Short Paper

Uni-Directional Alternating Group Graphs

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A class of uni-directional Cayley graphs based on alternating groups is proposed in this paper. It is shown that this class of graphs is strongly connected and recursively scalable. The analysis of the shortest distance between any pair of nodes in a graph of this class is also given. Based on the analysis, we develop a polynomial time routing algorithm which yields a path distance at most one more than the theoretic lower bound. Furthermore, comparisons among uni-directional hypercubes, uni-directional star graphs, and uni-directional alternating group graphs are given. These observations validate the superiority of uni-directional alternating group graphs among known uni-directional topologies.

Keywords: interconnection networks, uni-directional graphs, Cayley graphs, massive parallel computers, alternating group graphs

1. INTRODUCTION

Due to rapid advances in large *interconnection networks*, such as *massive parallel computers*, the search for large uni-directional graphs with fairly nice topological and symmetric properties has received much attention in the literature recently [5-9, 11, 16, 17]. Although most graphs studied as interconnection network models for parallel computers are un-directed, un-directed links in such models is normally realized by two directed links in practice. A generic architecture of a node in parallel computers is shown in Fig. 1, where each switch supports direct-in and direct-out channels. Usually a crossbar is used to allow all possible connections between input and output channels within the switch. Thus, the search for uni-directional graphs which possess properties similar to those of their un-directed counterparts is an immediate and meaningful design consideration. Other applications of uni-directional graphs include the design of *high speed networks* for *high performance computing* [17, 18].

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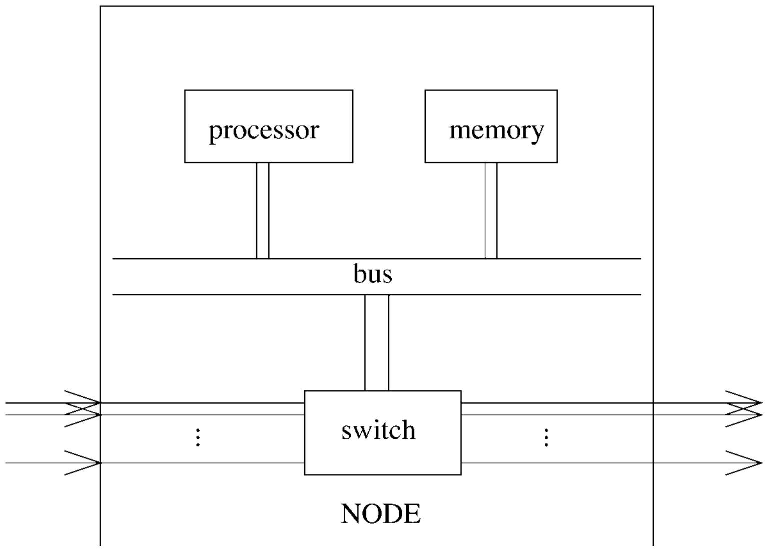

Fig. 1. A generic node architecture.

A class of graphs called *group* graphs or *Cayley* graphs provide a very natural and rich framework for interconnection networks. *Hypercubes* [13, 15], *star* graphs [1, 2] and *alternating group* graphs [12] are examples of Cayley graphs. Recently, Chou and Du [7] proposed a two different schemes to define the orientations of the edges of a hypercube. These two uni-directional schemes are called $UHC1_n$ and $UHC2_n$, respectively. Day and Tripathi [8] proposed a scheme called US_n to define the orientation of edges in a star graph. In this paper, our aim is to propose a uni-directional scheme for alternating group graphs [12] and to explore its properties. In addition, we shall compare some properties of the uni-directional alternating group graph with those of $UHC1_n$, $UHC2_n$ and US_n.

