JOURNAL OF INFORMATION SCIENCE AND ENGINEERING 23, 1213-1225 (2007)

Robust Fundamental Matrix Estimation with Accurate Outlier Detection

JING-FU HUANG, SHANG-HONG LAI AND CHIA-MING CHENG
Department of Computer Science
National Tsing Hua University
Hsinchu, 300 Taiwan

The estimation of fundamental matrix from two-view images has been an important topic of research in 3D computer vision. In this paper, we present an improved robust algorithm for fundamental matrix estimation via modification of the RANSAC algorithm. The proposed algorithm is based on constructing a voting array for all the point correspondence pairs to record the consistency votes for each point correspondence from a number of the fundamental matrix estimations determined from randomly selected subsets of correspondence pairs to facilitate the identification of outliers. The boundary between the inliers and outliers in the sorted voting array are determined through a hypothesis testing procedure. With this strategy, we can accurately determine the outliers from all pairs of point correspondences, thus leading to accurate and robust fundamental matrix estimation under noisy feature correspondences. Through experimental comparison with previous methods on simulated and real image data, we show the proposed algorithm in general outperforms other best-performed methods to date.

Keywords: fundamental matrix estimation, robust estimation, RANSAC, outlier detection, two-view geometry, stereo vision

1. INTRODUCTION

The main problem in 3D computer vision is to recover 3D information from multiple 2D images. Epipolar geometry provides basic foundation for computing 3D information from two viewed images [1, 2, 5, 6, 9]. A 3-by-3 fundamental matrix has been introduced to describe the epipolar geometry between two cameras [1, 2, 5, 6]. The estimation of fundamental matrix from point correspondences in two-view images of the same scene acquired by two cameras is essential to many tasks in 3D computer vision.

A number of researchers have proposed different algorithms for estimating the fundamental matrix from feature point correspondences in the past decade. They can be roughly divided into three different approaches; namely, the linear estimation approach, iterative estimation approach and robust estimation approach [1, 2]. We will very briefly review these three approaches in the following.

The idea of the linear estimation approach for fundamental matrix estimation is to take each fundamental matrix equation that corresponds to a pair of correspondence points as a linear equation and combine all these linear equations to form a homogeneous linear system, which is normally over-determined. Then, the seven-point method [2] or the eight-point method [5, 6, 10] has been proposed to compute the fundamental matrix from this homogeneous linear system. The advantages of the linear estimation approach are its simplicity for implementation and computational efficiency, but it is sensitive to

Received April 11, 2005; revised July 1, 2005; accepted September 29, 2005.
Communicated by Chin-Teng Lin.

noises in the point correspondences. This problem is especially serious when there are outliers in the point correspondences. In practice, outliers in pairs of corresponding feature points are inevitable especially for an automatic feature extraction and correspondence process, since inconsistency in feature extraction or mistake in feature correspondence are common problems in real-life applications.

The iterative estimation approach [2, 5] has been proposed to minimize the sum of geometric distances between the feature points and the corresponding epipolar lines, which are determined from the corresponding feature coordinates and the fundamental matrix. Due to the non-linearity of the distance function, the function minimization is achieved by updating the fundamental matrix iteratively. Some variants of the distance function have been proposed as the fundamental numerical scheme and constrained fundamental numerical scheme [2]. The advantage of this approach is its direct relation to a meaningful geometric measure. However, its weakness is still the noise sensitivity, especially when there are outliers in the feature correspondences.

To alleviate the effect of outliers to the fundamental matrix estimation, the robust estimation approach has been proposed in the past. Several different robust estimation methods have been employed to the fundamental matrix estimation problem. The three representative robust methods are the M-estimator [2, 6, 11, 12], LMedS (Least Median of Squares) [1, 6, 11] and RANSAC (RANdom SAmple Consensus) estimations [4-6, 13]. The robust estimation approach can deal with the problem of the false point correspondences and have a better tolerance to data noise, but it requires much more computational efforts than the other two approaches.

