Short Paper

The Optimal Solution of the Transportation Problem with Fuzzy Demand and Fuzzy Product

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In crisp transportation, trying to fuzzify the amount of supply of the ith origin a_i and the amount of demand of the jth destination b_j, we use level λ fuzzy numbers and level (λ, ρ) interval-valued fuzzy numbers to fuzzify a_i and b_j in the constraints. We get transportation problem in the fuzzy sense. We also cooperate some statistical concepts and corresponding to $(1 - \alpha) \times 100\%$ statistical confidence intervals of the amount of supply and the amount of demand. We use level $(1 - \beta, 1 - \alpha)$ interval-valued fuzzy numbers to fuzzify demand and product in the constraints. Then we get transportation problem in the fuzzy sense based on statistical data.

Keywords: fuzzy transportation problem, confidence interval, fuzzy numbers, interval-valued fuzzy sets, fuzzy linear programming

1. INTRODUCTION

In [1, 10, 13, 15, 16], they discussed crisp transportation problems without using fuzzification. In [3, 4, 8, 14], they considered fuzzy linear programming problems. The articles [2, 5-7] treated fuzzy transportation problems. These are special cases of fuzzy linear programming. In [3], they fuzzified the constraints in linear programming through fuzzy numbers and used α-cut to obtain linear programming in the fuzzy sense. In [4], they used ranking fuzzy numbers in the constraints and obtained linear programming in the fuzzy sense through α-cut. In [14], he changed the constant terms in the constraints of linear programming to intervals and derived two linear programming problems. The second method in his paper was to fuzzify the coefficients of the constraints and constant terms to fuzzy number. Then he had fuzzy number linear programming problems. Using α-cut, he got linear programming in the fuzzy sense. In [2], the objective functions had k transportation problems. They fuzzified objective functions by fuzzy numbers and used λ-cut to obtain the transportation problem in the fuzzy sense expressed in linear programming form. In [6], they used fuzzy numbers of the type L-L to fuzzify the cost c_{ij} in

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the objective function $\sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij}x_{ij}$ to $\tilde{c}_{ij}$. Then they used the λ-cut and got the interval $c_{ij}{}^{\lambda}$. In this way, they had objective function $\sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij}{}^{\lambda} x_{ij}$. That meant two objective functions which were transportation problems in the fuzzy sense. All the articles above did not use signed distance ranking, defuzzification by signed distance, interval-valued fuzzy sets and statistical data. In this paper, we use these to consider fuzzy transportation problems. Section 2 is for the preliminaries. We will define the signed distance of interval-valued fuzzy numbers and their ranking. In section 3, we fuzzify the product and demand to level (λ, 1) interval-valued fuzzy numbers. Using signed distance and ranking, we get the transportation problem in the fuzzy sense. We give an example in section 4 and section 5 is the discussion.

2. PRELIMINARIES

In order to use fuzzy numbers and level (λ, 1) interval-valued fuzzy sets in the transportations with fuzzy demand and fuzzy product, we give the following definitions.

Definition 1 [12] $\tilde{a}$ is a fuzzy point if $\tilde{a}$ is a fuzzy set on $R = (-\infty, \infty)$ with membership function

$$\mu_{\tilde{a}}(x) = \begin{cases} 1, & x = a \\ 0, & x \neq a \end{cases} \tag{1}$$

Definition 2 $[a, b; \alpha]$ is a level α fuzzy interval, $0 < \alpha \leq 1$ if $[a, b; \alpha]$ is a fuzzy set on R with membership function

$$\mu_{[a,b;\alpha]}(x) = \begin{cases} \alpha, & a \leq x \leq b \\ 0, & \text{otherwise} \end{cases}, \ a < b \tag{2}$$

Definition 3 $\tilde{A} = (a, b, c; \lambda)$ is a level λ fuzzy number, $0 < \lambda \leq 1$, if $\tilde{A}$ is a fuzzy set on R with membership function, $a < b < c$,

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{\lambda(x-a)}{b-a}, & a \leq x \leq b \\ \dfrac{\lambda(c-x)}{c-b}, & b \leq x \leq c \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

Definition 4 [9] $\tilde{A} = [\tilde{A}^L; \tilde{A}^U] = [(a, b, c; \lambda), (p, b, r; \rho)]$, $p < a < b < c < r$ (see Fig. 1). We call $\tilde{A}$ is a level (λ, ρ) $i - v$ fuzzy number.