2. THE UNI-DIRECTIONAL MODEL

Let $\langle n\rangle = \{1, 2,...., n\}$ and p be a permutation, such that $p = p_1 p_2....p_n$, where $p_i \in \langle n\rangle$ for all $1 \leq i \leq n$. Furthermore, let I denote the identity permutation of $\langle n\rangle$. If $p_i > p_j$ where $i < j$, the pair p_i and p_j constitute an *inversion*. A permutation is said to be *even* (resp. *odd*) if its number of inversions is even (resp. *odd*). A well known fact regarding permutation representation is that p can be represented by its *cycle* structure, i.e., $p = c_1c_2...c_ke_1e_2....e_l$, where c_i is a nontrivial cycle of length $|c_i| \geq 2$, for $1 \leq i \leq k$ and e_i is *invariant*, i.e., $|e_i| = 1$, for $1 \leq i \leq l$ [3]. As an example, consider the permutation $p = 3\ 1\ 2\ 4\ 6\ 5$. The cycle structure of p is (1 3 2) (4) (5 6), where (4) is an invariant.

Define an associative, binary operation (called a product) as $p \cdot q\ (x) = p(q(x))$. Let g_i be a permutation with only a cycle such that $g_i = (1\ 2\ i)$, $3 \leq i \leq n$. Define $\Omega = \{g_i \mid 3 \leq i \leq n\}$, and let each g_i in Ω be called a *generator* [3]. A *uni-directional* n-dimensional alternating group graph, denoted by $UAG_n(V_n, E_n)$, has the vertex set V_n of all the even permutations of $\langle n\rangle$ and the edge set $E_n = \{(p,q) \mid p, q \in V_n, q = p \cdot h$, for some $h \in \Omega\}$. Fig. 2 gives two examples of

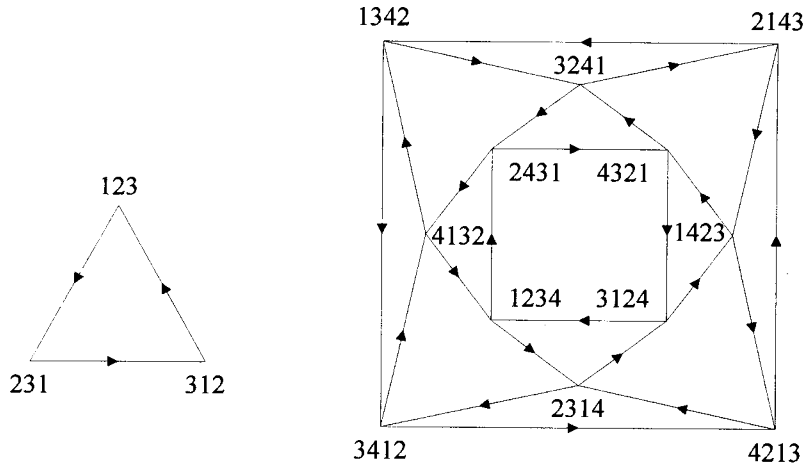

Fig. 2. (a) UAG_3 and (b) UAG_4.

UAG_n, where $n = 3$ and $n = 4$, respectively. Clearly, each edge in UAG_n is embedded in a circuit of length three. Thus if a packet is mis-routed from vertex *a*, it can always return to *a* in three hops.

The following Lemma is given without proof.

Lemma 1: UAG_n is strongly connected for $n \geq 3$.

Given a graph $G(V, E)$, an *automorphism* of *V* is a permutation α of *V* satisfying the property that if $(u, v) \in E$, then $(\alpha(u), \alpha(v)) \in E$. Observe that the set of all automorphisms of *V* forms a group under "**.**", the function composition operator. Furthermore, *G* is *vertex symmetric* if and only if, for $u, v \in V$, there exists an automorphism α such that $\alpha(u) = v$.

Lemma 2: UAG_n is vertex symmetric.

Recall that each vertex in V_n is an even permutation of $\langle n \rangle$. For $1 \leq i \leq n$, let $V_n^i = \{p_1 p_2 \cdots p_n \in V_n | p_n = i\}$. Consider the subgraph of UAG_n induced by V_n^i, denoted by UAG_n^i, whose edge set is represented by E_n^i. Without loss of generality, consider any two subgraphs UAG_n^i and UAG_n^j. Let $x \in V_n^i$ and $y \in V_n^j$ such that $x = p_1p_2...p_k\, j\, p_t...i$ and $x = p_2p_1...p_k\, i\, p_t...j$. Define an isomorphism $h(x) = y$. It can be easily verified that if $(p, q) \in E_n^i$ then $(h(p), h(q)) \in E_n^j$. Furthermore, it is obvious that by removing the last digit *n* from all the nodes in UAG_n^n the resulting graph is isomorphic to UAG_{n-1}. Thus, the following Lemma guarantees that UAG_n is recursively scalable.

