JOURNAL OF INFORMATION SCIENCE AND ENGINEERING 23, 885-892 (2007)

Short Paper

Optimal Frame Payload Size for Mobile Communications over Wireless Links of Cellular Networks[*]

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Internet accesses from mobile phones over cellular networks suffer from severe bandwidth limitations and high bit error rates in the wireless access links. Tailoring wireless link connections to best fit the characteristics of the link is thus very important. In this work, we propose a simple algorithm to find the optimal frame payload size in order to maximize the effective utilization of the wireless link. With some numerical examples, it is shown that the optimal frame payload size becomes a constant value when the window size of the wireless link exceeds a threshold. One can set the frame payload size of the wireless link to this optimal frame payload size for maximum efficiency on the expensive wireless link.

Keywords: optimal frame size, cellular network, mobile communications, wireless link, link utilization

1. INTRODUCTION

In recent years cell phone data services have proliferated in several countries, where the telecommunications operators' revenues from the data services is going to take a considerable share among the total revenues. This trend is now spreading all over the world. Typical examples of wireless data applications include transferring text messages, pictures, videos or MP3 music. However, data transmission over a wireless link usually shows serious performance degradation due to retransmissions on the wireless link, in which the bit error rate is high.

It has been widely studied to maximize the data transmission performance over the wireless link. A sliding window protocol with retransmission is a typical scheme used in the data link layer for error recovery. The sender can send *N* frames without acknowl-

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Received November 1, 2005; revised February 3 & May 29, 2006; accepted July 10, 2006.
Communicated by Yu-Chee Tseng.

[*] This work of GooYeon Lee was supported by grant No. B1220-0601-0032 from the University Fundamental Research Program of the Ministry of Information Communication in Republic of Korea. And the work of Hwa Jong Kim was supported in part by MIC, Korea under the ITRC program (C1090-0603-0035) supervised by IITA. This work was also partially supported by Kangwon Institute of Telecommunications and Information (KITI)

edgement, where N is determined by the flow control based on the receiver buffer size. The receiver requests retransmission for an error frame [1]. For a given window size, frame error rate is determined especially based on the size of the frame size.

Smaller MTU is preferred because smaller frame size increases the chance of successful transmission. For a fixed bit error rate, the probability of frame error increases exponentially and so does frame retransmission probability, as the frame length gets larger. However, too small a frame size would result in a low efficiency for a given frame overhead such as the header and trailer

In this work, we analyze the relation between the frame payload size and the wireless link utilization. A formula that gives an optimum frame payload size is derived in this analysis. There is a similar formula for the optimum frame payload size originally derived in [2] and used by [3]. However, that formula does not take the effects of finite window size and non-negligible propagation delay into account. Our analysis considers these factors in finding the optimum frame payload size. Our formula can be used for other studies to improve link utilization adaptively as in [4, 5]. This is not the only work on the link layer protocol with finite window size and non-negligible propagation delay. Ikegawa and Takahashi also studied similar models, however, their work was not finding optimal frame size but deriving the mean frame size under the given model [6].

The telecommunications operators would be interested in the result because it enables them to accommodate more users just by a carefully adjusting the frame payload size and thus maximizing the wireless link utilization. The result shows that a change in the frame payload size makes a considerable difference in the wireless link utilization.

2. MODEL AND ANALYSIS

Wireless link access between a cell phone and cellular networks is modeled in this section. In this work, we focus on the *effective* utilization defined by the average transmission rate of the payload data excluding wireless link overheads divided by the total link capacity. We will derive an expression of the steady state effective utilization for the wireless link, and suggest a guideline to get an optimal frame payload size in order to maximize the effective utilization. The effective utilization of the wireless link is affected by the retransmission probability, data link layer (frame) overhead, and window size. We assume that a frame with errors is retransmitted after a round-trip time. That is, the time needed to detect a frame error is ignored.