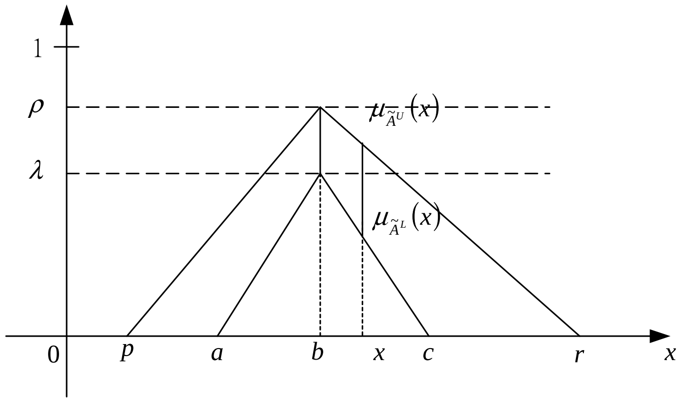

Fig. 1. level (λ, ρ) $i-v$ fuzzy number.

Let $F_{IV}(\lambda, \rho) = \{[(a, b, c; \lambda), (p, b, r; \rho)] \mid \forall p < a < b < c < r, p, a, c, r \in R\}$, $0 < \lambda < \rho \leq 1$, (4)

be the family of all level (λ, ρ) $i-v$ fuzzy numbers.

Remark 1: When $\lambda = 0$, $a = p$, $c = r$, $\tilde{A} = [\tilde{A}^L, \tilde{A}^U]$ will reduce to level ρ fuzzy number $\tilde{A}^U = (p, b, r; \rho)$.

We will use the signed distance [17] to consider the signed distance and ranking on $F_{IV}(\lambda, \rho)$ of level (λ, ρ) $i-v$ fuzzy numbers. We consider the signed distance on R first.

Definition 5 If $b, 0 \in R$, the signed distance of b from 0 is defined as $d^*(b, 0) = b$.

In the following we consider the signed distance and ranking of $i-v$ fuzzy numbers on $F_{IV}(\lambda, \rho)$.

Let $\tilde{A} = [(a, b, c; \lambda), (p, b, r; \rho)] = [\tilde{A}^L, \tilde{A}^U] \in F_{IV}(\lambda, \rho)$.

From Decomposition Theory (see Fig. 2), we obtain

$$\tilde{A} = \bigcup_{0 \leq \alpha < \lambda} ([A_l^U(\alpha), A_l^L(\alpha); \alpha] \cup [A_r^L(\alpha), A_r^U(\alpha); \alpha]) \cup \bigcup_{\lambda \leq \alpha \leq \rho} [A_l^U(\alpha), A_r^U(\alpha); \alpha] \quad (5)$$

The α-cut of $\tilde{A}$ is $A(\alpha) = [A_l^U(\alpha), A_l^L(\alpha)] \cup [A_r^L(\alpha), A_r^U(\alpha)]$ when $0 \leq \alpha \leq \lambda$, where

$$A_l^L(\alpha) = a + (b-a)\frac{\alpha}{\lambda}, \; A_r^L(\alpha) = c - (c-b)\frac{\alpha}{\lambda}$$

$$A_l^U(\alpha) = p + (b-p)\frac{\alpha}{\rho}, \; A_r^U(\alpha) = r - (r-b)\frac{\alpha}{\rho}$$

The α-cut of $\tilde{A}$ is $A(\alpha) = [A_l^U(\alpha), A_r^U(\alpha)]$ when $\lambda \leq \alpha \leq \rho$, where

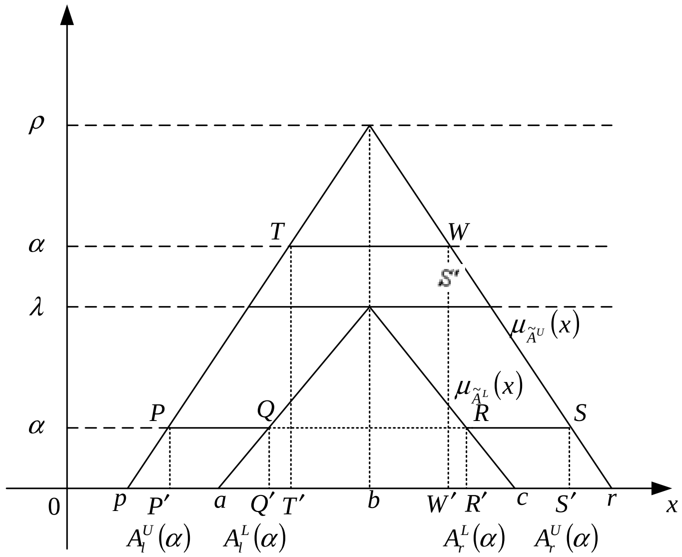

Fig. 2. α-cut of level (λ, ρ) $i-v$ fuzzy number.

$$A_l^U(\alpha) = p + (b-p)\frac{\alpha}{\rho},\ A_r^U(\alpha) = r - (r-b)\frac{\alpha}{\rho}$$

From (5) and Fig. 2 we have $A(\alpha) = [A_l^U(\alpha),\ A_l^L(\alpha)] \cup [A_r^L(\alpha),\ A_r^U(\alpha)]$ and $[A_l^U(\alpha),\ A_l^L(\alpha)] \cap [A_r^L(\alpha),\ A_r^U(\alpha)] = \phi$ when $0 \le \alpha \le \lambda$.

