Two-Player Perfect Information Games: A Brief Survey

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Abstract

- **Domain: two-player games.**
- **Which game characters are predominant when the solution of a game is the main target?**
  - It is concluded that **decision complexity** is more important than **state-space complexity**.
  - There is a trade-off between using **knowledge-based methods** and using **brute-force methods**.
  - There is a clear correlation between the first-player's **initiative** and the necessary effort to solve a game.
  - **Fairness** of a game.
- **A survey of related studies on two-player games.**
  - Mathematical properties.
    - ▷ *Strategy-stealing argument.*
  - Computational results.

Domain of studies

- **Domain: 2-person zero-sum games with perfect information.**
  - **Result: win, loss or draw.**
    - ▷ *Some games, such as EWN, have no draw.*
  - **Zero-sum means one player's loss is exactly the other player's gain, and vice versa.**
    - ▷ *There is no way for both players to win at the same time.*
    - ▷ *By the same token, there is no way for both players to lose at the same time.*
    - ▷ *In these cases, it is usually called a tie[1] or draw[2]*

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[1] "Tie" sometimes means both sides have chance to win.

[2] "Draw" sometimes means both sides have no chance to win.

Positions

- **An arrangement of pieces on the board with an indication of who is the next player.**
- **Sometimes a position includes some historical information such the location of Ko in Go.**
- **Classifications**
  - **Initial position(s)**
  - **Legal position: a position that can be reached from the initial position(s).**
  - **Unreasonable, but legal, positions**
    - ▷ *Example: if facing a position, one can capture the opponent's king and thus wins in Chess, but he makes other move and as a result his king is captured in the next ply.*
    - ▷ *How unreasonable is unreasonable is a question similar to how difficult a position is.*

Game-theoretic value

- **Game-theoretic value** of a game: the outcome, i.e., win, loss or draw, when all participants play optimally.
- Classification of games' solutions according to L.V. Allis [Ph.D. thesis 1994] if they are considered solved.
  - **Ultra-weakly solved**: the game-theoretic value of the initial position has been determined.
  - **Weakly solved**: for the initial position a strategy has been determined to achieve the game-theoretic value against any opponent.
    - ▷ *This strategy is called **optimal**.*
    - ▷ *The strategy must be efficient and practical in terms of resource usage.*
    - ▷ *If the game-theoretical value is win, then the optimal strategy has a role as the first player.*
    - ▷ *If the game-theoretical value is loss, then the optimal strategy has a role as the second player.*
    - ▷ *If the game-theoretical value is draw, then the optimal strategy must be able to **at least** draw any opponent from both roles as the first and the second player.*
  - **Strongly solved**: an optimal strategy has been determined for all legal positions.

Note on game-theoretic value

- **The game-theoretic values of most games are unknown or are only known for some legal positions.**
- **This is one of the most challenging and exciting research areas in games.**

Complexity of a game

- **State-space** complexity of a game: the number of **legal positions** in a game.
  - Often it is difficult to decide whether one can be reached from the initial position or not, instead we use all possible arrangements.
- **Game-tree** (or **decision**) complexity of a game: the number of nodes in a **solution search tree**.
  - Actually, it is usually a game **graph**, not tree.
  - A solution search tree is a tree where the game-theoretic value of the root position can be decided.
  - Each node in the tree is a legal position. The children of a parent node p are the positions that p can reach in one step.
    - ▷ *Some children of a node may not be in a solution search tree.*
    - ▷ *For example, if facing a position, one can capture the opponent's king and thus wins in Chess, but he makes other move and as a result his king is captured in the next ply.*
  - Some legal states may not be in an optimal solution search tree.
    - ▷ *These are **unreasonable** positions.*

Definitions

- **Initiative**: the right to move first.
  - **Many games are known to be favor to the first player.**
    - ▷ *Go-Moku.*
  - **Only very few games are known to be favor to the second player.**
    - ▷ *6 by 6 Othello.*

- **A fair game**: the game-theoretic value is draw and both players have roughly an equal probability to make a mistake.
  - **People normally enjoy playing fair games over unfair ones.**
  - **Examples:**
    - ▷ *Paper-scissor-stone.*
    - ▷ *Roll a dice and the one getting a larger number wins.*
    - ▷ *Nine Men's Morris (proven in 1995).*
    - ▷ *Checkers (proven in 2007).*
  - **Many popular games are not fair or are unknown of their fairness.**
  - **It is difficult to prove a non-trivial game is fair or to design a non-trivial fair one, which is also an exciting research area.**

- **An asymmetric game**
  - **An asymmetric game is one that has different rules for the two players.**
  - **Examples: Renju, Go with a non-zero Komi value.**

