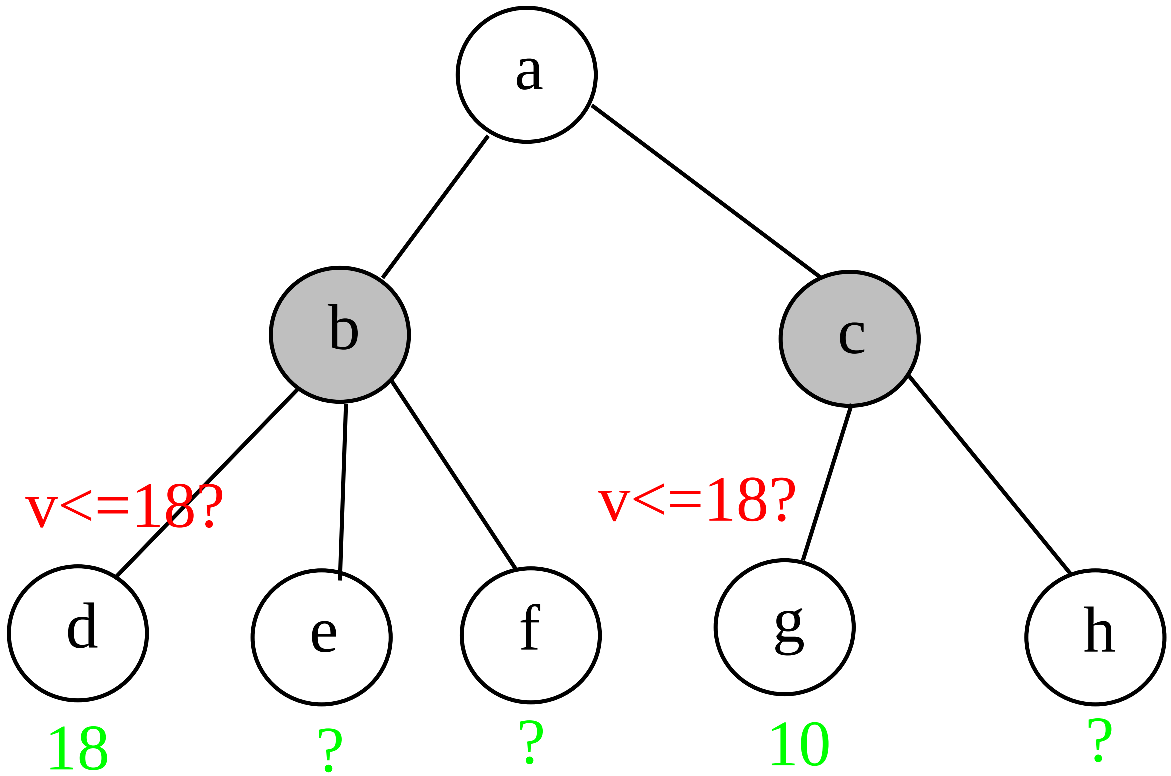
Multi-Valued game Tree

- **The values of the leaves may not be binary.**
  - **Assume the values are non-negative integers.**
  - **Note: it can be in any finite countable domain.**
- **Revision of the proof and disproof numbers.**
  - $proof_v(u)$: **the minimum number of leaves needed to visited in order for the value of** u **to** $\geq v$.
    - ▷ $proof(u) \equiv proof_1(u)$.
  - $disproof_v(u)$: **the minimum number of leaves needed to visited in order for the value of** u **to** $< v$.
    - ▷ $disproof(u) \equiv disproof_1(u)$.

Illustration



Illustration



Multi-Valued proof number

- u **is a leaf:**
  - **If** $value(u)$ **is unknown, then** $proof_v(u)$ **is cost of evaluating** u**.**
  - **If** $value(u) \geq v$**, then** $proof_v(u) = 0$**.**
  - **If** $value(u) < v$**, then** $proof_v(u) = \infty$**.**
- u **is an internal node with all of the children** $u_1, \ldots, u_b$**:**
  - **if** u **is a MAX node,**

$$proof_v(u) = \min_{i=1}^{i=b} proof_v(u_i);$$

  - **if** u **is a MIN node,**

$$proof_v(u) = \sum_{i=1}^{i=b} proof_v(u_i).$$

Multi-Valued disproof number

- u **is a leaf:**
  - **If** $value(u)$ **is unknown, then** $disproof_v(u)$ **is cost of evaluating** u.
  - **If** $value(u) \geq v$, **then** $disproof_v(u) = \infty$.
  - **If** $value(u) < v$, **then** $disproof_v(u) = 0$.
- u **is an internal node with all of the children** $u_1, \ldots, u_b$**:**
  - **if** u **is a MAX node,**

$$disproof_v(u) = \sum_{i=1}^{i=b} disproof_v(u_i);$$

  - **if** u **is a MIN node,**

$$disproof_v(u) = \min_{i=1}^{i=b} disproof_v(u_i).$$

Revised PN-search(v): algorithm (1/2)