From Fig. 2, to obtain the signed distance from P', Q', R', S' to 0, by definition 5, we may define the signed distance from $[A_l^U(\alpha),\ A_l^L(\alpha)]$ to 0 as $d^*([A_l^U(\alpha),\ A_l^L(\alpha)],\ 0) = \frac{1}{2}[d^*(A_l^U(\alpha),\ 0) + d^*(A_l^L(\alpha),\ 0)] = \frac{1}{2}[A_l^U(\alpha) + A_l^L(\alpha)] = \frac{1}{2}[a + p + (b-a)\frac{\alpha}{\lambda} + (b-p)\frac{\alpha}{\rho}]$.

Similarly, $d^*([A_r^L(\alpha),\ A_r^U(\alpha)],\ 0) = \frac{1}{2}[c + r - (c-b)\frac{\alpha}{\lambda} - (r-b)\frac{\alpha}{\rho}]$. Since $[A_l^U(\alpha),\ A_l^L(\alpha)] \cup [A_r^L(\alpha),\ A_r^U(\alpha)] \leftrightarrow [A_l^U(\alpha),\ A_l^L(\alpha);\ \alpha] \cup [A_r^L(\alpha),\ A_r^U(\alpha);\ \alpha]$ is a one-one mapping, we can define the signed distance from $[A_l^U(\alpha),\ A_l^L(\alpha);\ \alpha] \cup [A_r^L(\alpha),\ A_r^U(\alpha);\ \alpha]$ to $\tilde{0}$ (y axis) as $d_0([A_l^U(\alpha),\ A_l^L(\alpha);\ \alpha] \cup [A_r^L(\alpha),\ A_r^U(\alpha);\ \alpha],\ \tilde{0}) = \frac{1}{2}[d^*([A_l^U(\alpha),\ A_l^L(\alpha)],\ 0) + d^*([A_r^L(\alpha),\ A_r^U(\alpha)],\ 0)] = \frac{1}{4}[a + c + p + r + (2b - a - c)\frac{\alpha}{\lambda} + (2b - p - r)\frac{\alpha}{\rho}]$. d_0 is a continuous function on $0 \le \alpha \le \lambda$. We can define the signed distance from $\tilde{A}$ to $\tilde{0}$ as the mean value on $[0, \lambda]$ through integration. Similarly, we get $d_0([A_l^U(\alpha),\ A_r^U(\alpha);\ \alpha],\ \tilde{0}) = \frac{1}{2}[p + r + (2b - p - r)\frac{\alpha}{\rho}]$, $\lambda \le \alpha \le \rho$. This is also a continuous function $[\lambda, \rho]$. We may define the signed distance from $\tilde{A}$ to $\tilde{0}$ as the mean value on $[0, \rho]$ through integration.

Definition 6 $\tilde{A} = [(a,\ b,\ c;\ \lambda),\ (p,\ b,\ r;\ \rho)] = [\tilde{A}^L,\ \tilde{A}^U] \in F_{IV}(\lambda,\ \rho),\ \forall 0 < \lambda < \rho \le 1.$ The signed distance from $\tilde{A}$ to $\tilde{0}$ is defined as

$$d_0(\tilde{A},\ \tilde{0}) = \frac{1}{\lambda}\int_0^\lambda \frac{1}{4}[a + c + p + r + (2b - a - c)\frac{\alpha}{\lambda} + (2b - p - r)\frac{\alpha}{\rho}]d\alpha +$$
$$\frac{1}{\rho - \lambda}\int_\lambda^\rho \frac{1}{2}[p + r + (2b - p - r)\frac{\alpha}{\rho}]d\alpha$$
$$= \frac{1}{8}[6b + a + c + 4p + 4r + 3(2b - p - r)\frac{\lambda}{\rho}] \tag{6}$$

Remark 2: If $\tilde{a} = [(a,\ a,\ a;\ 1),\ (a,\ a,\ a;\ 1)]$, then (6) reduces to $d_0(\tilde{a},\ \tilde{0}) = 2a$.

Definition 7 If $\tilde{A} = [(a,\ b,\ c;\ \lambda),\ (p,\ b,\ r;\ \rho)]$, $\tilde{B} = [(d, e, g; \lambda),\ (m, e, q; \rho)] \in F_{IV}(\lambda, \rho)$, then

$$\tilde{B} \prec \tilde{A} \text{ iff } d_0(\tilde{B},\ \tilde{0}) < d_0(\tilde{A},\ \tilde{0})$$
$$\tilde{B} \approx \tilde{A} \text{ iff } d_0(\tilde{B},\ \tilde{0}) = d_0(\tilde{A},\ \tilde{0})$$

Property 1 $(F_{IV}(\lambda, \rho), \approx, \prec)$ satisfies the law of trichotomy.

