A Modified Approach to Determine the Most Appropriate Spreading Codes for a DS/CDMA System with Exponentially Weighted Despreading Sequences

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Exponential chip weighting waveform used in a direct sequence/code division multiple access (DS/CDMA) receiver is an effective method to reject multiple access interference (MAI). Recently, we introduced a strategy to determine the most appropriate spreading codes (MASC) which might be used as references in a given code set for a DS/CDMA system employing exponentially weighted despreading sequences. In the strategy it was shown that lower bit error rates might be achieved when the MASC, which were differentiated by the strategy, are used as references. The main limitation of the strategy was the requirement for dividing a code set into sub-sets to distinguish the resultant MASC. In this study, we present a modified approach that eliminates this requirement and also simplifies the determination of the MASC by ignoring one of the calculations, which was taken into consideration in the previous strategy on the features of the spreading codes in a given code set. Numerical results show that the MASC for a given code set can be detected simply and precisely by using the proposed approach.

Keywords: spread spectrum communications, DS/CDMA systems, optimal code selection, multiple access interference, weighted despreading sequences

1. INTRODUCTION

DS/CDMA is a spread spectrum technique for simultaneously transmitting a number of signals representing information messages from a multitude of users over a channel employing a common carrier. In a DS/CDMA system with perfect power control, one of the major limitations in performance, and hence in capacity, is due to MAI [1, 2]. With the intention of MAI rejection, an optimum multiuser receiver has been proposed in [3]. Although this receiver significantly outperforms the conventional single-user receiver, its computational complexity grows exponentially with the number of users. Various suboptimum multiuser receivers have been proposed to achieve a performance near that of the optimum receiver but with reduced complexity [4-7].

Because these suboptimum receivers require locking and despreading some or all of the co-user signals to optimally remove MAI, they are still too complex to be implemented in practice. Based on a noise whitening approach, a simple structure called the integral equation receiver (IER) has been introduced in [8, 9]. The IER is constrained to operate bit-by-bit, that is, to make bit decisions based on observation of the received
waveform over approximately one bit time in the absence of knowledge of the co-users’ spreading codes, chip timing, and carrier phase.

Since the IER operates without knowledge of the co-users’ spreading codes, it rejects MAI mainly by taking advantage of coloration in the power spectrum of the received chip waveform. This approach is different from multiuser detection strategies proposed in [4-7]. It does not require knowledge of the spreading codes of the co-users; it does not require locking and despreading the multiple arriving signals. It is also not required that the same set of transmitters be active across many bit times. As might be expected, this approach can be applied under a broader set of circumstances than typical multiuser detection, and with much less complexity, but it results in poorer performance. However, it is not necessarily mutually exclusive with multiuser detection techniques. The goal is to improve the performance of a single-user detector with respect to the interference caused by the co-users (MAI) [8, 9].

The IER employs a despreading function, which is the solution of a Fredholm integral equation of the second kind. The resulting despreading function consists of $2N^2$ exponential terms with the number of coefficients proportional to $N^2$, where $N$ is the processing gain. Moreover, it is still not easy to find the optimum despreading function in practice if $N$ is relatively large. Because the despreading function given in [8, 9] emphasizes the transitions in the received signal of the reference user for MAI rejection, it has been proposed to weight the despreading sequence with exponential chip weighting waveforms [10, 11]. In this method weighting of the despreading sequences has been applied at the receiver in order to utilize the spectral shape of the interference for improvement of the signal-to-interference-plus-noise ratio (SINR). The weighting functions found in [11] are a simple approximation of the solution to the Fredholm integral equation found in [8, 9]. These weighting functions have been shown to be better suited for practical implementations at only minor loss of performance in terms of MAI reduction by noise-whitening. The success of this rejection scheme, especially in relatively high signal-to-noise ratios where the bit error rates (BER’s) are mainly caused by MAI, has been demonstrated by the same authors in [11].

In our previous work [12], we showed that better BER performances might be achieved for a DS/CDMA system employing exponentially weighted despreading sequences by selecting proper codes as references in a given code set. For this purpose, we had presented an efficient strategy that enables determining the MASC. Despite this ability, there was a requirement for dividing a given code set into sub-sets to distinguish the resultant MASC. However, it should be noted that a number of calculations were required on the features of the spreading codes representing possible candidates to be employed as references [12, 13].

In this paper a detailed analysis on the strategy [12] is carried out and, a modified approach, which not only removes the requirement of dividing a code set into sub-sets but also ignores one of the calculations taken into consideration in the previous strategy, is presented [14]. Throughout this paper, it is assumed that the spreading codes in a given code set have equal cross-correlation properties.