Although the robust estimation approach can provide more robust estimation of the fundamental matrix in the presence of outliers or incorrect point correspondence pairs, the current robust estimation methods still have their limitations. For example, the M-estimator [6] requires a good initial guess for the fundamental matrix or works under low percentages of outliers. The RANSAC algorithm [4, 5] needs the information of the percentage of outliers, which is usually not available. The LMedS [1] estimation does not require such information, but it is very time-consuming.

In this paper, we develop a modified robust algorithm for estimating the fundamental matrix and detecting outliers simultaneously. In our algorithm, we modify the voting concept in RANSAC to build a voting array for the point correspondence pairs to indicate the possibility of being an outlier for each pair of point correspondence. Then, we propose an effective hypothesis testing procedure to precisely separate the inliers from outliers based on this voting array. By using this new robust algorithm, we can not only accurately estimate the fundamental matrix but also precisely identify the outliers from the set of point correspondence pairs. Experimental results prove the robustness and accuracy of the proposed algorithm on benchmarking datasets under different levels of noises and outliers.

The rest of this paper is organized as follows. In the next section, we describe the two traditional robust algorithms for fundamental matrix estimation; namely, the M-estimator and RANSAC method. Then, we describe the proposed robust algorithm for fundamental matrix estimation and outlier identification in section 3. Some experiment results of applying the proposed algorithm with comparisons to previous methods on benchmarking datasets are given in section 4. Finally, we conclude this paper in the last section.

2. TRADITIONAL ROBUST FUNDAMENTAL MATRIX ESTIMATION

In two-view epipolar geometry, a point in one image must lie on the corresponding epipolar line in the other image. This property is very helpful in finding point correspondences in stereo vision. This epipolar geometry can be described by the equation of the fundamental matrix given as follows:

$$m_k'^T F m_k = 0 \tag{1}$$

where m_k and m'_k are the kth pair of corresponding points in the two-viewed images represented in homogeneous coordinate, and F is the 3×3 fundamental matrix describing this epipolar geometry. Thus, the problem of fundamental matrix estimation is to estimate the 3×3 fundamental matrix from a set of point correspondences (m_k, m'_k), $k = 1, \ldots, n$, such that Eq. (1) can be satisfied as best as possible.

It is obvious that the above problem is a linear estimation problem, which can be easily solved by linear least-square estimation methods. However, automatic methods for finding point correspondence make mistakes very often in practice, thus introducing outliers in the point correspondences for the fundamental matrix estimation. It is well-known that least-square estimation is very sensitive to outliers [11], therefore several robust estimation techniques have been applied to overcome the outlier problem. Among them, M-estimator and RANSAC technique have been widely used in many computer vision problems. Since these two methods are closely related to our proposed algorithm for robust fundamental matrix estimation, we briefly describe them in the following.

2.1 M-Estimation

Let the residue of the fundamental matrix constraint for the kth point correspondence pair be denoted by $r_k = m_k'^T F m_k$. The least-square estimation is to find the matrix F such that the sum of squared residues is minimized, *i.e.*,

$$\hat{F} = \arg \min_F \sum_{k=1}^{n} r_k^2. \tag{2}$$

As mentioned before, least-square estimation is sensitive to outliers. The M-estimators try to reduce the influence of outlier by replacing the square function by some robust ρ functions. Two of the well-known ρ functions are the Cauchy function ρ_C and the Huber function ρ_H given as follows [11]:

$$\rho_C(r) = \frac{b^2}{2} \log\left[1 + \left(\frac{r}{b}\right)^2\right], \ \rho_H(r) = \begin{cases} \frac{1}{2} r^2, & |u| \le c \\ \frac{1}{2} c(2|r| - c), & c < |r| \end{cases}.$$