More definitions (1/2)

- **A convergent game:** the size of the state space decreases as the game progresses.
  - Start with many pieces on the board and pieces are gradually removed during the course of the game.
    - ▷ *Example: Checkers.*
  - It means the number of possible configurations decreases as the game progresses.
- **A divergent game:** the size of the state space increases as the game progresses.
  - May start with an empty board, and pieces are gradually added during the course of the game.
    - ▷ *Example: Connect-5 before the board is almost filled.*
  - It means the number of possible configurations increases as the game progresses.
    - ▷ *For Chinese chess, a rook can visit more places when it is away from its initial location.*

More definitions (2/2)

- **A game may be convergent at one stage and then divergent at other stage.**
  - Most games are dynamic.
  - For the game of Tic-Tac-Toe, assume you have played x plys where x is even.
    - ▷ *Then you have a possible of*

$$\binom{9}{x/2}\binom{9-x/2}{x/2}$$

  - *different configurations.*
    - ▷ *This number is not monotonously increasing or decreasing.*

Predictions made in 1990

- **Predictions were made in 1990 [Allis et al. 1991] for the year 2000 concerning the expected playing strength of computer programs.**

solved	over champion	world champion	grand master	amateur
Connect-four	Checkers $(8*8)$	Chess	Go $(9*9)$	Go $(19*19)$
Qubic	Renju	Draughts $(10*10)$	Chinese chess	
Nine Men's Morris	Othello		Bridge	
Go-Moku	Scrabble			
Awari	Backgammon			

   ▷ *Over champion means definitely over the best human player.*
   ▷ *World champion means equaling to the best human player.*
   ▷ *Grand master means beating most human players.*

A double dichotomy of the game space

$\log\log(\text{state-space complexity}) \rightarrow$

category 3 if solvable at all, then by knowledge-based methods	**category 4** currently unsolvable by any method
category 1 solvable by any method	**category 2** if solvable at all, then by brute-force methods

$\log\log(\text{game-tree complexity}) \rightarrow$

Questions to be researched

- **Can perfect knowledge obtained from solved games be translated into rules and strategies which human beings can assimilate?**
- **Are such rules generic, or do they constitute a multitude of ad hoc recipes?**
- **Can methods be transferred between games?**
  - More specifically, are there generic methods for all category-i games, or is each game in a specific category a law unto itself?

Convergent games (1/2)

- **Since most games are dynamic, here we consider games whose ending phases are convergent.**
  - Can be solved by the method of **endgame databases** if we can enumerate and store all possible positions at a certain stage.
- **Problems solved (mostly using retrograde analysis):**
  - Nine Men's Morris: in the year 1995, a total of 7,673,759,269 states.
    - ▷ *The game theoretic value is* **draw**.
  - Mancala games
    - ▷ *Awari: in the year 2002 whose game theoretic value is* **draw**.
    - ▷ *Kalah: in the year 2011[3] upto Kalah(6,6) whose game theoretic value is* **first player win**.
  - Checkers
    - ▷ *By combining endgame databases, middle-game databases and verification of opening-game analysis.*
    - ▷ *Solved the so called 100-year position in 1994.*
    - ▷ *The game is proved to be a* **draw** *in 2007.*
    - ▷ *A fair game.*

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[3]http://kalaha.krus.dk/

Convergent games (2/2)

- **More problems solved (mostly using retrograde analysis):**
  - Chess endgames
  - Chinese chess endgames
  - Chinese dark chess endgames
  - $\cdots$

Divergent games

- **Since most games are dynamic, here we consider games whose INITIAL phases are divergent.**
- **Connection games**
  - **Connect-four ($6 * 7$)**
  - **Qubic ($4 * 4 * 4$)**
  - **Go-Moku ($15 * 15$)**
  - **Renju**
  - **k-in-a-row games**
  - **Hex ($10 * 10$ or $11 * 11$)**
- **Polyomino games: place pieces inside a board without overlapping and alternatively until one cannot place more.**
  - **Pentominoes**
  - **Domineering**
- **Othello**
- **Chess**
- **Chinese chess**
- **Shogi**
- **Go**

Connection games (1/2)

- **Connect-four ($6 * 7$)**
  - Solved by J. Allen in 1989 using a brute-force depth first search with alpha-beta pruning, a transposition table, and killer-move heuristics.
  - Also solved by L.V. Allis in 1988 using a knowledge-based approach by combining 9 strategic rules that identify potential threats of the opponent.
    - ▷ *Threats are something like forced moved or moves you have little choices.*
    - ▷ *Threats are moves with predictable counter-moves.*
  - It is first-player-win.
  - Weakly solved on a SUN-4 workstation using 300+ hours.
- **Qubic ($4 * 4 * 4$)**
  - A three-dimensional version of Tic-Tac-Toe.
  - Connect-four played on a $4 * 4 * 4$ game board.
  - Solved in 1980 by O. Patashnik by combining the usual depth-first search with expert knowledge for ordering the moves.
    - ▷ *It is first-player-win for the 2-player version.*