This paper is organized as follows. In section 2, we provide some background material needed in the rest of the paper. Section 3 presents the modified approach, and the numerical results on the accuracy of the new approach are given in section 4. Finally, section 5 contains some concluding remarks.
2. BACKGROUND

2.1 Signal Model

We consider a binary DS/CDMA system with $K$ independent users simultaneously sharing a channel, each transmitting with power $P$ at a common carrier frequency $\omega_c$, using a data rate $R_b = 1/T_b$ and a chip rate $R_c = 1/T_c$. The $k$th user, ($k = 1, 2, \ldots, K$), is assigned a unique spreading code sequence $\{a^{(k)}_j\}$ of chip elements (+1, -1), so that its code waveform can be given by

$$a_k(t) = \sum_{j=-\infty}^{\infty} a^{(k)}_j P_{c_k}(t - jT_c)$$

(1)

where the function $P_{c_k}(\cdot)$ denotes the chip pulse of duration $T_c$. The spreading code sequence $\{a^{(k)}_j\}$ is assumed to be periodic with one period equal to the processing gain $N = T_b/T_c$. The data signal waveform $b_k(t)$ given by

$$b_k(t) = \sum_{j=-\infty}^{\infty} b^{(k)}_j P_{b_k}(t - jT_b)$$

(2)

is binary phase-shift-keyed (BPSK) onto the carrier at $\omega_c$, which is then spread by that user’s spreading code sequence (reference) and transmitted over the channel. The resulting $k$th user’s transmitted signal is thus given by

$$s_k(t) = \sqrt{2P}b_k(t)a_k(t)\cos(\omega_c t + \theta_k)$$

(3)

where the $\theta_k$ is the phase angle of the $k$th transmitter.

The received signal $r(t)$ at the base station can be expressed as

$$r(t) = \sum_{k=1}^{K} \sqrt{2P}b_k(t - \tau_k)a_k(t - \tau_k)\cos(\omega_c t + \phi_k) + n(t)$$

(4)

The random time delays and phases along the communication links between the $K$ transmitters and the receiver are denoted by $\tau_k$ and $\phi_k = \theta_k - \omega_c \tau_k$ for $1 \leq k \leq K$, respectively. The ambient channel noise $n(t)$ is modeled as an additive white Gaussian noise (AWGN) process with two-sided spectral density $N_0/2$. Random variables $\tau_k$ and $\phi_k$ are independent of one another and uniformly distributed in $[0, T_b]$ and $[0, 2\pi]$, respectively.

Based on the definitions given in [11], the weighted despreadng sequence for the $k$th receiver is

$$\hat{a}_k(t) = \sum_{j=-\infty}^{\infty} a^{(k)}_j w^{(k)}_j \left[ \delta(t - \tau_k) c^{(k)}_j c^{(k)}_{j+1} \right] P_{c_k}(t - jT_c)$$

(5)

where $c^{(k)}_j = a^{(k)}_{j-k}$ and $w^{(k)}_j \left[ c^{(k)}_j c^{(k)}_{j+1} \right]$, for $0 \leq t \leq T_c$, is the $j$th chip weighting
waveform for the $k$th receiver, conditioned on the status of three consecutive chips \( a_{i-1}^{(k)}, a_i^{(k)}, a_{i+1}^{(k)} \). Each \( c_{j}^{(k)} \) is a random variable, indicating whether or not the next element of the $k$th spreading signal is the same as the preceding element \( (c_{j}^{(k)} = -1 \text{ if } a_{i-1}^{(k)} \neq a_i^{(k)} \text{ and } c_{j}^{(k)} = 1 \text{ if } a_{i-1}^{(k)} = a_i^{(k)}) \). The $j$th chip conditional weighting waveform for the $k$th receiver is defined as \cite{11}

\[
W_j^{(k)}(c_j^{(k)}, c_{j+1}^{(k)}) = \begin{cases} 
  m_1(t) & \text{if } c_j^{(k)} = +1 \text{ and } c_{j+1}^{(k)} = +1 \\
  m_3(t) & \text{if } c_j^{(k)} = -1 \text{ and } c_{j+1}^{(k)} = -1 \\
  m_5(t) & \text{if } c_j^{(k)} = -1 \text{ and } c_{j+1}^{(k)} = +1 \\
  m_7(t) & \text{if } c_j^{(k)} = +1 \text{ and } c_{j+1}^{(k)} = -1 
\end{cases}
\]