In a selective repeat ARQ model which is widely used in modeling a link layer, the link utilization, U, is well known [7] and is given by

$$U = 1 - P_e \qquad \text{when } N \geq 2a + 1 \tag{1}$$

$$U = \frac{N}{2a+1}(1 - P_e) \qquad \text{when } N \leq 2a + 1 \tag{2}$$

where N is the window size (in transmission frames), P_e is the frame-error probability, and a is the normalized propagation delay defined by

$$a = \frac{\textit{propagation time}}{\textit{transmission time}}. \tag{3}$$

We now derive the effective utilization of the wireless link using Eqs. (1) and (2). The effective utilization can be expressed as the product of the wireless link utilization and the link efficiency on the wireless link.

The wireless link utilization can be calculated from Eqs. (1) or (2) if we use the selective ARQ model. In this calculation P_e should be interpreted as the probability that a frame has errors. The frame format considered in the derivation is shown in Fig. 1.

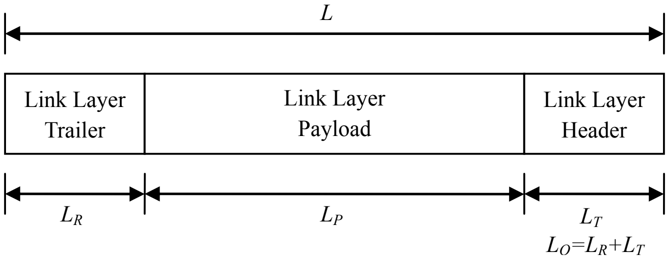

Fig. 1. Frame format on the wireless link.

The normalized propagation delay a can be written as $a=\frac{t_p}{t_r}$, where t_p is the propagation delay and t_r is the frame transmission time. The frame transmission time is given by $\frac{L}{R}$, where L is the frame size, and R is the transmission rate on the wireless link. Therefore, we have

$$a = \frac{t_p}{t_r} = \frac{t_p}{L/R} = \frac{R \cdot t_p}{L}. \tag{4}$$

We will find the optimum frame payload size that maximizes the effective utilization. The effective utilization, U_A, can be expressed as

$$U_A = \frac{L_P}{L}U = \frac{L_P}{L_P + L_O}U. \tag{5}$$

From Eq. (4), the condition for the Eq. (1), $N \geq 2a + 1$, can be expressed as

$$N \geq \frac{2R \cdot t_p}{L} + 1. \tag{6}$$

Since $L = L_P + L_O$, we have $L_P \geq \frac{2Rt_p}{N-1} - L_O$, and we let

$$L_B = \frac{2Rt_p}{N-1} - L_O. \tag{7}$$

For $N \leq 2a + 1$, on the other hand, we have $L_P \leq L_B$. To summarize,

$$L_P \geq L_B \text{ for } N \geq 2a + 1 \tag{8}$$

$$L_P \le L_B \text{ for } N \le 2a + 1. \tag{9}$$

Now let us consider the frame error probability P_e in Eqs. (1) and (2). P_e is determined by the bit error probability of the wireless link B_e, the coding scheme used on the wireless link, and the frame size L. B_e is a parameter determined by the wireless link, which we assume to be fixed. P_e is a function of L_P, where L_P is the frame payload size which is determined by subtracting the overhead from the frame size L. Generally the curve of P_e versus L_P changes according to the coding scheme used on the wireless link. However, regardless of the coding scheme, P_e converges always to 1 as L_P increases to infinity. In numerical notation, $\lim_{L_P\to\infty} P_e = 1$. Therefore, we can say that $\lim_{L_P\to\infty} U_A = 0$ from Eqs. (1) and (2). Furthermore, we notice that $\lim_{L_P\to 0} U_A = 0$ from Eq. (5). Since U_A is always non-negative, U_A has a maximum at some point of L_P as L_P increases. That point corresponds to the optimal frame size that maximizes the effective utilization.