Lemma 3: UAG_n can be decomposed into *n* subgraphs, namely, UAG_n^1, UAG_n^2, $\ldots UAG_n^n$ each of which is isomorphic to UAG_{n-1}.

3. DESIGN AND ANALYSIS OF THE ROUTING ALGORITHM

For $u,v \in V_n$, it is clear that $(u, v) \in E_n$ if and only if $(I, u^{-1} \cdot v) \in E_n$. By extending the notion of an edge in UAG_n, we can see that if $v = u \cdot h_1 \cdot h_2...h_t$, where $h_i \in \Omega$ for $1 \leq i \leq t$, then $I = v^{-1} \cdot u \cdot h_1 \cdot h_2....h_t$. Therefore, the path from u to v is equivalent to the path from v_u^{-1} to I. For $p = v^{-1} \cdot u = p_1 p_2...p_n \in V_n$, it is our ultimate goal to find a sequence of generators drawn from Ω so that each p_i in p will eventually be migrated to the p_i-th position in I. This process essentially sorts p_i in p to the right (i. e., p_i-th) position.

Lemma 4: Let $p \in V_n$. 1 and 2 in p will automatically be sorted when all the other entries in p are sorted.

Given $p_1 p_2....p_n \in V_n$ with some i satisfying $3 \leq i \neq p_i \geq 3$, we can easily see that it takes three steps to move p_i back to the right position. Assume that $(r_0\, r_1...r_{x\text{-}1})$ and $(s_0\, s_1....s_{y\text{-}1})$ are two cycles of p containing neither 1 nor 2. In order to sort each and every element in the first cycle, we first apply the generator $g_{r_{(i\text{-}1)\ mod\ x}}$ to move r_i, where $0 \leq i \leq x - 1$, to the second position. Then, for the second cycle, we apply $g_{s_{(j\text{-}1)\ mod\ y}}$ to move s_j, where $0 \leq j \leq y - 1$, to the second position, and r_i is accordingly moved to the first position. Now, applying the corresponding generator g_{r_i} will sort r_i, we move s_j and r_{i+1} to the first and second positions, respectively. Next, we apply g_{s_j} to sort s_j, and move r_{i+1} and s_{j+1} to the first and second positions, respectively. The number of steps needed to sort all the elements in either cycle (of interest) by alternating the above process between the two cycles is at least one more than the respective cycle length. Thus, the following lemma is immediately obtained.

Lemma 5: If cycle $c = (t_1\, t_2...t_r)$ does not contain 1 or 2, then at least $r + 1$ steps are needed to sort all the elements in c back to their correct positions.

Suppose each nontrivial cycle is associated with a weight which is one more than its cycle length. Following the above development, the shortest distance between any two nodes can be obtained by first partitioning the nontrivial cycles into 2 groups such that the total weight in one group is as close to the total weight of the other group as possible, and by then alternating the sorting process discussed in the previous paragraph between these 2 groups. The former step is the weighted set partition problem, which is NP-complete (p. 223, [10]). Before giving a heuristic to efficiently and effectively perform obtain the partition, we shall explore the issue of the shortest distance between two nodes p and I in UAG_n a bit further. Let $p = c_1... c_k\, e_1...e_l$, and let D_p denote the shortest distance from p to I.