(6)

where $m_p(t)$ for $p \in \{1, 2, 3, 4\}$ are the chip weighting waveforms. The elements of the chip weighting waveforms $m_1(t), m_2(t), m_3(t)$ and $m_4(t)$ are given by the following equations \cite{11}:

\[
\begin{align*}
  m_1(t) &= e^{-\gamma t/2} P_{\Psi}(t) \\
  m_2(t) &= e^{-\gamma t/2} P_{\Psi}(t) + e^{-\gamma (t-T_c)/2} P_{\Psi}(t-T_c/2) \\
  m_3(t) &= e^{-\gamma t/2} P_{\Psi}(t) + e^{-\gamma t/2} P_{\Psi}(t-T_c/2) \\
  m_4(t) &= e^{-\gamma t/2} P_{\Psi}(t) + e^{-\gamma (t-T_c)/2} P_{\Psi}(t-T_c/2)
\end{align*}
\]

(7)

where $\gamma \in [0, \infty)$ is a parameter of the exponential chip weighting waveforms. Without loss of generality, it is assumed that user $i$ is the reference user ($T_c = 0$ and $\phi_i = \theta_i = a_i = 0$) and the $s$th spreading code in a code set, having $\Psi$ spreading codes, used as a reference. The $SINR_{i(s)}$, conditioned on \( \{c_j^{(i,s)}\} \), is given by

\[
SINR_{i(s)} = \frac{\frac{1}{2} \chi(1-e^{-\gamma}) + \gamma(1-\chi)e^{-\gamma}}{2 \chi [1-e^{-\gamma/2} + \gamma(1-\chi)e^{-\gamma/2}]} + \frac{(K-1)\Xi(G^0_{j(s)}, \gamma)}{2N[2\chi(e^{\gamma/2} - 1) + (1-\chi)]^2}
\]

(8)

where $\bar{P}_i = E_b / N_0$, $E_b = PT_b$, $\chi = \hat{N}_{i(s)}/N$ and, $\hat{N}_{i(s)}$ is a random variable which represents the number of times that $c_j^{(i,s)} = -1$ for all $j \in [0, N-1]$. The term $\Xi(G^0_{j(s)}, \gamma)$ in Eq. 8 is given by

\[
\Xi(G^0_{j(s)}, \gamma) = \Xi^0 = \frac{1}{N} \left[ 4 + \frac{12}{\gamma} + \frac{16e^{\gamma/2}}{\gamma} + \frac{4e^\gamma}{\gamma} \right] G^0_{j(s), 0} + \frac{3}{2} \frac{\gamma^2}{4} + \gamma^2 + \frac{19e^{\gamma/2}}{2\gamma} + \frac{19e^\gamma}{\gamma} \left( \frac{5}{\gamma/2} + \frac{5e^\gamma}{\gamma} \right).
\]

(9)
where $r_{(i,u)}^{(i,u)}$ is the number of occurrences of $\{ c_{j-1}^{(i,u)}, c_{j}^{(i,u)}, c_{j+1}^{(i,u)} \}$ for all $j$ in the $u$th spreading sequence, $u = (1, 2, 3, \ldots, \Psi)$, selected as a reference and each $v_{in}, n \in [1, 2, 3]$, takes values $+1$ or $-1$ with equal probabilities. It is worth noting that parameter $\gamma$ of the exponential chip weighting waveforms should be tuned with respect to each signal to noise ratio $P_0$, so as to maximize the $\text{SINR}(i,u)$ [11, 15, 16].

2.2 The Importance of Employing Proper Codes as References

In order to demonstrate the advantages to be gained with proper codes as references in a given code set, it is useful to assess the BER performance of the $i$th user’s receiver when different spreading codes are employed as a reference, sequentially. It is well known that the codes used for spreading should have low cross-correlation values and be unique to every user. The Gold codes have only three cross-correlation peaks, which tend to get less important as the length of the code increases. At the same time, they have a single auto-correlation peak at zero [17].

Table 1 shows a sample code set. The spreading codes in this table are chosen from a Gold code set of length 31 [18]. The normalized cross-correlation matrix $R$ between the spreading codes is given in Eq. 10. As can be seen, the candidate codes have equal cross-correlation properties (7/31).

Table 1. Sample Code Set, $N = 31$ and $\Psi = 5.$

<table>
<thead>
<tr>
<th>code #1</th>
<th>111101100010011101110001</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>10111111111001010000110101</td>
</tr>
<tr>
<td>3</td>
<td>11000010111010111111110101</td>
</tr>
<tr>
<td>4</td>
<td>011001101111010001110001110001</td>
</tr>
<tr>
<td>5</td>
<td>1101011010010101101110110111</td>
</tr>
</tbody>
</table>

$$R = \frac{1}{31} \begin{bmatrix} 31 & 7 & 7 & 7 \\ 7 & 31 & 7 & 7 \\ 7 & 7 & 31 & 7 \\ 7 & 7 & 7 & 31 \end{bmatrix} \quad (10)$$