Connection games (2/2)

- **Go-Moku ($15 * 15$)**
  - First-player-win.
  - Weakly solved by L.V. Allis in 1995 using a combination of threat-space search and database construction.
- **Renju**
  - Does not allow the first player to play certain moves.
  - An **asymmetric** game.
  - Weakly solved by Wágner and Viráag in 2000 by combining search and knowledge.
    - ▷ *Took advantage of an iterative-deepening search based on threat sequences up to 17 plies.*
    - ▷ *It is still first-player-win.*
- **k-in-a-row games**
  - mnk-**Game:** a game playing on a board of m rows and n columns with the goal of obtaining a straight line of length k.
  - Variations: first ply picks only one stone, the rest of the plys pick two stones at one time.
    - ▷ *Connect6.*
    - ▷ *Try to balance the advantage of the initiative!*

Hex ($10 * 10$ or $11 * 11$)

- **Invented in 1942 by Peit Hein and John Hash (source: wiki)**
- **Rules**
  - Two players place one piece of each one's color alternatively.
  - A player connects the NW and SE sides, while the other one connects the NE and SW sides.
  - One who can do the assigned connection wins.
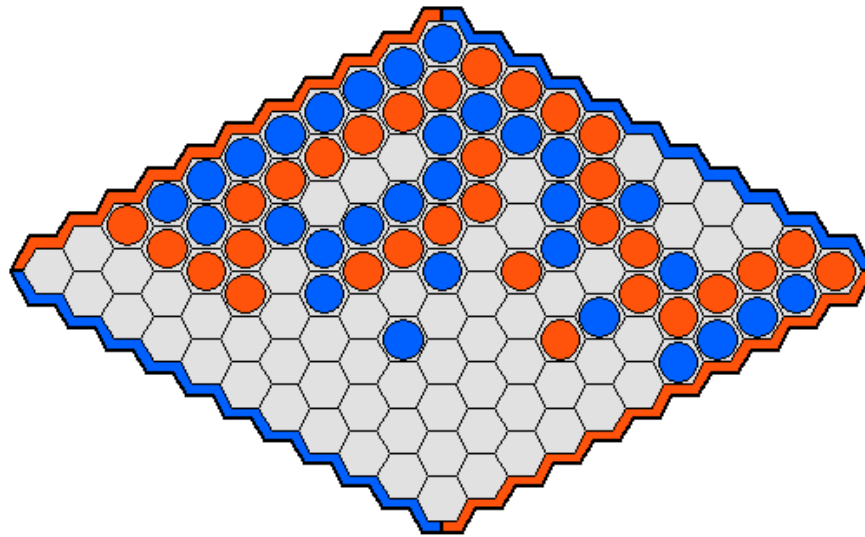
Red won

Courtesy of ICGA web site

HEX: properties

- **Revised rules:**
  - ▷ *When the first player wins, allow the second player to play one more time. If the second player also wins, then the game is tie.*
  - ▷ *When the board is full and no one wins, then it is a draw.*