We consider a simple case where there is no coding on the wireless link layer in this work. Then, the frame error probability is given by

$$P_e = 1 - (1 - B_e)^L = 1 - (1 - B_e)^{L_P+L_O}. \tag{10}$$

When a coding scheme is used, once the relation between P_e and L_P is given, the remaining procedure can be applied the same way.

First let us consider the case of $N \ge 2a + 1$. By combining Eqs. (1), (5), and (10), we have

$$U_A = \frac{L_P}{L_P + L_O}U = \frac{L_P}{L_P + L_O}(1 - P_e) = \frac{L_P}{L_P + L_O}(1 - B_e)^{L_P+L_O}. \tag{11}$$

U_A in Eq. (11) is always positive and has a unique maximum value. The maximum value and the corresponding value of L_P is obtained from $\frac{dU_A}{dL_P} = 0$ which makes

$$L_P^2 + L_O L_P + \frac{L_O}{\ln(1 - B_e)} = 0. \tag{12}$$

Eq. (12) has only one solution since L_P should be positive. We call the positive one L_{P1}, which is given by

$$L_{P1} = \frac{-L_O + \sqrt{L_O^2 - 4 \cdot \frac{L_O}{\ln(1 - B_e)}}}{2}. \tag{13}$$

The maximum U_A can be obtained from Eqs. (13) and (11). However, we note that Eq. (13) should satisfy the condition $L_{P1} \ge L_B$ in Eq. (8). If $L_{P1} \le L_B$, the maximum U_A is obtained when $L_P = L_B$, since U_A is monotonically decreasing in the region. To summarize, the maximum U_A for the case of $N \ge 2a + 1$, $U_{A,\max}$, is given by

$$U_{A,\max 1} = U_A \big|_{L_P = L_{P1}} \quad \text{when } L_{P1} \ge L_B \tag{14}$$

$$U_{A,\max 1} = U_A \big|_{L_P = L_B} \quad \text{when } L_{P1} \le L_B. \tag{15}$$

Next we consider the case of $N \leq 2a + 1$. Combining (2), (4), (5) and (10), we have

$$U_A = \frac{L_P}{L} \cdot \frac{N}{2 \cdot \frac{R \cdot t_p}{L} + 1} \cdot (1 - B_e)^L = \frac{N \cdot L_P}{2 \cdot R \cdot t_p + L} \cdot (1 - B_e)^L$$

$$= \frac{N \cdot L_P}{L_P + L_O + 2 \cdot R \cdot t_p} \cdot (1 - B_e)^{L_P + L_O}. \tag{16}$$

U_A in Eq. (16) is always positive and has a unique maximum value. The maximum value and the corresponding value of L_P is obtained from $\frac{dU_A}{dL_P} = 0$ which makes

$$L_P{}^2 + (L_O + 2 \cdot R \cdot t_p)L_P + \frac{L_O + 2 \cdot R \cdot t_p}{\ln(1 - B_e)} = 0. \tag{17}$$

Eq. (17) has only one solution since L_P should be positive. We call the positive one L_{P2}, which is given by

$$L_{P2} = \frac{-(L_O + 2 \cdot R \cdot t_p) + \sqrt{(L_O + 2 \cdot R \cdot t_p)^2 - 4 \cdot \frac{L_O + 2 \cdot R \cdot t_p}{\ln(1 - B_e)}}}{2}. \tag{18}$$

The maximum U_A can be obtained from Eqs. (18) and (16). However, we note that Eq. (18) should satisfy the condition $L_{P2} \leq L_B$ in Eq. (9). If $L_{P2} \geq L_B$, the maximum U_A is obtained when $L_P = L_B$, since U_A is monotonically increasing in the region. To summarize, the maximum U_A for the case of $N \leq 2a + 1$, $U_{A,\max 2}$, is given by

$$U_{A,\max 2} = U_A \big|_{L_P = L_{P2}} \text{ when } L_{P2} \leq L_B \tag{19}$$

$$U_{A,\max 2} = U_A \big|_{L_P = L_B} \text{ when } L_{P2} \geq L_B. \tag{20}$$

From Eqs. (14), (15), (19) and (20), the maximum effective utilization, $U_{A,\max}$ is given by $U_{A,\max} = \max(U_{A,\max 1}, U_{A,\max 2})$ and its corresponding L_P will be the optimal frame payload size.