- {* **Compute and update proof$_v$ and disproof$_v$ numbers of the root in a bottom up fashion until it is proved or disproved.** *}
- *loop:*
  - **If** $proof_v(root) = 0$ **or** $disproof_v(root) = 0$**, then we are done, otherwise**
    - ▷ $proof_v(root) \leq disproof_v(root)$: ***we try to prove it.***
    - ▷ $proof_v(root) > disproof_v(root)$: ***we try to disprove it.***
  - $u \leftarrow root$; {* **find a leaf to prove or disprove** *}
  - **if we try to prove, then**
    - ▷ ***while*** u ***is not a leaf do***
    - ▷ ***if*** u ***is a MAX node, then***
      $u \leftarrow$ ***leftmost child of*** u ***with the smallest non-zero proof$_v$ number;***
    - ▷ ***else if*** u ***is a MIN node, then***
      $u \leftarrow$ ***leftmost child of*** u ***with a non-zero proof$_v$ number;***
  - **else if we try to disprove, then**
    - ▷ ***while*** u ***is not a leaf do***
    - ▷ ***if*** u ***is a MAX node, then***
      $u \leftarrow$ ***leftmost child of*** u ***with a non-zero disproof$_v$ number;***
    - ▷ ***else if*** u ***is a MIN node, then***
      $u \leftarrow$ ***leftmost child of*** u ***with the smallest non-zero disproof$_v$ number;***

PN-search: algorithm (2/2)

- {$*$ **Continued from the last page** $*$}
  - **solve** u;
  - **repeat** {$*$ **bottom up updating the values** $*$}
    - ▷ ***update*** $proof_v(u)$ ***and*** $disproof_v(u)$
    - ▷ $u \leftarrow u$***'s parent***

    until u **is the root**
  - **go to** $loop$;

Multi-valued PN-search: algorithm

- **When the values of the leaves are not binary, use an open value binary search to find an upper bound of the value.**
  - **Set the initial value of v to be 1.**
  - $loop$: **PN-search(v)**
    - ▷ ***Prove the value of the search tree is $\geq v$ or disprove it by showing it is $< v$.***
  - **If it is proved, then double the value of v and go to $loop$ again.**
  - **If it is disproved, then the true value of the tree is between $\lfloor v/2 \rfloor$ and $v - 1$.**
  - **{$*$ Use a binary search to find the exact returned value of the tree. $*$}**
  - $low \leftarrow \lfloor v/2 \rfloor$; $high \leftarrow v - 1$;
  - **while** $low \leq high$ **do**
    - ▷ ***if*** $low = high$, ***then return*** low ***as the tree value***
    - ▷ $mid \leftarrow \lfloor (low + high)/2 \rfloor$
    - ▷ ***PN-search***(mid)
    - ▷ ***if it is disproved, then*** $high \leftarrow mid - 1$
    - ▷ ***else if it is proved, then*** $low \leftarrow mid$

Comments

- **Can be used to construct opening books.**
- **Appear to be good for searching certain types of game trees.**
  - **Find the easiest way to prove or disprove a conjecture.**
  - **A dynamic strategy depends on work has been done so far.**
- **Performance has nothing to do with move ordering.**
  - **Performances of most previous algorithms depend heavily on whether good move orderings can be found.**
- **Searching the "easiest" branch may not give you the best performance.**
  - **Performance depends on the value of each internal node.**
- **Commonly used in verifying conjectures, e.g., first-player win.**
  - **Partition the opening moves in a tree-like fashion.**
  - **Try to the "easiest" way to prove or disprove the given conjecture.**
- **Take into consideration the fact that some nodes may need more time to process than the other nodes.**

More research topics

- **Does a variation of a game make it different?**
  - **Whether Stalemate is draw or win in chess.**
  - **Japanese and Chinese rules in Go.**
  - **Chinese and Asia rules in Chinese chess.**
  - **...**
- **Why a position is easy or difficult to human players?**
  - **Can be used in tutoring or better understanding of the game.**

Unique features in games

- **Games are used to model real-life problems.**
- **Do unique properties shown in games help modeling real applications?**
  - **Chinese chess**
    - *Very complicated rules for loops: can be draw, win or loss.*
    - *The usage of cannons for attacking pieces that are blocked.*
  - **Go: the rule of Ko to avoid short cycles, and the right to pass.**
  - **Chinese dark chess: a chance node that makes a deterministic ply first, and then followed by a random toss.**
  - **EWN: a chance node that makes a random toss first, and then followed with a deterministic ply later.**
  - **Shogi: the ability to capture an opponent's piece and turn it into your own.**
  - **Chess: stalemate is draw.**
  - **Promotion: a piece may turn into a more/less powerful one once it satisfies some pre-conditions.**
    - *Chess*
    - *Shogi*
    - *Chinese chess: the mobility of a pawn is increased once it advances twice, but is decreased once it reaches the end of a column.*

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