In Table 2, the quantities $r_{(i,u)}^{(i,u)}$ and $\hat{N}_{(i,u)}$ are listed for each code given in Table 1. Since MAI is modeled as a zero-mean, colored, Gaussian process, the probability of error $P_e$ in all the BER curves is defined as

$$P_e = Q(\sqrt{\text{max}[\text{SINR}_{(i,u)}]}) \quad Q(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp(-t^2/2)dt \quad (11)$$

where $\text{max}[\text{SINR}_{(i,u)}]$ is the maximum value of $\text{SINR}_{(i,u)}$. It is worth noting that by the central limit theorem, the accuracy of the Gaussian assumption should increase as the processing gain $N$ and number of users $K$ increase [11].
Table 2. The quantities $\Gamma_{[\nu,\nu',\nu]}^{(\nu)}$, $\Gamma_{[\nu,\nu',\nu]}^{(-\nu,\nu')}$, $\Gamma_{[\nu,\nu',\nu]}^{(-\nu,'\nu)}$, $\Gamma_{[\nu,\nu',\nu]}^{(\nu')}$, $\hat{N}_{(i,u)}$ for each code given in Table 1.

<table>
<thead>
<tr>
<th>Code #</th>
<th>Code in Table 1</th>
<th>$\Gamma_{(-1,-1,-1)}^{(\nu)}$</th>
<th>$\Gamma_{(-1,-1,1)}^{(\nu)}$</th>
<th>$\Gamma_{(-1,1,-1)}^{(\nu)}$</th>
<th>$\Gamma_{(-1,1,1)}^{(\nu)}$</th>
<th>$\Gamma_{(-1,1,-1)}^{(-\nu)}$</th>
<th>$\Gamma_{(1,1,-1)}^{(-\nu)}$</th>
<th>$\hat{N}_{(i,u)}$</th>
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<tr>
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<td>6</td>
<td>10</td>
<td>3</td>
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<td>6</td>
<td>12</td>
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</table>

Fig. 1. BER performance of the $i$th user’s receiver when various codes given in Table 1 are used as a reference ($K=5$).

To emphasize the importance of employing proper spreading codes as references, we consider using the codes (code # 1 – code # 5) given in Table 1 as a reference. Fig. 1 shows the BER performance of the $i$th user’s receiver, when these codes are assigned to the $i$th user as a reference. As can be seen, the worst BER performance (the lowest $\max [\text{SINR}_{(i,u)}]$) would be achieved by using the code # 1 having the smallest $\hat{N}_{(i,u)}$ among these codes. Also, it is readily seen that the best BER performance (the biggest $\max [\text{SINR}_{(i,u)}]$) would be achieved when the code # 5 having the biggest $\hat{N}_{(i,u)}$ is assigned as a reference for the $i$th user. In addition to these observations, we should also consider the following two cases:
(1) Even though \( \hat{N}_{(i,u)} \) of some codes are identical (Table 2), such as code \# 3 and code \# 4 in Table 1, the achieved BER performance is not exactly the same. Therefore, there is a need for taking the quantities \( \Gamma_{[y,v,x]}^{(i,u)} \) into consideration for each code to determine the MASC among such codes.

(2) Although the quantities \( \Gamma_{[y,v,x]}^{(i,u)} \) belonging to certain spreading codes are not exactly the same, such as code \# 2 and code \# 3 in Table 1, it is possible to achieve the same BER performance.

In the light of these observations, there is no doubt that the more suitable codes selected as references are, the better the BER performances will be. Clearly, the codes having the biggest \( \max [\text{SINR}_{(i,u)}] \) are the MASC for using as references but unfortunately, the \( \max [\text{SINR}_{(i,u)}] \) must be computed for each code in a code set to make a simple performance comparison on the \( i \)th user’s receiver which use different spreading codes as a reference. So, it is obvious that the procedures summarized below should be followed to determine the MASC in a given code set:

(i) Compute the quantities \( \Gamma_{[y,v,x]}^{(i,u)} \) and \( \hat{N}_{(i,u)} \) for \( u = (1, 2, 3, \ldots, \Psi) \).

(ii) For a certain \( \gamma_{(u)} \geq 15 \text{ dB} \), where the BER’s are mostly caused by MAI, compute the optimal value of the parameter \( \gamma_{(u)} \) which maximize the \( \text{SINR}_{(i,u)} \), for \( u = (1, 2, 3, \ldots, \Psi) \).

(iii) For all \( u \), compute the \( \max [\text{SINR}_{(i,u)}] \).

(iv) Mark \( K \) spreading codes with the biggest \( \max [\text{SINR}_{(i,u)}] \) as the MASC.

In view of Eqs. 8 and 9, there is no doubt that it is not so easy to find \( \gamma_{(u)} \) for each code in a given code set (procedure ii). Furthermore, the computational complexities of these expressions are obviously very high. As a consequence, this way is not practical enough for a spread spectrum designer to distinguish the MASC in a given code set.