- **Properties:**
  - **Property 0: It is a finite game.**
    - ▷ *Trivially true.*
  - **Property 1: It is impossible for both players to win at the same time.**
    - ▷ *No chance of tie.*
  - **Property 2: There is at least one winner.**
    - ▷ *No chance of draw.*
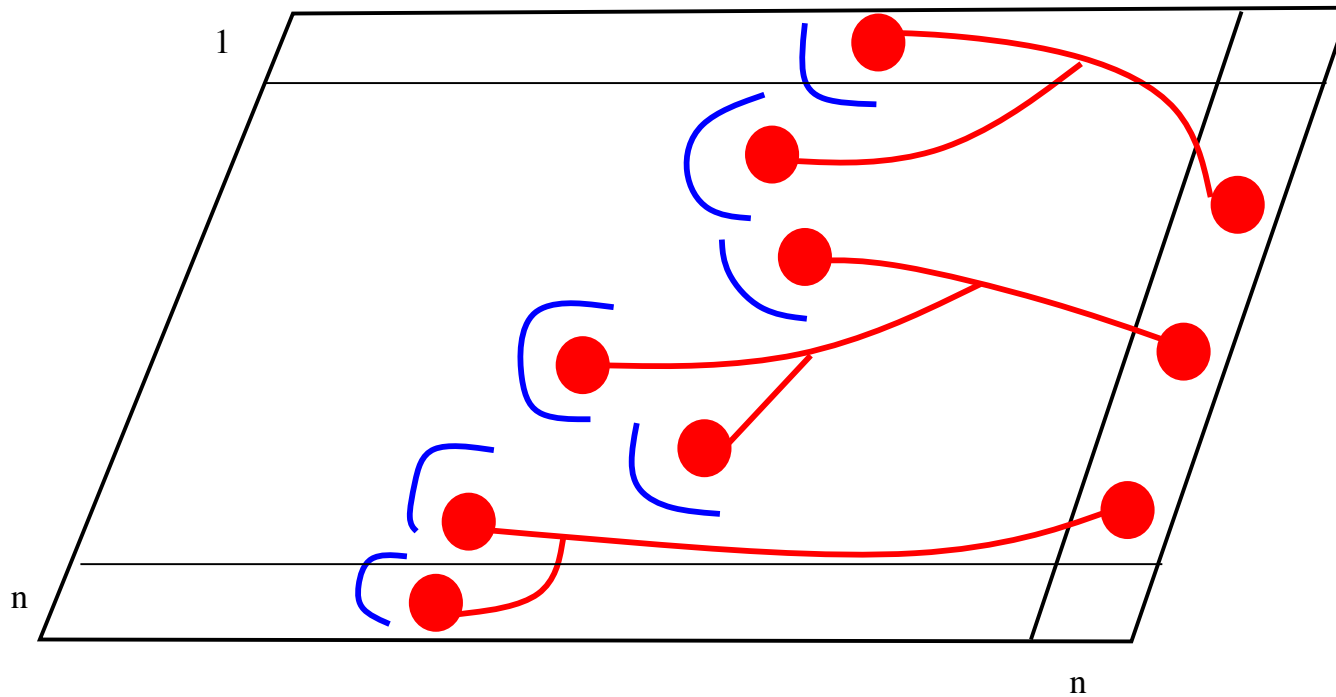 - **Property 3: It is a first-player-win game.**

It cannot have two winners

- **Property 1: there cannot be two winners.**
- **A topological argument.**
  - **A vertical chain can only be cut by a horizontal chain and vice versa because each cell is connected with 6 adjacent cells.**
    - ▷ *Note if a cell has 4 neighbors as in the case of Go, then it is possible to cut off a vertical chain by cells that are not horizontally connected and vice versa.*

- **Other arguments such as one using graph theory exist.**

There is at least one winner

- **Property 2: there is at least one winner.**
- **Assume there is no winner when the board is full.**
- **W.l.o.g. let R be the set of red cells that can be reached by chains originated from the rightmost column.**
  - R **must contain a cell of the rightmost column; otherwise we have a contradiction which means the blue wins.**
- **Let $N(R)$ be the blue cells that can be reached by R originated from the rightmost column.**
  - $N(R)$ **must contain a cell in the top row.**
    - ▷ *Otherwise, R contains all cells in the first row, which is a contradiction.*
  - $N(R)$ **must contain a cell in the bottom row.**
    - ▷ *Otherwise, R contains all cells in the bottom row, which is a contradiction.*
  - $N(R)$ **must be connected.**
    - ▷ *Otherwise, R can advance further.*
  - **Hence $N(R)$ is a blue winning chain.**
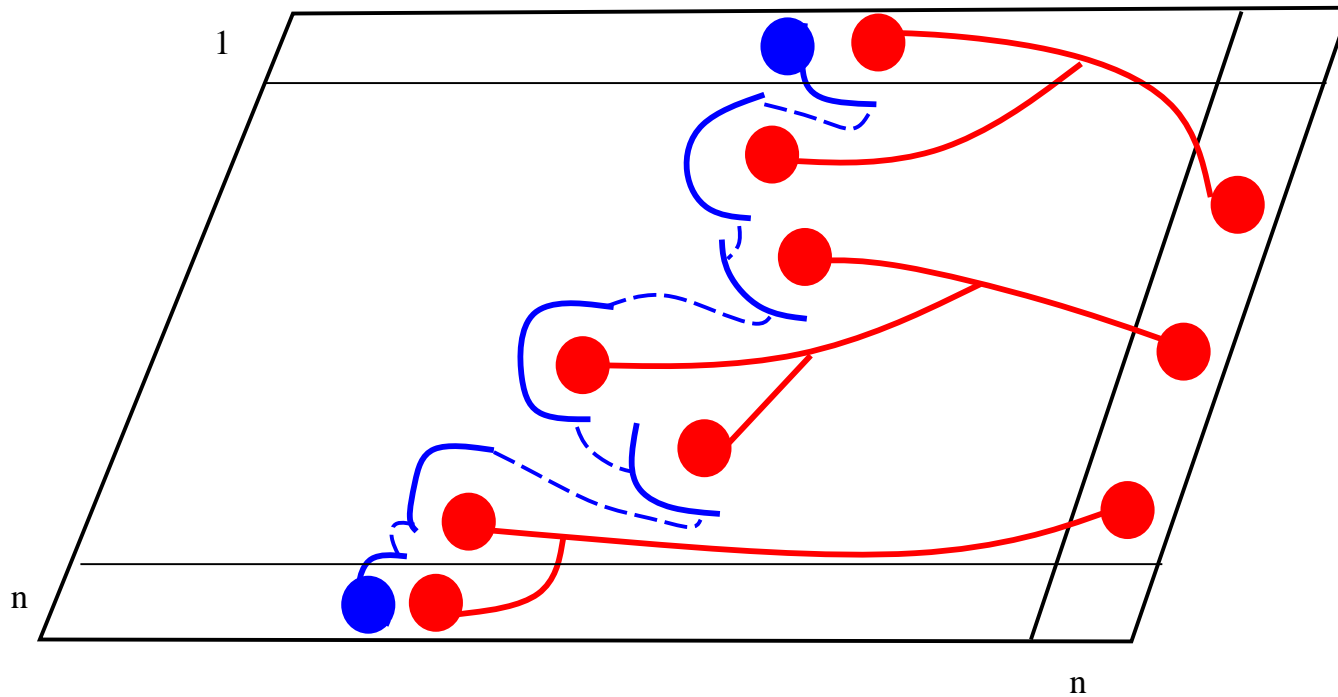- **Corollary: only a chain of a direction can stop a chain of the other direction.**