3. NUMERICAL RESULTS AND DISCUSSION

As a numerical example we consider the effective utilization of the wireless link when $t_p = 0.02$ ms, $R = 2$ Mbps, $L_O = 5$ bytes, $N = 5$. The effective utilization versus the frame payload size in this case is shown in Fig. 2.

In this case, L_B is calculated to be -20 bits from Eq. (7). The optimal L_P should be one of the values of $\max(L_{P1}, L_B)$ and $\min(L_{P2}, L_B)$ from Eqs. (14), (15), (19) and (20). Since $L_B = -20$ in this case, the optimal L_P is L_{P1}. The optimal L_P values in Fig. 2 are 6,304 bits, 1,980 bits, 612 bits and 180 bits at $B_e = 0.000001$, 0.00001, 0.0001 and 0.001, respectively.

Fig. 3 shows the effective utilization when $t_p = 0.05$ ms, $R = 10$ Mbps, $L_O = 5$ bytes, $N = 5$. In this case, L_B is 210 bits. In the figure, in the case of $B_e = 0.001$, $L_{P1} = 181$ bits

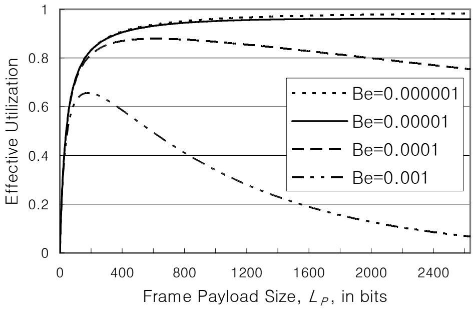

Fig. 2. Effective utilization versus frame payload size when $t_p = 0.02$ ms, $R = 2$ Mbps, $L_O = 5$ bytes and $N = 5$.

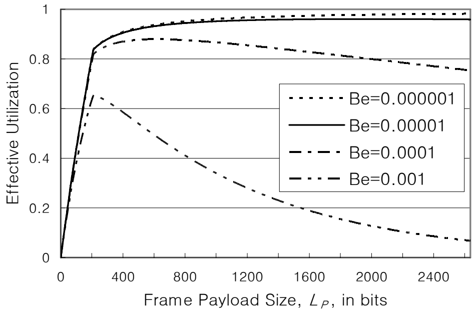

Fig. 3. Effective utilization versus frame payload size when $t_p = 0.05$ ms, $R = 10$ Mbps, $L_O = 5$ bytes and $N = 5$.

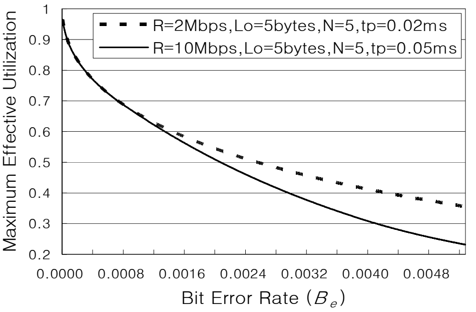

Fig. 4. Maximum effective utilization vs. bit error rate.

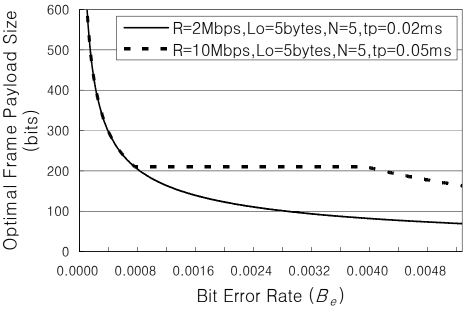

Fig. 5. Optimal frame payload size vs. bit error rate.

and L_{P2} = 625 bits. Therefore, $L_{P1} \le L_B \le L_{P2}$ and the value of max(L_{P1}, L_B) and min(L_{P2}, L_B) is L_B. Thus, we have the optimal frame payload size at $L_P = L_B =$ 210 bits. For the other cases of the figure, L_{P1} and L_{P2} are both greater than L_B. Thus, the optimal frame payload size is L_{P1}.