Proof: By $(R, =, <)$ satisfies the law of trichotomy and definition 7.

Let $\tilde{A} = [(a_1,\ b_1,\ c_1;\ \lambda),\ (p_1,\ b_1,\ r_1;\ \rho)] = [\tilde{A}^L,\ \tilde{A}^U]$,

$\tilde{B} = [(a_2,\ b_2,\ c_2;\ \lambda),\ (p_2,\ b_2,\ r_2;\ \rho)] = [\tilde{B}^L,\ \tilde{B}^U] \in F_{IV}(\lambda, \rho), k > 0.$

From the addition of level λ and level ρ fuzzy numbers, we get

$$\tilde{A}^L \oplus \tilde{B}^L = (a_1 + a_2,\ b_1 + b_2,\ c_1 + c_2;\ \lambda),\ \tilde{A}^U \oplus \tilde{B}^U = (p_1 + p_2,\ b_1 + b_2,\ c_1 + c_2;\ \rho),$$
$$\tilde{k}(\cdot)\tilde{A}^L = (ka_1,\ kb_1,\ kc_1;\ \lambda),\ \tilde{k}(\cdot)\tilde{A}^U = (kp_1,\ kq_1,\ kr_1;\ \lambda),\ \text{ for } k > 0. \tag{7}$$

Hence we define $\tilde{A} \oplus \tilde{B} = [\tilde{A}^L \oplus \tilde{B}^L,\ \tilde{A}^U \oplus \tilde{B}^U],\ \tilde{k}(\cdot)\tilde{A} = [\tilde{k}(\cdot)\tilde{A}^L,\ \tilde{k}(\cdot)\tilde{A}^U].$

Property 2 Let $\tilde{A} = [(a_1,\ b_1,\ c_1;\ \lambda),\ (p_1,\ b_1,\ r_1;\ \rho)]$,

$\tilde{B} = [(a_2,\ b_2,\ c_2;\ \lambda),\ (p_2,\ b_2,\ r_2;\ \rho)], \in F_{IV}(\lambda, \rho)$. Then

$d_0(\tilde{A} \oplus \tilde{B},\ \tilde{0}) = d_0(\tilde{A},\ \tilde{0}) + d_0(\tilde{B},\ \tilde{0})$ and $d_0(\tilde{k}(\cdot)\tilde{A},\ \tilde{0}_1) = kd_0(\tilde{A}),\ k > 0$

Proof: By (7) and definition 6.

3. TRANSPORTATION PROBLEM WITH FUZZY DEMAND AND FUZZY PRODUCT BASED ON LEVEL (λ, 1) $i - v$ FUZZY NUMBERS

We use i ($i = 1, 2, \ldots, m$) and j ($j = 1, 2, \ldots, n$) to denote the origin and destination. Let a_i and b_j be the amount of supply of the ith origin and the amount of demand of the jth destination, respectively. The unit transportation cost from ith origin to jth destination is c_{ij}. Let x_{ij} represent the amount transported from origin i to destination j. We have the crisp transportation model:

$$\text{min. } Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \tag{8}$$

$$\text{s.t. } \sum_{j=1}^{n} x_{ij} = a_i,\ a_i > 0,\ i = 1, 2, \ldots, m \tag{9}$$

$$\sum_{i=1}^{m} x_{ij} = b_j,\quad b_j > 0,\ j = 1, 2, \ldots, n \tag{10}$$

$$x_{ij} \geq 0,\ i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n \tag{11}$$

For consistency, one must have

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \tag{12}$$

If we know c_{ij}, a_i, b_j, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$, then we can use simplex method, north-west corner rule, the least-cost method, Hungarian Algorithm or Vogel's approximation method [1, 15, 16] to solve it. Suppose the transportation plan does not execute once. Since the amount of supply at the ith origin may not be exactly a_i each time, it could fluctuate a little. Hence, the decision maker would decide the amount of supply to lie in the interval $[a_i - \Delta_{1i}, a_i + \Delta_{2i}]$, $0 < \Delta_{1i} < a_i$, $0 < \Delta_{2i}$. Since $[a_i - \Delta_{1i}, a_i + \Delta_{2i}]$ is not a value, by the fuzzy sense, corresponding to this interval, we set the following level 1 fuzzy number (13). We can fuzzify a_i to a level 1 fuzzy numbers.

$$\tilde{a}_i = (a_i - \Delta_{1i},\ a_i,\ a_i + \Delta_{2i};\ 1),\ \ 0 < \Delta_{1i} < a_i,\ 0 < \Delta_{2i},\ i = 1, 2, \ldots, m \tag{13}$$

Similarly, we fuzzify b_j to a level 1 fuzzy numbers,

$$\tilde{b}_j = (b_j - \omega_{1j},\ b_j,\ b_j + \omega_{2j};\ 1),\ \ 0 < \omega_{1j} < b_j,\ 0 < \omega_{2j},\ j = 1, 2, \ldots, n \tag{14}$$