Illustration of the ideas (1/3)

Strategy-stealing argument

- **Property 3: The *unrestricted form* of Hex is a first-player-win game.**
- **Proof is constructed using the "strategy-stealing" argument made by John Nash in 1949.**
  - **If there is a winning strategy for the second player, the first player can still win by making an arbitrary first move and using the second-player strategy from then on.**
    - ▷ *The first player ignores the arbitrary first move by assuming that move does not exist.*
    - ▷ *Hence the second move made by the second player becomes the first move.*
    - ▷ *The third move made by the first player becomes the second move.*
  - **If using the second-player strategy requires playing the chosen first move or any move played before, then make another arbitrary move.**
    - ▷ *An arbitrary extra move can never be a disadvantage in Hex.*
  - **We have obtained a contradiction, and thus the second player cannot win from the initial empty board.**
  - **Since we have proved there is no draw, and there is always a winner, and both players cannot win at the same time, the first player must have a winning strategy from the initial empty board.**

Strategy-stealing argument: proof (1/3)

- **Assume the second player P_2 has a winning function $f(B)$ that tells the next ply towards winning when seeing the board B.**
  - **Assume the initial board position is B_0 which is an empty board.**
  - **$f(B)$ has a value only for the case B is a legal position for the second player.**
    - ▷ *$f(B)$ returns the x-y coordinates of a location and the color of the piece to play.*
  - **$rev(m)$: flip the color and coordinate of a ply m.**
    - ▷ ***Let $m = (x_m, y_m)$ be the location to play.***
    - ▷ ***Let c be the color of the piece to play.***
    - ▷ ***Let $\bar{c}$ be the color flipped.***
    - ▷ ***Return the location (y_m, x_m) and the color $\bar{c}$.***

Strategy-stealing argument: proof (2/3)

- **The steps taken by the first player P_1 to also win:**
  - P_1 **makes an arbitrary 1st ply** m_1. **Call it** m^*.
  - P_2 **makes the 2nd ply** $m_2 = f(B_0 + m_1)$.
  - P_1 **makes the 3rd ply** $m_3 = rev(f(B_0 + rev(m_2)))$.
    - ▷ *If $m_3 = m^*$, then make another arbitrary ply and let it be the new m^*.*
  - P_2 **makes the 4th ply** $m_4 = f(B_0 + m_1 + m_2 + m_3)$.
  - P_1 **makes the 5th ply** $m_5 = rev(f(B_0 + rev(m_2) + rev(m_3) + rev(m_4)))$.
    - ▷ *If $m_5 = m^*$, then make another arbitrary ply and let it be the new m^*.*
  - P_2 **makes the 6th ply** $m_6 = f(B_0 + m_1 + m_2 + m_3 + m_4 + m_5)$.
  - $\cdots$