Thus, the M-estimation is to find the solution such that the following cost function is minimized:

$$\sum_{k=1}^{n} \rho(r_k). \tag{3}$$

The above non-linear energy function can be minimized by an iteratively reweighed least square algorithm when a good initial guess is available. In short, we solve the following weighted least square minimization problem in the ith iteration [2, 11, 12].

$$\hat{F}_i = \arg\min_F \sum_{k=1}^{n} w(r_k)\, r_k^2 \tag{4}$$

where the weight associated with each pair of point correspondence is given by

$$w(x) = \frac{\psi(x)}{x} \tag{5}$$

and $\psi(x) = \frac{d\rho(x)}{dx}$ is called the influence function in robust estimation [2, 11, 12]. In each iteration of the iteratively re-weighted least square algorithm, we first compute the weights for all the correspondence pairs based on their residues computed with the current estimate of the fundamental matrix, and then the fundamental matrix estimate is updated with the weighted least square estimation in Eq. (4). Note that the M-estimator requires a good initial guess for the fundamental matrix, which is usually provided by the standard least square estimation. When the outlier percentage is high, the initial guess for the fundamental matrix can be far away from the true solution. Thus, M-estimator cannot work well for the case with high percentage of outlier. In other words, M-estimator has a low breakdown point [11, 14].

2.2 RANSAC

The random sampling consensus (RANSAC) technique originally proposed by Fischler and Bolles [13] has been widely used for robust parameter estimation to overcome the outlier problem. The basic idea of using RANSAC for fundamental matrix estimation is as follows: randomly selecting a number of minimal subsets of point correspondences to determine the fundamental matrix for each subset, and then find the best fundamental matrix that is most consistent with the entire set of point correspondences. Given a set of point correspondences, denoted by $M = \{(m_k, m'_k) \mid k = 1, \ldots, n\}$, the RANSAC algorithm for fundamental matrix estimation consists of the following steps [5, 6, 13]:

1. Randomly select a number of subsets of seven [2] or eight [5, 6, 10] point correspondences from the entire set M. (Note that the minimal size of the subset depends on the Fundamental matrix estimation method in the next step.)
2. For each subset, indexed by j, compute the corresponding fundamental matrix F_j by using a linear estimation algorithm [2, 5, 6, 10].
3. For each estimate F_j, compute the residue r_k for each point correspondence and count the total number of consistent correspondences, *i.e.* $r_k^2 < \sigma^2$, where σ^2 is a predefined constant.
4. Keep the fundamental matrix F_j that yields the most consistent correspondences.
5. Refine the fundamental matrix estimation by applying the linear estimation algorithm or the M-estimator on the set of consistent point correspondences only.

Assume the outlier percentage in the entire correspondence set M is ε. Thus, the total number of randomly selected subsets N required in RANSAC to achieve a probability P that at least one selected subset does not contain any outlier is given by [5, 6, 13]

$$N = \frac{\log(1-P)}{\log[1-(1-\varepsilon)^q]} \tag{6}$$

where q is the size of the minimal subset, *i.e.* 7 or 8 in this case.

3. PROPOSED ROBUST ESTIMATION ALGORITHM

The proposed algorithm is based on modification of the RANSAC algorithm to achieve more precise identification of outliers, thus leading to more robust and accurate fundamental matrix estimation. RANSAC has better ability in detecting outliers and M-estimator can provide very accurate estimation under the condition when there are very few outliers. In our robust algorithm, we modify the RANSAC method to accumulate the votes for each pair of point correspondence into a voting array that intuitively indicates the probability of being an inlier. Then, we use a hypothesis-and- testing scheme to precisely identify outliers from the set of correspondence pairs. Finally, an M-estimator is applied to the set of identified inliers to estimate the fundamental matrix. Fig. 1 gives the complete procedure of the proposed robust algorithm.