Let T be the time period on which c_{ij}, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$, does not change in the crisp transportation problem (8) - (12). In this period, the transportation plan does not execute once. In real situations as in (13), (14), the amount of supply in the ith origin may not be a fixed situations, the amount of supply in the ith origin may not be a fixed number a_i. Then we fuzzify a_i to $\tilde{a}_i$. In time period T, the grade of membership of a_i is not in general 1. We may assume the grade of membership of a_i falls into the interval $[\lambda, 1]$, $0 < \lambda < 1$. In other words, we let $\tilde{a}_i$ be a level $(\lambda, 1)$ $i - v$ fuzzy number.

$$\tilde{a}_i = [(a_i - \Delta_{3i},\ a_i,\ a_i + \Delta_{4i};\ \lambda),\ (a_i - \Delta_{1i},\ a_i,\ a_i + \Delta_{2i};\ 1)] \tag{15}$$

where $0 < \Delta_{3i} < \Delta_{1i} < a_i$, $0 < \Delta_{4i} < \Delta_{2i}$, $i = 1, 2, \ldots, m$.

Similarly, we fuzzify b_j to the following level $(\lambda, 1)$ fuzzy number,

$$\tilde{b}_j = [(b_j - \omega_{3j},\ b_j,\ b_j + \omega_{4j};\ \lambda),\ (b_j - \omega_{1j},\ b_j,\ b_j + \omega_{2j};\ 1)] \tag{16}$$

where $0 < \omega_{3j} < \omega_{1j} < b_j$, $0 < \omega_{4j} < \omega_{2j}$, $j = 1, 2, \ldots, n$.

Let $\tilde{1}^* = [(1, 1, 1; \lambda), (1, 1, 1; 1)]$.

We obtain the fuzzy transportation model.

$$\text{min. } Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \tag{17}$$

$$\text{s.t. } \sum_{j=1}^{n} x_{ij}\cdot\tilde{1}^* \approx \tilde{a}_i,\ \ i = 1, 2, \ldots, m \tag{18}$$

$$\sum_{i=1}^{m} x_{ij}\cdot\tilde{1}^* \approx \tilde{b}_j,\ \ j = 1, 2, \ldots, n \tag{19}$$

$$x_{ij} \geq 0,\ i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n \tag{20}$$

$$\text{From (12) we get } \sum_{i=1}^{m}\tilde{a}_i \approx \sum_{j=1}^{n}\tilde{b}_j \tag{21}$$

where $\approx$ is defined in definition 7.

Property 3 By fuzzy transportation (17) - (21), (15), (16), definitions 6 and 7, we have transportation problem in the fuzzy sense.

$$\text{min. } Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \tag{22}$$

$$\text{s.t. } \sum_{j=1}^{n} x_{ij} = a_i + \frac{1}{16}[\Delta_{4i} - \Delta_{3i} + (4-3\lambda)(\Delta_{2i} - \Delta_{1i})],\ \ i = 1, 2, \ldots, m \tag{23}$$

$$\sum_{i=1}^{m} x_{ij} = b_j + \frac{1}{16}[\omega_{4j} - \omega_{3j} + (4-3\lambda)(\omega_{2j} - \omega_{1j})],\ \ j = 1, 2, \ldots, n \tag{24}$$

$$x_{ij} \geq 0,\ i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n \tag{25}$$

For consistency,

$$\sum_{i=1}^{m}[\Delta_{4i} - \Delta_{3i} + (4-3\lambda)(\Delta_{2i} - \Delta_{1i})] = \sum_{j=1}^{n}[\omega_{4j} - \omega_{3j} + (4-3\lambda)(\omega_{2j} - \omega_{1j})] \tag{26}$$

Proof: From (15) and definition 6, $d_0(\tilde{a}_i, \tilde{0}) = 2a_i + \frac{1}{8}[\Delta_{4i} - \Delta_{3i} + (4-3\lambda)(\Delta_{2i} - \Delta_{1i})]$, and $d_0\left(\sum_{j=1}^{n} x_j \cdot \tilde{1}^*,\ 0\right) = 2\sum_{j=1}^{n} x_{ij}$. By definition 7, (18), $d_0\left(\sum_{j=1}^{n} x_{ij} \cdot \tilde{1}^*,\ \tilde{0}\right) = d_0(\tilde{a}_i,\ \tilde{0})$, $i = 1, 2, \ldots, m$. This is (23). Similarly, we can obtain (24), (26).

Remark 3: If $\Delta_{4i} = \Delta_{3i}$, $i = 1, 2, \ldots, m$, $\omega_{4j} = \omega_{3j}$, $j = 1, 2, \ldots, n$ and $\lambda = 0$, property 6 reduces to the case of level 1 fuzzy numbers of $(a_i - \Delta_{1i}, a_i, a_i + \Delta_{2i}; 1)$, $(b_j - \omega_{1j}, b_j, b_j, b_j + \omega_{2j}; 1)$.