Strategy-stealing argument: proof (3/3)

- **Hence we know the first player also has a winning connection.**
- **We also know the followings.**
  - **The game is finite and deterministic.**
  - **There is at least one winner before or when the board is completely filled.**
  - **There cannot be two winners.**
  - **Hence we can enumerate the whole solution search tree.**
    - ▷ *In this solution search tree, there is a way for one player to win all of the times no matter what the opponent reacts.*

- **Thus the assumption of the second player having a winning strategy f is not valid.**

Strategy-stealing argument: comments

- **This is not a constructive proof.**
- **It only shows the first player has a winning strategy from the initial empty board, not from an arbitrary position.**
- **The strategy-stealing argument may not be good for other games.**
- **The argument works for any game when**
  - there is a way for the first player not to lose at the first ply,
  - the second player cannot win at the second ply,
  - it is symmetric,
  - it is **history independent**,
  - it always has exactly one winner, and
    - ▷ *namely, it cannot have a draw by having no winner or two winners,*
  - **an arbitrary extra move can always be made and can never be a disadvantage.**
    - ▷ *Note: it requires that a player is always possible to place an arbitrary move which may not be true for some games.*

Properties of Hex

- **Variations of Hex**
  - The **one-move-equalization** rule: one player plays an opening move and the other player then has to decide which color to play for the reminder of the game.
    - ▷ *The revised version is a second-player-win game (ultra-weakly).*

- **Hex exhibits considerable mathematical structure.**
  - Hex in its general form has been proved to be **PSPACE**-complete by Even and Tarjan in 1976 by converting it to a Shannon switching game.
  - The state-space and decision complexities are comparable to those of Go on an **equally-sized** board.

- **Solutions**
  - (Weakly or strongly) solved on a $6*6$ board in 1994.
  - Maybe possible to solve the $7*7$ case.
    - ▷ *The $7*7$ case was solved in 2001. [Yang et al. 2001]*
  - Not likely to solve the $8*8$ case without fundamental breakthroughs.
    - ▷ *The $8*8$ case was solved in 2009. [Henderson et al. 2009]*
  - The $9*9$ case was solved in 2013. [Pawlewicz et al. 2013]
  - The most-central opening was solved for the $10*10$ case in 2014. [Pawlewicz et al. 2013]

More divergent games (1/3)

- **Polyomino games: placing 2-D pieces of a connected subset of a square grid to construct a special form.**
  - **Pentominoes**
  - **Domineering**
  - **Games on smaller boards have been solved.**
- **Othello**
  - **M. Buro's LOGISTELLO beat the resigning World Champion by 6-0 in 1997.**
  - **Weakly solved on a $6 * 6$ board by J. Feinstein in 1993.**
    - ▷ *Second-player-win*
- **Chess**
  - **DEEP BLUE beat the human World Champion in 1997!**

More divergent games (2/3)

- **Chinese chess**
  - Professional 7-dan since 2007.
  - Still in progress, but lack of reporting recent advancement due to complications facing the using of deep learning based approaches.
- **Shogi**
  - Claimed to be professional 2-dan in 2007.
  - Defeat a lady professional player in 2010.
  - Defeat a 68-year old 1993 Meijin during 2011 and 2012.
  - Still in progress, but lack of reporting recent advancement due to complications facing the using of deep learning vased approaches.

More divergent games (3/3)

- **Go**
  - **5 by 5 Go was solved in 2002.**
    - ▷ *First player wins and takes all cells using 22 plys.*
  - **Recent success and breakthrough using Monte Carlo UCT based methods between 2004 and 2012.**
  - **Amateur 1 − 4 kyu in 2008.**
    - ▷ *Beat a professional 8-dan by having an 8-stone advantage.*
    - ▷ *Beaten by a professional 9-dan by giving a 7-stone advantage.*
  - **Amateur 1 dan in 2010.**
  - **Amateur 3 dan in 2011.**
  - **The program Zen beat a 9-dan professional master at March 17, 2012.**
    - ▷ *First game: Five stone handicap and won by 11 points.*
    - ▷ *Second game: four stones handicap and won by 20 points.*
  - **Go (19 by 19) is now over human: AlphaGo beat a human top player by a margin of 4:1 at March 2016, and beat the human top player by 3:0 at May 2017.**