3. MODIFIED APPROACH FOR THE DETERMINATION OF THE MASC

In this section, we define the strategy introduced in [12] first, and then concentrate on modifying it, rather than directly introducing the modified approach. In order to proceed, Eq. 9 can be rewritten as

\[
\Xi_{(i,u)} = \frac{1}{N} \left[ A \Gamma_{[-1,-1,-1]}^{(i,u)} + B \left( \Gamma_{[-1,-1,1]}^{(i,u)} + \Gamma_{[1,-1,-1]}^{(i,u)} \right) + C \left( \Gamma_{[-1,1,1]}^{(i,u)} + \Gamma_{[1,1,-1]}^{(i,u)} \right) + D \Gamma_{[-1,-1,-1]}^{(i,u)} + E \Gamma_{[-1,1,1]}^{(i,u)} + F \Gamma_{[1,1,1]}^{(i,u)} \right]
\]

where

\[
A = \left( 4 + \frac{12}{\gamma} - \frac{16e^{\gamma/2}}{\gamma} + \frac{4e^\gamma}{\gamma} \right)
\]

\[
B = \left( \frac{5}{2} \gamma + \frac{\gamma^2}{4} + \frac{19}{24} + \frac{e^\gamma}{\gamma} - \frac{12e^{\gamma/2}}{\gamma} + \frac{5e^\gamma}{2\gamma} \right)
\]

In view of Eqs. 8 and 9, there is no doubt that it is not so easy to find \( \gamma_{(u)} \) for each code in a given code set (procedure ii). Furthermore, the computational complexities of these expressions are obviously very high. As a consequence, this way is not practical enough for a spread spectrum designer to distinguish the MASC in a given code set.
In Fig. 2, A, B, C, D, E and F are plotted as a function of parameter $\gamma$ in the interval (0, 4). As seen in this figure, the superiority state of these expressions compared with those in $\Xi$ vary as the value of $\gamma$ increases. To be more specific, if $\gamma$ is set to 2, then $D > C > F > A > B > E$. If $\gamma$ is fixed to 3.5, then $D > A > C > B > F > E$.

To elaborate further, let us assume that two different spreading codes, $x$ and $y$, which have the features given below, are used as a reference for the $i$th user,

$$\left( \Gamma^{(i,1)}_{[-1,-1]} + \Gamma^{(i,3)}_{[-1,1]} \right) = \left( \Gamma^{(i,1)}_{[-1,1]} + \Gamma^{(i,3)}_{[-1,-1]} \right) = \left( \Gamma^{(i,1)}_{[-1,1]} + \Gamma^{(i,3)}_{[1,1]} \right) = \left( \Gamma^{(i,1)}_{[1,-1]} + \Gamma^{(i,3)}_{[1,1]} \right)$$

(13.f)
\[ \Gamma_{[-1,1), [1,1)} = \Gamma_{[-1,1), [1,1)} \], and \\
\hat{N}_{(i,x)} = \hat{N}_{(i,y)} \]

Also, let us assume that \( \Gamma_{[-1,1], [1,1]} = \Gamma_{[-1,1], [1,1]} = r, \quad \Gamma_{[1,1], [1,1]} = \Gamma_{[1,1], [1,1]} = z (r > z) \), and the \( i \)th user’s receiver is tuned to the same value of the parameter, \( \chi \), for these reference codes. Under these circumstances, the difference between \( \text{SNIR}_{(u,i)} \) and \( \text{SNR}_{(i,y)} \) will be in the \( \Xi^{(i,y)} \) and \( \Xi^{(i,x)} \) values for a certain \( \gamma \). As seen in Fig. 2, it is obvious that if the receiver is tuned with a \( \chi \) which was selected from the interval (2, 2.6), then \([rA(\chi) + z F(\gamma)] < [z A(\chi) + r F(\gamma)] \), which implies that \( \Xi^{(i,y)} < \Xi^{(i,x)} \). In view of Eq. 8, it is clear that the highest value of the \( \text{SNIR}_{(u,i)} \), in other words, the lowest BER, would be achieved by using a reference code which contributes the lowest ratio of the \( \Xi^{(i,y)} \) value to the \( \text{SNR}_{(u,i)} \). Therefore, it seems more suitable to use code \( y \) as a reference rather than code \( x \) when the receiver is tuned with a \( \chi \) value selected from the interval mentioned above.