4. EXAMPLE

A company has two factories F_1, F_2 and three retail stores W_1, W_2 and W_3. The production quantities per month F_1 and F_2 are 10 and 8 tons respectively. The demands per month for W_1, W_2 and W_3 are 5, 6, and 7 tons respectively. The transportation cost per ton c_{ij}, $i = 1, 2$, $j = 1, 2, 3$ are the following. $c_{11} = 16$, $c_{12} = 15$, $c_{13} = 25$, $c_{21} = 19$, $c_{22} = 24$ and $c_{23} = 12$. In our notation $a_1 = 10$, $a_2 = 8$, $b_1 = 5$, $b_2 = 6$ and $b_3 = 7$.

We have the crisp transportation problem (8) - (12).

4.1 Crisp Case

$$\begin{aligned}
&\text{min. } Z = 16x_{11} + 15x_{12} + 25x_{13} + 19x_{21} + 24x_{22} + 12x_{23} \\
&\text{s.t. } x_{11} + x_{12} + x_{13} = 10 \\
&\quad x_{21} + x_{22} + x_{23} = 8 \\
&\quad x_{11} + x_{21} = 5 \\
&\quad x_{12} + x_{22} = 6 \\
&\quad x_{13} + x_{23} = 7 \\
&\quad x_{ij} \geq 0,\ i = 1, 2,\ j = 1, 2, 3.
\end{aligned}$$

Using simplex method or north-west corner rule [14] or [15], we obtain the optimal solution $x_{11} = 4$, $x_{12} = 6$, $x_{13} = 0$, $x_{21} = 1$, $x_{22} = 0$, and $x_{23} = 7$. The minimal transportation cost is $Z_0 = 257$.

4.2 Fuzzy Case

Case 1. Let $\tilde{a}_1 = (10 - 4, 10, 10 + 8)$, $\tilde{a}_2 = (8 - 2, 8, 8 + 6)$, $\tilde{b}_1 = (5 - 2, 5, 5 + 6)$, $\tilde{b}_2 = (6 - 2, 6, 6 + 10)$ and $\tilde{b}_3 = (7 - 6, 7, 7 + 2)$.

From remark 3 we have transportation problem in the fuzzy sense.

$$\begin{aligned}
&\text{min. } Z = 16x_{11} + 15x_{12} + 25x_{13} + 19x_{21} + 24x_{22} + 12x_{23}\\
&\text{s.t. } x_{11} + x_{12} + x_{13} = 11\\
&\quad x_{21} + x_{22} + x_{23} = 9\\
&\quad x_{11} + x_{21} = 6\\
&\quad x_{12} + x_{22} = 8\\
&\quad x_{13} + x_{23} = 6\\
&\quad x_{ij} \geq 0,\ i = 1, 2,\ j = 1, 2, 3.
\end{aligned}$$

We obtain the optimal solution $x_{11} = 3$, $x_{12} = 8$, $x_{13} = 0$, $x_{21} = 3$, $x_{22} = 0$ and $x_{23} = 6$. The minimal transportation cost is $Z_1 = 297$.

Case 2. Let $\tilde{a}_1 = (10 - 0.6, 10, 10 + 9)$, $\tilde{a}_2 = (8 - 0.4, 8, 8 + 1.4)$, $\tilde{b}_1 = (5 - 0.5, 5, 5 + 4.5)$, $\tilde{b}_2 = (6 - 0.6, 6, 6 + 1)$ and $\tilde{b}_3 = (7 - 0.4, 7, 7 + 5.4)$.

From remark 3 we have transportation problem in the fuzzy sense.

$$\begin{aligned}
&\text{min. } Z = 16x_{11} + 15x_{12} + 25x_{13} + 19x_{21} + 24x_{22} + 12x_{23}\\
&\text{s.t. } x_{11} + x_{12} + x_{13} = 12.1\\
&\quad x_{21} + x_{22} + x_{23} = 8.25\\
&\quad x_{11} + x_{21} = 6\\
&\quad x_{12} + x_{22} = 6.1\\
&\quad x_{13} + x_{23} = 8.25\\
&\quad x_{ij} \geq 0,\ i = 1, 2,\ j = 1, 2, 3.
\end{aligned}$$

We obtain the optimal solution $x_{11} = 6$, $x_{12} = 6.1$, $x_{13} = 0$, $x_{21} = 0$, $x_{22} = 0$, $x_{22} = 8.25$. The minimal transportation cost is $Z_2 = 286.5$.

Case 3. Let $\tilde{a}_1^U = (10 - 4, 10, 10 + 8; 1)$, $\tilde{a}_2^U = (8 - 2, 8, 8 + 6; 1)$, $\tilde{b}_1^U = (5 - 2, 5, 5 + 6; 1)$, $\tilde{b}_2^U = (6 - 2, 6, 6 + 10; 1)$ and $\tilde{b}_3^U = (7 - 6, 7, 7 + 2; 1)$.