1. $p = 1\ 2\ p_3...p_n$, and each cycle c_i is associated with a number $l_i = |c_i| + 1$. It suffices to study the following three cases for dealing with the problem of partitioning $\{l_1, l_2,..., l_k\}$ into two sets A and B such that $\Sigma_{l_i \in A}\, l_i \geq \Sigma_{l_i \in B}\, l_i$.
   (a) $\Sigma_{l_i \in A}\, l_i - \Sigma_{l_i \in B}\, l_i = 0$: $D_p = \Sigma_{l_i \in A \cup B}\, l_i = (n - \ell) + k = n + k - \ell$.
   (b) $\Sigma_{l_i \in A}\, l_i - \Sigma_{l_i \in B}\, l_i = 1$: Since p is an even permutation, this case cannot occur.
   (c) Otherwise: It takes one step more than case (1)(a) does. That is, $D_p = (n + k - \ell + 1)$.
2. $p = 2\ 1\ p_3...p_n$, and $c_1 = (1\ 2)$. As before, let $l_i = |c_i| + 1$, for $1 \leq i \leq k$. It suffices to study the following 3 cases for dealing with the problem of partitioning $\{l_2, l_3,..., l_\ell\}$ into two sets A and B such that $\Sigma_{l_i \in A}\, l_i \geq \Sigma_{l_i \in B}\, l_i$. Consider the following three cases:

(a) $\Sigma_{l_i \in A} l_i - \Sigma_{l_i \in B} l_i = 0$: The number of even length cycles is even. Since cycle c_1 will be automatically sorted, the number of odd length cycles is odd in A and B. This case will not occur.

(b) $\Sigma_{l_i \in A} l_i - \Sigma_{l_i \in B} l_i = 1$: $D_p = (n - \ell - 2) + (k - 1) = n + k - \ell - 2$.

(c) Otherwise: Using the same argument as for case (1)(c), $D_p = (n - \ell - 2) + (k - 1) + 1 = n + k - \ell - 2$.

3. $p = 1\, p_2\, p_3 ... p_{j-1}\, 2\, p_{j+1} .... p_n$ and $c_1 = (2\, p_2 ... j)$. Define $l_i = |c_i| + 1$ for $2 \le i \le l$ and $l_1 = |c_1| - 1$. It suffices to study the following 3 cases for dealing with the problem of partitioning $\{l_1, l_2, l_3, ..., l_\ell\}$ into two sets A and B such that $\Sigma_{l_i \in B} l_i \ge \Sigma_{l_i \in A} l_i$ and l_1 is in A. Consider the following three cases:

(a) $\Sigma_{l_i \in A} l_i - \Sigma_{l_i \in B} l_i = 0$: $D_p = \Sigma_{l_i \in A \cup B} l_i = (n - \ell - 1) + (k - 1) = n + k - \ell - 2$.

(b) $\Sigma_{l_i \in B} l_i - \Sigma_{l_i \in A} l_i = 1$: It can be seen that this case can not occur.

(c) Otherwise: Using the same argument as for case (1)(c), $D_p = (n - \ell - 1) + (k - 1 + 1) = n + k - \ell - 1$.

4. Let $p = p_1\, 2\, p_3 ... p_{j-1}\, 1\, p_j + 1 ... p_n$ and $c_1 = (1\, p_1\, p_i ... j)$. Move p_1 back to the correct position. Then, the case can be :

(a) $2\, 1\, p_3 ... p_{j-1}\, p_1\, p_{j+1} ... p_n$. if $p_1 = j$. In this case, the analysis is similar to that for case (2).

(b) $2\, p_i ... p_{j-1}\, 1\, p_{j+1} ... p_n$. if $p_1 \ne j$. The analysis of this case is similar to that for case (3).

5. Let $p = 2\, p_2 ... p_{j-1}\, 1\, p_{j+1} ... p_n$ and c_1 $(1\, 2\, p_1\, p_i ... j)$. Following the same argument as for case (3), let $l_1 = |c_1| - 2$. The following discussion is straightforward:

(a) $\Sigma_{l_i \in A} l_i - \Sigma_{l_i \in B} l_i = 0$: This case will not occur.

(b) $\Sigma_{l_i \in B} l_i - \Sigma_{l_i \in A} l_i = 1$: $D_p = (n - \ell - 2) + (k - 1) = n + k - \ell - 3$.

(c) Otherwise: $D_p = (n - \ell - 2) + (k - 1 + 1) = n + k - \ell - 2$.