Let us now focus on the interval (2.6, 3), shown in Fig. 2. If we assume that the receiver is tuned to a value which belongs to this interval, then \([rA(\chi) + z F(\gamma)] > [z A(\chi) + r F(\gamma)] \), which implies that \( \Xi^{(i,y)} > \Xi^{(i,x)} \). In this case, code \( x \) is a more powerful candidate to use as a reference in place of the code \( y \) on the condition that \( \gamma \) is chosen from the interval (2.6, 3). Likewise, we expect that the same BER performances can be produced by using these codes as references when the \( i \)th user’s receiver is tuned to \( \chi = 2.6 \), which implies \( \Xi^{(i,y)} = \Xi^{(i,x)} \).

These observations are important because they imply that a performance comparison on the \( i \)th user’s receiver employing various spreading codes would not be accurate due to ignoring \( \gamma^u \) for each code given above. Therefore, it seems that it is mandatory to determine the \( \text{max} \left[ \text{SNIR}_{(u,i)} \right] \) which uses the \( \Xi^{(i,y)} \) for each code, \( u = 1, 2, 3, \ldots, \Psi \), in a given code set to distinguish the MASC as references.

In order to proceed, we have searched for a starting point after which the greatness state of the \( A, B, C, D, E \) and \( F \) in the \( \Xi^{(i,y)} \) remains fixed. For this purpose, we have examined the variations of these expressions (Eqs. 13.a, …, 13.f) versus \( \gamma \) in a large observation window, \( \gamma \in [0, 8] \) (Fig. 3). It is interesting to note that, after a specific value of the parameter, \( \gamma = 5.6 \), the greatness state of these expressions in \( \Xi^{(i,y)} \) remains invariant as depicted in Fig. 3. As seen, the final change in superiority was observed between \( C \) and \( E \) at \( \gamma = 5.6 \). After this point, it was observed that the greatness state of the expressions remains as \( A > B > D > E > C > F \).

After the stable order of these expressions is determined, it is now possible to explain the differences among codes having the same quantity of \( \hat{N}_{(i,y)} \), when we assume that the \( i \)th user’s receiver was tuned to the same value of \( \chi \) for each code in a code set, by using the following expression \( (\gamma > 5.6) \):

\[
\Delta^{(i,u)} \left[ \Gamma_{[-1,1], [1,1]} \right] = 2A(\gamma) \Gamma_{[-1,1], [1,1]} + 2B(\gamma) \Gamma_{[-1,1], [1,1]} + C(\gamma) \Gamma_{[1,1], [1,1]} + D(\gamma) \Gamma_{[-1,1], [1,1]} + E(\gamma) \Gamma_{[-1,1], [1,1]} + F(\gamma) \Gamma_{[-1,1], [1,1]}
\]

\[ (14) \]
Fig. 3. A, B, C, D, E and F as a function of $\gamma$ in the range of $\gamma \in [0, 8]$.

It is worth mentioning that because Eq. 14 is mainly derived from $\Xi^{(i, u)}$ in Eq. 9, the MASC should be a combination of codes having values of $\Delta^{(i, u)}$ lower than the other candidate codes in a code set (lower values of $\Delta^{(i, u)}$ correspond to lower $\Xi^{(i, u)}$). Therefore, the following steps should be performed for determining the MASC in a given code set [12, 13]:

(i) Compute the quantities $\Gamma^{(i, u)}_{\{v_1, v_2, \ldots, v_r\}}$ and $\hat{N}^{(i, u)}_{\{v_1, v_2, \ldots, v_r\}}$ for a given code set, $u = (1, 2, 3, \ldots, \Psi)$.
(ii) Divide the code set into sub-sets in which the spreading codes have an equal quantity of $\hat{N}^{(i, u)}_{\{v_1, v_2, \ldots, v_r\}}$ and compute $\Delta^{(i, u)}$ for all $u$ using Eq. 14.
(iii) First, begin with the sub-set having the biggest quantity of $\hat{N}^{(i, u)}_{\{v_1, v_2, \ldots, v_r\}}$.
(iv) If the number of the active users ($K$) is less than the number of codes in this sub-set ($\Psi_k$), mark $K$ of the codes having $\Delta^{(i, u)}$ lower than the others in the set as the MASC.
(v) Finally, if $K > \Psi_k$, follow the previous procedure (iv) for the next sub-set which has $\hat{N}^{(i, u)}_{\{v_1, v_2, \ldots, v_r\}}$ closest to the previous sub-set to determine the remaining $K - \Psi_k$ MASC.