Let $\tilde{a}_1^L = (10 - 1, 10, 10 + 1; 0.9)$, $\tilde{a}_2^L = (8 - 1, 8, 8 + 5; 0.9)$, $\tilde{b}_1^L = (5 - 1, 5, 5 + 2; 0.9)$, $\tilde{b}_2^L = (6 - 1, 6, 6 + 7; 0.9)$ and $\tilde{b}_3^L = (7 - 4, 7, 7 + 1; 0.9)$. We get $\tilde{a}_i = [\tilde{a}_i^L, \tilde{a}_i^U]$, $i = 1, 2$ and $\tilde{b}_j = [\tilde{b}_j^L, \tilde{b}_j^U]$, $j = 1, 2, 3$. Through property 3, we have the following transportation problem in the fuzzy sense.

From property 3 we have transportation problem in the fuzzy sense.

$$\begin{aligned}
&\text{min. } Z = 16x_{11} + 15x_{12} + 25x_{13} + 19x_{21} + 24x_{22} + 12x_{23}\\
&\text{s.t. } x_{11} + x_{12} + x_{13} = 10.325\\
&\quad x_{21} + x_{22} + x_{23} = 8.575\\
&\quad x_{11} + x_{21} = 5.3875
\end{aligned}$$

$x_{12} + x_{22} = 7.025$
$x_{13} + x_{23} = 6.4875$
$x_{ij} \geq 0$, $i = 1, 2$, $j = 1, 2, 3$.

We obtain the optimal solution $x_{11} = 3.3$, $x_{12} = 7.025$, $x_{13} = 0$, $x_{21} = 2.0875$, $x_{22} = 0$, $x_{23} = 6.4875$. The minimal transportation cost is $Z_3 = 275.6875$.

6. DISCUSSION

6.1 The Comparisons among Crisp and Fuzzy Case

As shown in Table 1, the transportation cost is significant whenever the fluctuation is large in the fuzzy case and the transportation cost is close to the crisp case whenever the variation is also small in the fuzzy case. The level (0.9, 1) $i - v$ fuzzy number in case 3 is composed of level 1 fuzzy number in case 1 and level 0.9 fuzzy number. We see that $0 < r_{30} < r_{10}$.

Table 1. The comparisons among crisp case and properties 3, remark 3.

j	case	Transportation cost Z_j	$r_{j,\,j-1} = \dfrac{Z_j - Z_{j-1}}{Z_{j-1}} \times 100\%$	Remark
0	crisp case	257		no statistical data in case 0
1	fuzzy numbers	297	$r_{10} = 15.56$	the fluctuation is significant for this fuzzy case, case 1
2	fuzzy numbers	286.5	$r_{20} = 0.51$ $r_{21} = 11.48$	the fluctuation is significant for this fuzzy case, case 2
3	level (0.9, 1) $i - v$ fuzzy numbers	275.6875	$r_{30} = 7.27$ $r_{31} = -7.18$ $r_{32} = 6.81$	combined by level 1 fuzzy number in case 1 and level 0.9 fuzzy number, case 3

6.2 Transportation Problem based on Statistical Data and Statistical Confidence Intervals

In crisp transportation problem (8) - (12), if a_i, $i = 1, 2, \ldots, m$ and b_j, $j = 1, 2, \ldots, n$ are unknown, then we can get the point estimates $\overline{a}_i = \frac{1}{N}\sum_{p=1}^{N} a_{ip}$ and $\overline{b}_j = \frac{1}{N}\sum_{p=1}^{N} b_{jp}$ of a_i and b_j, respectively, from the statistical data in the past. Let their variances be S_{ia}^2 and S_{ib}^2, respectively. Since the probabilities of the error between $\overline{a}_i$ and a_i and the error between $\overline{b}_j$ and b_j are unknown, we use statistical confidence interval instead in statistics. Using the same method in section 3, we set an $i - v$ fuzzy number corresponding to this interval. For any $i \in \{1, 2, \ldots, m\}$ and $j \in \{1, 2, \ldots, n\}$, corresponding to $(1 - \alpha) \times$

100% and $(1-\beta)\times 100\%$ confidence intervals of a_i and b_j, we set level $1-\alpha$ fuzzy numbers $\tilde{a}_i^U$, $\tilde{b}_j^U$ and level $1-\beta$ fuzzy numbers $\tilde{a}_i^L$, $\tilde{b}_j^L$. Then we obtain the following level $(1-\beta, 1-\alpha)$ $i-v$ fuzzy numbers $\tilde{a}_i = [\tilde{a}_i^L, \tilde{a}_i^U]$ and $\tilde{b}_j = [\tilde{b}_j^L, \tilde{b}_j^U]$, (27)
where

$$\tilde{a}_i^L = (\bar{a}_i - Z_i(\beta_1), \bar{a}_i, \bar{a}_i + Z_i(\beta_2); 1-\beta),\ \tilde{a}_i^U = (\bar{a}_i - Z_i(\alpha_1), \bar{a}_i, \bar{a}_i + Z_i(\alpha_2); 1-\alpha)$$

$$\tilde{b}_j^L = (\bar{b}_j - Z_j(\beta_3), \bar{b}_j, \bar{b}_j + Z_j(\beta_4); 1-\beta),\ \tilde{b}_j^U = (\bar{b}_j - Z_j(\alpha_3), \bar{b}_j, \bar{b}_j + Z_j(\alpha_4); 1-\alpha)$$