6. Let $p = p_1\, p_2 ... p_{i-1}\, 1\, p_{i+1} ... p_{j-1}\, 2\, p_{j+1} ... p_n$ and $c_1 = (1\, \underbrace{p_1 \cdots j}_{m_1}\, 2\, \underbrace{p_2 \cdots i}_{m_2})$. Continue to route the element in the first position back to its correct position until $p_1 \in \{1, 2\}$. Consider the following cases:

(a) $p = 2\, 1\, p_3 ... p_n$, if $m_1 = m_2$: The discussion is similar to that for case (2).

(b) $p = 1\, 2\, p_3 ... p_n$, if $m_1 = m_2 + 1$: The discussion is similar to that for case (1).

(c) $p = 1\, p_2\, p_3 ... p_n$, if $m_1 > m_2 + 1$: The discussion is similar to that for case (3).

(d) $p = 2\, p_2\, p_3 ... p_n$, if $m_1 < m_2$: The discussion is similar to that for case (5).

7. Let $p = p_1\, p_2 ... p_{i-1}\, 1\, p_{i+1} ... p_{j-1}\, 2\, p_{j+1} ... p_n$ and $c_1 = (1\, \underbrace{p_1 \cdots i}_{m_1})$ and $c_2 = (2\, \underbrace{p_2 \cdots j}_{m_2})$. The argument of this case is quite similar to that for case (6). We leave this as an exercise to the readers.

Theorem 6 summarizes the above discussion.

Theorem 6: If $p (\in V_n)$ is of the form $c_1\, c_2 ... c_k\, e_1\, e_2 .... e_\ell$, then

$D_p = n + k - \ell$ or $n + k - \ell + 1$. if $p_1 = 1$ and $p_2 = 2$.
$= n + k - \ell - 3$ or $n + k - \ell - 2$. if $p_1 = 2$ and $p_2 = 1$.
$= n + k - \ell - 2$ or $n + k - \ell - 1$. if $p_1 \ne 1$ and $p_2 = 2$.
$= n + k - \ell - 2$ or $n + k - \ell - 1$. if $p_1 = 1$ and $p_2 \ne 2$.
$= n + k - \ell - 3$ or $n + k - \ell - 2$. if $1, 2 \in c_i$, $1 \le i \le k$, $3 \le |c_i|$, and 1, 2 are not successive.
$= n + k - \ell - 3$ or $n + k - \ell - 2$. if $1, 2 \in c_i$,

$1 \leq i \leq k$, $3 \leq |c_i|$
and 1, 2 are successive.
$= n + k - \ell - 4$ or $n + k - \ell - 3$. if $1 \in c_i$, $2 \in c_j$ and $i \neq j$.

Since the cycle set partition is NP-complete, we shall develop a polynomial time greedy heuristic to perform distributed routing from $p = p_1 p_2 ... p_n$ to I in the following. This heuristic always produces a routing distance at most one more than the theoretical lower bound [12]. Let vertex $r = r_1 r_2 ... r_n$ be a node in the routing path from p to I. Its immediate successor r' can be determined by using Algorithm 1 given below.

Algorithm 1 : Let $c_i = (q_1 q_2 ... q_m)$ and $q_1 < q_i$, $2 \leq j \leq m$.