Despite the success of the strategy defined above, a major criticism that can be raised against it is that the requirement for separating a given code set into sub-sets for a simple performance comparison on a DS/CDMA receiver, which uses different spreading codes as references [12, 13]. It is obvious that this requirement can easily be avoided and a generalization of Eq. 14 can be obtained by including the knowledge of $\hat{N}^{(i, u)}_{\{v_1, v_2, \ldots, v_r\}}$ into Eq. 14 [14]. The expression in Eq. 14, with the determined coefficient values, can be modified as
CODE SELECTION FOR DS/CDMA SYSTEM WITH WEIGHTED DESPREADING SEQUENCES

\[
\Delta_m^{(\mu)} \left( r_{\{v_1,v_2,v_3\}}^{(\mu)}, \hat{N}_{(\mu)} \right) = \frac{1}{10^{-x} N_{(\mu)}} \left[ 2.21 \Gamma_{[-1,-1,-1]}^{(\mu)} + 1.52 \Gamma_{[-1,\ldots,-1]}^{(\mu)} + 0.76 \Gamma_{[-1,\ldots,-1]}^{(\mu)} + 1.16 \Gamma_{[-1,\ldots,-1]}^{(\mu)} + 0.82 \Gamma_{[-1,\ldots,-1]}^{(\mu)} + 0.36 \Gamma_{[-1,\ldots,-1]}^{(\mu)} \right]
\]

(15)

Based on the detailed investigations given above, we have chosen \( \gamma_t \) to be 6 to determine the coefficients in \( \Delta_m^{(\mu)} \). It should be noted that the other values which satisfy the condition \( \gamma_t > 5.6 \) will also support the validation of Eq. 15 [12].

The flexibility in the estimation of \( \gamma_t \) in Eq. 14 leads us to consider deriving an expression which reduces computational complexity compared to \( \Delta_m^{(\mu)} \). Because the coefficient of the last term, \( \Gamma_{[-1,\ldots,-1]}^{(\mu)} \), in \( \Delta_m^{(\mu)} \) is the lowest of the terms, we hoped to eliminate this term without any significant loss (i.e., causing erroneous decisions) in the determining the MASC. To gain some insight, we have analyzed the variation of \( F \) according to each expression \( A, B, C, D \) and \( E \). As shown in Fig. 4, the proportional variations decrease rapidly to zero as the values of \( \gamma_t \) increases. Therefore, it seems suitable to ignore \( \Gamma_{[-1,\ldots,-1]}^{(\mu)} \) throughout the determination of the MASC, on the condition that \( \gamma_t \) is chosen greater than 12. As a consequence, \( \Delta_m^{(\mu)} \) can be simplified as to

\[
\Delta_m^{(\mu)} \left( r_{\{v_1,v_2,v_3\}}^{(\mu)}, \hat{N}_{(\mu)} \right) = \frac{1}{10^{-x} N_{(\mu)}} \left[ 13.53 \Gamma_{[-1,-1,-1]}^{(\mu)} + 8.55 \Gamma_{[-1,-1,-1]}^{(\mu)} + 1.78 \Gamma_{[-1,-1,-1]}^{(\mu)} + 3.54 \Gamma_{[-1,-1,-1]}^{(\mu)} + 3.49 \Gamma_{[-1,-1,-1]}^{(\mu)} \right]
\]

(16)

Fig. 4. \( F / G, F / B, F / A, F / C \) and \( F / D \) depicted as a function of \( \gamma_t \).
Note that $\gamma$ is chosen to be 13, and other values which satisfy the condition $\gamma > 13$ will lessen the importance of the lack of $\Gamma_{\{1,1,1\}}$ in Eq. 16 (Fig. 4). Finally, we summarize the procedures to determine the MASC in a given code set by using the modified approach presented in this work [14]:

**Step 1** Compute $\Gamma_{\{1,1,1\}}^{(i,u)}$ and $\hat{N}_{\{1,1\}}$, except for $\Gamma_{\{1,1,1\}}^{(i,u)}$ for $u = (1, 2, 3, \ldots, \Psi)$.

**Step 2** Compute $\Delta_{ms}^{(i,u)}$ for all $u$, using Eq. 16.

**Step 3** Mark $K$ spreading codes having lower $\Delta_{ms}^{(i,u)}$ values than the others as the MASC.

In summary, the modified approach has advantages over the strategy proposed in [12]:

(i) The modified approach does not require dividing a given code set into sub-sets,

(ii) The necessity of computing the quantities $\Gamma_{\{1,1,1\}}$ is removed.

### 4. NUMERICAL RESULTS

In this section, numerical results of the accuracy of the proposed approach are presented. To indicate the usefulness of the approach, various code sets (code set # 1, code set # 2 and code set # 3) with different code lengths are taken into consideration. The spreading codes in these code sets are chosen from Gold sets having lengths of $N = 31$, $N = 63$ and $N = 127$, respectively [18]. The selected codes in each set have equal cross-correlation properties. The quantities $\Gamma_{\{1,1,1\}}$ and $\hat{N}_{\{1,1\}}$ belonging to each code in code set # 1, code set # 2 and code set # 3 are given in Tables 3, 4 and 5, respectively.