Table of complexities

Game	$\log_{10}$(state-space)	$\log_{10}$(game-tree size)
Nine Men's Morris	10	50
Pentominoes	12	18
Awari	12	32
Kalak(6,4)	13	18
Connect-four	14	21
EinStein würfelt nicht!	15	22
Domineering $(8 * 8)$	15	27
Dakon-6	15	33
Checkers	21	31
Othello	28	58
Qubic	30	34
Draughts	30	54
Chinese Dark Chess	37	135
Chess	46	123
Chinese chess	48	150
Hex $(11 * 11)$	57	98
Shogi	71	226
Renju $(15 * 15)$	105	70
Go-Moku $(15 * 15)$	105	70
Connect6 $(19 * 19)$	172	70
Go $(19 * 19)$	172	360

State-space versus game-tree size

- **In 1994, the boundary of solvability by complete enumeration was set at 10^{11}.**
  - The current estimation is about 10^{13} (since the year 2007).
- **It is often possible to use heuristics in searching a game tree to cut the number of nodes visited tremendously when the structure of the game is well studied.**
  - Example: Connect-Four.
  - Good heuristics for some games are easier to design than the others.

Remarks

- **With the same complexities, a stochastic game is usually more interesting and challenging than a deterministic game.**
  - Example 1: EinStein würfelt nicht! (EWN) is definitely more challenging than Connect Four.
  - Example 2: Chinese Dark Chess (CDC) is more interesting than Chess.
- **Rules of playing games also affect the fundamental properties of games.**
  - Example: in Shogi you own the captured pieces and may turn them into yours in later plays.

Methods developed for solving games

- **Brute-force methods**
  - Retrograde analysis
  - Enhanced transposition-table methods
- **Knowledge-based methods**
  - Threat-space search and λ-search
  - Proof-number search
  - Depth-first proof-number search
  - Pattern search
    - ▷ *To search for **threat patterns**, which are collections of cells in a position.*
    - ▷ *A threat pattern can be thought of as representing the **relevant area** on the board, an area that human players commonly identify when analyzing a position.*

- **Recent advancements:**
  - Monte Carlo UCT based game tree simulation.
    - ▷ *Monte Carlo method has a root from statistic.*
    - ▷ *Biased sampling.*
    - ▷ *Using methods from machine learning.*
    - ▷ *Combine domain knowledge with statistics.*
  - Combine searching with deep learning.

Brute-force versus knowledge-based methods

- **Games with both a relative low state-space complexity and a low game-tree complexity have been solved by both methods.**
  - **Category 1**
  - Connect-four and Qubic
- **Games with a relative low state-space complexity have mainly been solved with brute-force methods.**
  - **Category 2**
  - Namely by constructing endgame databases
  - Nine Men's Morris
- **Games with a relative low game-tree-complexities have mainly been solved with knowledge-based methods.**
  - **Category 3**
  - Namely, by intelligent (heuristic) searching
  - Sometimes, with the helps of endgame databases
  - Go-Moku, Renju, and k-in-a-row games

Advantage of the initiative

- **Theorem (or argument) made by Singmaster in 1981: The first player has advantages.**
  - **Two kinds of positions**
    - ▷ P-positions: the previous player can force a win.
    - ▷ N-positions: the next player can force a win.
  - **Arguments**
    - ▷ For the first player to have a forced win, just one of the moves must lead to a P-position.
    - ▷ For the second player to have a forced win, all of the moves must lead to N-positions.
    - ▷ It is easier to the first player to have a forced win **assuming all positions are randomly distributed**.
    - ▷ Can be easily extended to games with draws.

- **Remarks:**
  - **On small boards, the second player is able to draw or even to win for certain games.**
  - **Cannot be applied to an infinite board.**
  - **The assumption of all positions are randomly distributed may not be true for all games.**

How to make use of the initiative

- **A potential universal strategy for winning a game:**
  - **Try to obtain a small advantage by using the initiative.**
    - ▷ *The opponent must react adequately on the moves played by the other player.*
  - **To reinforce the initiative the player searches for threats, and even a sequence of threats using an evaluation function E.**
  - **Force the opponent to always play the moves you expected.**
- **Threat-space search**
  - **Search for threats only!**

Offsetting the initiative

- **An example of a game with a huge initiative:**
  - **A connection $mn1$-game.**
    - ▷ *Connect-1 was mentioned in a famous Sci-fi novel published 1979.*
  - **A connection $mn2$-game.**
  - **A connection $mn3$-game.**
  - **For a connection mni-game, you can have a feeling that the advantage given to the first player through initiative is gradually lessened when i gets larger.**
- **Need to offset the initiative.**
  - **The offsetting rule must be simple.**
  - **The revised game must be as fair as possible.**
    - ▷ *It is difficult to prove a game is fair.*
    - ▷ *Example: Paper-scissor-stone is fair.*
  - **The revised game needs to be fun to play with.**
  - **The revised game cannot be too much different from the original game.**
- **Knowing how to properly offsetting the initiative may uncover some fundamental properties of the game such as its level of difficulty.**