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evaluate the cycle structure of $r$;
if ($r_1 \in \{1, 2\}$) then
    if ($r_2 \in \{1, 2\}$) then
        if ($r = I$) then
            the destination is reached;
        else
            Select any nontrivial cycle $c_i$ in $r$ and $c_i \neq (1\ 2)$;
            $r' = r \cdot g_{q_m}$;
        end
    else
        if (exist a cycle $c_i$ in $r$ and $r_2 \notin c_i$) then
            $r' = r \cdot g_{q_m}$;
        else
            There exists only one nontrivial cycle $c_i$ in $r$ and $q_j = r_2$;
            $\ell = |c_i - \{1, 2\}|$;
            if ($\ell > 1$) then
                $r' = r \cdot g_{q_{(j+\lfloor \frac{m}{2} \rfloor)^{-1}}}$;
            else
                $r' = r \cdot g_{r_2}$;
            end
        end
    end
else
    $r' = r \cdot g_{r_1}$.
end
```

The worst case analysis of employing Algorithm 1 successively to route vertex p back to vertex I leads to the following Lemma.

Lemma 7: Let D be the diameter of UAG_n; then,

$$
\begin{aligned}
D &= 3 * \frac{n-2}{2}, \quad \textit{if } n \textit{ is even and } \frac{n-2}{2} \textit{ is even.} \\
&= 3 * \frac{n-2}{2} + 1, \quad \textit{if } n \textit{ is even and } \frac{n-2}{2} \textit{ is odd.} \\
&= 3 * \left\lfloor \frac{n-2}{2} \right\rfloor + 1, \quad \textit{if } n \textit{ is odd and } \left\lfloor \frac{n-2}{2} \right\rfloor \textit{ is even.} \\
&= 3 * \left\lfloor \frac{n-2}{2} \right\rfloor + 2, \quad \textit{if } n \textit{ is odd and } \left\lfloor \frac{n-2}{2} \right\rfloor \textit{ is odd.}
\end{aligned}
$$

4. CONCLUDING REMARKS

Table 1 shows comparisons among UAG_n, $UHC1_n$ and $UHC2_n$. It can be seen that if we fix the cardinality of the vertex set for the four uni-directional schemes theoretically,then (1) uni-directional alternating group graphs have the smallest diameter, and (2) uni-directional alternating group graphs have far less indegree and outdegree than uni-directional hypercubes (of interest) do. The vertex symmetric property of UAG_n allows a distributed computing structure in one region of the network to be readily *translated* into another region without affecting the quality of the original embedding [12]. $UHC1_n$, $UHC2_n$ and US_n do not have the vertex symmetric property though $UHC1_n$ and $UHC2_n$ can be decomposed into two vertex-symmetric components [6]. These observations show the superiority of uni-directional alternating group graphs.

Several extensions of this study are possible. (a) More topological properties for UAG_n need to be explored. (b) Communication mechanisms, such as broadcating and multicasting, on UAG_n need be studied.

Table 1. A comparison among UAG_n, $UHC1_n$, $UHC2_n$ and US_n.

Networks	No. vertices	Indegree (Outdegree)	Vertex Symmetry	Diameter
UAG_n	$\frac{n!}{2}$	$n-2$ $(n-2)$	yes	$3*\frac{n-2}{2}$. if n is even and $\frac{n-2}{2}$ is even, $3*\frac{n-2}{2}+1$. if n is even and $\frac{n-2}{2}$ is odd, $3*\left\lfloor\frac{n-2}{2}\right\rfloor+1$, if n is odd and $\left\lfloor\frac{n-2}{2}\right\rfloor$ is even, $3*\left\lfloor\frac{n-2}{2}\right\rfloor+2$, if n is odd and $\left\lfloor\frac{n-2}{2}\right\rfloor$ is odd.
$UHC1_n$	2^n	$\left\lceil\frac{n}{2}\right\rceil$ or $\left\lfloor\frac{n}{2}\right\rfloor$ ($\left\lfloor\frac{n}{2}\right\rfloor$ or $\left\lceil\frac{n}{2}\right\rceil$)	no	n+1, if n is even, n+2, if n is odd.
$UHC2_n$	2^n	$\left\lceil\frac{n}{2}\right\rceil$ or $\left\lfloor\frac{n}{2}\right\rfloor$ ($\left\lfloor\frac{n}{2}\right\rfloor$ or $\left\lceil\frac{n}{2}\right\rceil$)	no	n+2, if n = 3, n+1, if $n \neq 3$.
US_n	$n!$	$\left\lfloor\frac{n-1}{2}\right\rfloor$ or $\left\lceil\frac{n-1}{2}\right\rceil$ ($\left\lceil\frac{n-1}{2}\right\rceil$ or $\left\lfloor\frac{n-1}{2}\right\rfloor$)	no	$\leq 5 * (n-2) +1$.

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