In the Figs. 2 and 3, we can see that the effective utilization and the optimal frame payload size decrease (or non-increase for some interval) as the bit error rate on the wireless link gets large. The tendency can be also seen in Figs. 4 and 5. Note that the low effective utilization at a small frame payload size is due to the relatively large frame overhead and the low effective utilization at a large frame payload size is due to the high frame error rate.

From Figs. 2 and 3 we see that the effective utilizations are fairly large (more than 0.8) for a broad range of frame payload size when the bit error rate is 10^{-6}. On the contrary, when $B_e > 0.00001$, the maximum effective utilization decreases fast as the frame payload size increases. Therefore decision of the frame payload size is very important when the bit error rate is relatively large, as in the wireless link.

Figs. 4 and 5 show the maximum effective utilization and the optimal frame payload size of the wireless link, respectively for varying bit error rate. In the figures, we see that the maximum effective utilization and the optimal frame payload size decreases as the bit error rate increases. In Fig. 5, there is a flat region of solid line. It is the case of $L_{P1} \le L_B$

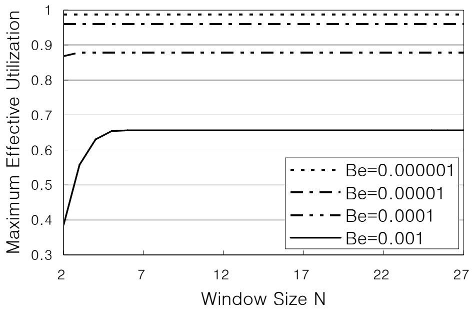

Fig. 6. Maximum effective utilization versus window size (t_p = 0.05 ms, R = 10 Mbps, L_O = 5 bytes).

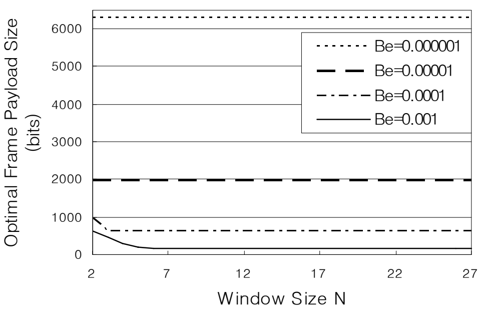

Fig. 7. Optimal frame payload size versus window size (t_p = 0.05 ms, R = 10 Mbps, L_O = 5 bytes).

$\leq L_{P2}$. Therefore, in the region, the value of max(L_{P1}, L_B) and min(L_{P2}, L_B) is L_B. Thus, we have the optimal frame payload size at $L_P = L_B = 210$ bits in the region.

Figs. 6 and 7 show the maximum effective utilization and the corresponding optimal frame payload size for varying window size. When the window size is small, the effective utilization increases and the optimal frame payload size decreases as the window size gets large. This is the window-limited region. However, when the window size grows over a threshold, both of the maximum effective utilization and the optimal frame payload size curves remain flat at fixed values, in which region the data flow is limited by the wireless link bandwidth. In other words, the optimal frame payload size is independent of the window size. Thus, to get the highest maximum effective utilization, we can set the window size to the threshold window size and design the receiver buffer capacity by the product of the threshold window size and the optimal frame size.

4. CONCLUSION

The maximum effective utilization of a wireless link for cellular networks can be obtained by setting an optimal frame payload size which can be calculated from the proposed algorithm. The possibility of increasing the effective bandwidth of a wireless link just by adjusting the frame payload size is very attractive. The result of this work will be very useful for the telecommunications operators who provide data access over the cell phone wireless links.

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