$0<\alpha<\beta<1$, $0<\alpha_k<\beta_k<1$, $k=1,2,3,4$, $\alpha_1+\alpha_2=\alpha$, $\alpha_3+\alpha_4=\alpha$, $\beta_1+\beta_2=\beta$, $\beta_3+\beta_4=\beta$. $t_{N-1}(r_k)$ is the r_k point of the t-distribution with degree $N-1$, i.e., $Z_i(r_k) = t_{N-1}(r_k)\dfrac{S_{ia}}{\sqrt{N}}$, $k=1,2$ and $Z_j(r_k) = t_{N-1}(r_k)\dfrac{S_{jb}}{\sqrt{N}}$, $k=3,4$, $r_k=\alpha_k$ or β_k.

Through (27) and the property 3 in section 3, by the similar treatment, we get the transportation problem in the fuzzy sense.

6.3 The Determination of α_k, β_k, $k = 1, 2, 3, 4$

(1) The determination of α_j, $j=1,2$, $0<\alpha<1$, $0<\alpha_k<1$, $k=1,2$, $\alpha_1+\alpha_2=\alpha$.

Consider $\tilde{a}_i = (\bar{a}_i - Z_i(\alpha_1), \bar{a}_i, \bar{a}_i + Z_i(\alpha_2); 1-\alpha)$, where $Z_i(\alpha_k) = t_{N-1}(\alpha_k)\dfrac{S_i}{\sqrt{N}}$, $k=1,2$. The decision maker can determine α_1 and α_2 as follows. Let $a_i^* \equiv d(\tilde{a}_i, \tilde{0}) = \bar{a}_i + \frac{1}{4}(Z_i(\alpha_2) - Z_i(\alpha_1))$. a_i^* is the estimate of a_i in the fuzzy sense. We divide the statistical data a_{ip} into two sets. Let $\bar{a}_i = \frac{1}{N}\sum_{p=1}^{N} a_{ip}$, $A = \{P \mid \bar{a}_i \le a_{ip}\}$ and $B = \{P \mid a_{ip} < \bar{a}_i\}$. Suppose the numbers of the element in A and B are n_1 and n_2 respectively. Then $S_A^2 = \frac{1}{n_1}\sum_{p\in A}(a_{ip}-\bar{a}_i)^2$ and $S_B^2 = \frac{1}{n_2}\sum_{p\in B}(a_{ip}-\bar{a}_i)^2$.

(1-1) If $S_B^2 < S_A^2$, the statistical data is skew to the right.
The decision maker can choose $0<\alpha_2<\alpha_1<1$ and $\alpha_1+\alpha_2=\alpha$, $0<\alpha<1$. Then $t_{N-1}(\alpha_1) < t_{N-1}(\alpha_2)$. It follows that $\bar{a}_i < a_i^*$. Since the statistical data is skew to the right, it will be better to choose a_i^*, the estimate of a_i, to be greater than $\bar{a}_i$.

(1-2) If $S_A^2 < S_B^2$, the statistical data is skew to the left.
The decision maker can choose $0<\alpha_1<\alpha_2<1$ and $\alpha_1+\alpha_2=\alpha$, $0<\alpha<1$. Then $t_{N-1}(\alpha_2) < t_{N-1}(\alpha_1)$. It follows that $a_i^* < \bar{a}_i$. Since the statistical data is skew to the left, it will be better to choose a_i^*, the estimate of a_i, to be less than $\bar{a}_i$.

(1-3) If $S_A^2 = S_B^2$, the decision maker can choose $\alpha_1 = \alpha_2 = \dfrac{\alpha}{2}$. Then $\bar{a}_i = a_i^*$.

(2) Consider $\tilde{b}_j = (\bar{b}_j - Z_i(\alpha_3), \bar{b}_j, \bar{b}_j + Z_i(\alpha_4); 1-\alpha)$, $0<\alpha_k<1$, $k=3,4$, $\alpha_3+\alpha_4=\alpha$.

Let $b_j^* \equiv d(\tilde{b}_j,\ \tilde{0}) = \bar{b}_j + \frac{1}{4}(Z_j(\alpha_4) - Z_j(\alpha_3)),$ where α_3 and α_4 can be decided by the decision maker as described in (1).

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