Examples (1/2)

- **Enforce rules so that the first player cannot win by selective patterns.**
  - **Renju.**
    - ▷ *Still first-player-win.*
  - **Go ($19 * 19$).**
    - ▷ *The first player must win by more than 7 stones.*
    - ▷ *Komi = 7.5 in 2011.*
    - ▷ *The value of Komi changes with the time and may be different because of using different set of rules.*

- **The one-move-equalization rule: one player plays an opening move and the other player then has to decide which color to play for the reminder of the game.**
  - ▷ *Hex*
  - ▷ *Second-player-win if Hex uses this variant*

Examples (2/2)

- **The first move plays one stone, the rest plays two stones each.**
  - ▷ *Connect6.*
  - ▷ *Intuitively, in each turn the initiative goes to different players alternatively.*
  - ▷ *Still not able to prove the game is fair (upto the year 2021).*

- **The first player uses less resources.**
  - • **For example: using less time.**
    - ▷ *Chinese chess.*
  - • **A resource-auctioning scheme.**

- **General rules to redesign a game to make it fair are unknown which makes a very good and challenging research topic.**

Conclusions

- **The knowledge-based methods mostly inform us on the structure of the game, while exhaustive enumeration rarely does.**
- **Many ad-hoc recipes are produced currently.**
  - The database can be used as a corrector or verifier of strategies formulated by human experts.
- **Monte-Carlo searching technique with deep learning seems to open a new avenue of researching.**
- **It may be hopeful to use data mining techniques to obtain cross-game methods.**
  - Currently not very successful.
  - It is noted that Alpha Zero claimed on having a universal solution by using techniques from deep learning.

Comments

- **Can combine knowledge-based method with exhaustive enumeration.**
  - **For converging games, build endgame databases when the remaining state spaces is manageable.**
    - ▷ *Example: build endgames with at most 5 pieces in Chess and stop searching when the number of pieces on the board is less than 6.*
  - **For diverging games, pre-compute all possible opening moves and solve them one by one in sequence or in parallel.**
- **This is different from the usage of pattern databases in solving one-player games.**
  - **Patterns are used to guide the search in solving one-player games.**
  - **Endgame databases are used here to stop the search earlier which has a flavor like that of bi-directional search.**

- **Predictions were made in 1990 [Allis et al. 1991] for the year 2000 concerning the expected playing strength of computer programs.**

solved	over champion	world champion	grand master	amateur
Connect-four	**Checkers** $(8 * 8)$	**Chess**	**Go** $(9 * 9)$	**Go** $(19 * 19)$
Qubic	**Renju**	**Draughts** $(10 * 10)$	**Chinese chess**	
Nine Men's Morris	**Othello**		**Bridge**	
Go-Moku	**Scrabble**			
Awari	**Backgammon**			

- **Very successful in using exhaustive enumeration.**
- **color code**
  - **Green: Performs much better than expected**
  - **Red: right on the target.**
  - **Black: have some progress towards the target.**
  - **Blue: not so.**

Predictions for 2010

- **Predictions were made at the year 2000 for the year 2010 concerning the expected playing strength of computer programs.**

solved	over champion	world champion	grand master	amateur
Awari	Chess	Go $(9 * 9)$	Bridge	Go $(19 * 19)$
Othello	Draughts $(10 * 10)$	Chinese chess	Shogi	
Checkers $(8 * 8)$	Scrabble	Hex		
	Backgammon	Amazons		
	Lines of Action			

Predictions for 2010 − Status

- My personal opinion about the status of Prediction-2010 at October, 2010, right after the Computer Olympiad held in Kanazawa, Japan.

solved	over champion	world champion	grand master	amateur
Awari	Chess	Go $(9 * 9)$	Bridge	Go $(19 * 19)$
Othello	Draughts $(10 * 10)$	Chinese chess	Shogi	
Checkers $(8 * 8)$	Scrabble	Hex		
	Backgammon	Amazons		
	Lines of Action			

- Successful in using knowledge based methods.
- color code
  - Red: right on the target.
  - Black: have some progress towards the target.
  - Blue: not so.

Concluding remarks

- **Very diverse development since year 1940 until 2010. New methods are techniques are discovered every couple of years.**
  - Accompany the development of Computing Science.
- **Between years 2010 and 2020, a lot of progress us achieved in Go-related research.**
  - Mostly using deep learning techniques.
  - Mostly on better winning games.
  - Not much is done with regard to pure game-theoretical researches.
- **A better link of game related research and bettering human life is needed.**

References and further readings (1/2)

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