**Table 3.** The quantities $\Gamma_{\{1,1,1\}}$ and $\hat{N}_{\{1,1\}}$ for each selected code in the code set # 1 ($N = 31$ and $\Psi = 15$).

<table>
<thead>
<tr>
<th>Code # $u$</th>
<th>$\Gamma_{{1,1,1}}^{(i,u)}$</th>
<th>$\Gamma_{{1,1,1}}^{(i,u)}$</th>
<th>$\Gamma_{{1,1,1}}^{(i,u)}$</th>
<th>$\Gamma_{{1,1,1}}^{(i,u)}$</th>
<th>$\Gamma_{{1,1,1}}^{(i,u)}$</th>
<th>$\hat{N}_{{1,1}}$</th>
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<tr>
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<td>9 10 1</td>
<td>1 1 0</td>
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<tr>
<td>12</td>
<td>6 4 6</td>
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<td>13</td>
<td>10 5 3</td>
<td>1 1 1</td>
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Table 4. The quantities $\Gamma_{(i,a)}$ and $\hat{N}_{(i,a)}$ for each selected code in the code set #2 ($N = 63$ and $\Psi = 15$).

<table>
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<th>code # $u$</th>
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Table 5. The quantities $\Gamma_{(i,a)}$ and $\hat{N}_{(i,a)}$ for each selected code in the code set #3 ($N = 127$ and $\Psi = 15$).

<table>
<thead>
<tr>
<th>code # $u$</th>
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<th>$\Gamma_{(i,a)}$</th>
<th>$\Gamma_{(i,a)}$</th>
<th>$\hat{N}_{(i,a)}$</th>
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Table 6: The MASC determined by using both the strategy in [12] and the modified approach presented in this work for various code sets ($\gamma = 20$ dB)

<table>
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<td>max SINR</td>
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<td>max SINR</td>
<td>38.0</td>
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</table>

Notes:
- BER = Bit Error Rate
- max SINR = Maximum Signal-to-Noise Ratio
- $\gamma = 20$ dB indicates the signal-to-noise ratio in decibels.
Table 6 shows the results of the expression $\Delta_{ms}^{(i,u)}$ and the spreading codes distinguished as the MASC in the modified approach for code set # 1, code set # 2 and code set # 3. In this table the code # u with (×) must be selected as the reference for each set to achieve better BER performance. Note that the marked codes have lower $\Delta_{ms}^{(i,u)}$ values than the others in the same code set. In order to emphasize that the proposed approach finds the MASC, we present in Table 6 the calculated $\max\left[SINR_{i,u}\right]$ values and the BER’s when all of the spreading codes in the sets are used as a reference. Also shown for comparison is the MASC distinguished by using the strategy in [12].

As clear from Table 6, the spreading codes determined by our approach correspond to the ones having a larger $\max\left[SINR_{i,u}\right]$ than the others in the relevant code sets. Furthermore, we can see that better BER performances (lower BER’s) can be obtained by the use of the spreading codes distinguished as references.

Finally it should be stressed that although the $\Gamma_{[1,1]}^{(i,u)}$ quantities have not been taken into consideration in the determination (see Eq. 16), the modified approach has the ability to discern the codes that achieve fairly close BER’s, such as code # 3 and code # 4 in code set # 2 and, also code # 10 and code # 11 in code set # 3.

5. CONCLUSIONS

In this study, we have introduced a new approach to simplify the determination of the MASC that should be preferred as references in a given code set for a DS/CDMA system using exponentially weighted despreading sequences.

Two important conclusions can be made from the details of the numerical results. First, the requirement for dividing a given code set into sub-sets is successfully removed by using the modified approach. Second, even though the proposed approach does not employ one of the features belonging to a spreading code that is a possible candidate to be used as a reference, it exhibits a competitive performance for the determination of the MASC. Furthermore, the numerical results show that the modified approach can be applied to various code sets which have different code lengths for the same task, which was the goal of this effort.

It is well known that users in DS/CDMA systems are able to transmit information simultaneously over the same bandwidth by modulating a spreading code unique to each user. The necessity of storing a determined MASC in DS/CDMA receivers is the price to be paid in single-user detection for the resulting improvement in BER.

This paper is limited to only an AWGN channel. In practice, multipath fading also has a major effect on BER performance, and needs to be taken into consideration. In future work the authors plan to investigate the effects of multipath fading on the determination of the MASC. Even if the approach proposed in this work is suitable for the code sets, including spreading codes with equal cross-correlation, it is not in general form. Generalizing the approach for the code sets which include codes having non-equal cross-correlation is currently under investigation.

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REFERENCES

CODE SELECTION FOR DS/CDMA SYSTEM WITH WEIGHTED DESPREADING SEQUENCES

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