1. Normalize coordinates of all match points. The data used later are in the normalized coordinates.
2. Initialize $i = 1$ and the voting array to 0's. Select a random subset of 8 pairs of corresponding points and compute the fundamental matrix F_i on this subgroup of correspondence pairs by the eigenvalue minimization method.
3. Compute the geometric distances between all data points and their corresponding epipolar lines. Determine inliers or outliers for each pair of match points based on the standard deviation of the geometric distances. For each pair that was termed an inlier, increment the corresponding element of the voting array by 1.
4. Set $i = i + 1$ and repeat steps 2 and 3 until i reaches N.
5. Sort the voting array in a non-increasing order. This sorted array can be viewed as the probability of being an inlier.
6. Determine the boundary of inliers and outliers from the sorted voting array.
7. Take all pairs of match points that were classified as inliers to compute the fundamental matrix by using the M-estimator.
8. Denormalize the fundamental matrix based on the transformation in step 1.

Fig. 1. Proposed robust fundamental matrix estimation algorithm.

At the first step, we normalize the coordinates of all match points. The data used in the later steps are in the normalized coordinate. We adopt the coordinate normalization process proposed by Hartley [5]. The normalization consists of two stages. At the first stage, we compute the mean of all match points and then take it to be the origin of the normalized coordinate system. This is accomplished by subtracting the mean from the coordinates of all data points. At the second stage, we multiply a scale on these translated coordinates to make the root mean square distance between these data points and the origin to be $\sqrt{2}$. Note that this normalization is performed on each view of the image separately.

At the second step, we select a random subgroup of 8 pairs of corresponding points and estimate the fundamental matrix F_i by the eigenvalue minimization. For each F_i, we compute the geometric distances between all data points and their corresponding epipolar lines. The geometrical distance between the kth match points and the corresponding epipolar lines are defined as follows

$$r_k = \frac{\left|X_k'^T \cdot F_i \cdot X_k\right|}{\sqrt{l_{k1}^2 + l_{k2}^2}} + \frac{\left|X_k'^T \cdot F_i \cdot X_k\right|}{\sqrt{l_{k1}'^2 + l_{k2}'^2}}, \text{ for } k = 1, \ldots, n \tag{7}$$

where X_k and X_k' are the kth pair of correspondence points in a normalized homogeneous coordinate, and the corresponding epipolar lines are given by

$$l_k = F_i^T X_k' = (l_{k1}, l_{k2}, l_{k3}),\ l_k' = F_i X_k = (l_{k1}', l_{k2}', l_{k3}'). \tag{8}$$

We assume that the error distribution of the measured points excluding the outliers can be approximated by a Gaussian function. From robust statistics [12, 15], we can obtain a robust estimate of the standard deviation of the geometric distances by the following equation,

$$\sigma = 1.4826(1 + 5/(n - q)) \underset{k}{median} |r_k|. \tag{9}$$

Note that the above equation is the median absolute deviation (MAD) scale estimate [15]. The first magic number is obtained from the inverse of the cumulative normal distribution, and the term $(1 + 5/(n - q))$ is the finite sample correction factor with the total number of parameters $q = 8$ in this case. The details of the derivation can be found in [15]. According to the distribution model, we distinguish the inliers from their geometric distances of each pair of corresponding points. We declared it as an inlier while the geometric distance is less than 3σ, since 99.14% of the data points lies within $\pm 3\sigma$ under the assumption of the Gaussian distribution error model.

A voting array accumulates the number of votes that a pair of correspondence points is declared to be an inlier during the random subset sampling with fundamental matrix estimation and consistency checking process in our proposed algorithm. After repeating the random sampling and the voting process for certain trials, we sort the voting array in a non-increasing order. Note that this sorted voting array can be regarded as the sorted probability of being an inlier. An example of a sorted voting array is depicted in Fig. 2.

To determine a precise cut-point between inliers and outliers from the voting array, we develop a hypothesis-testing scheme to find the best boundary between them. In this hypothesis-testing scheme, if the first i pairs of match points in the sorted voting array are regarded as inliers and the rest as outliers, then we compute the fundamental matrix

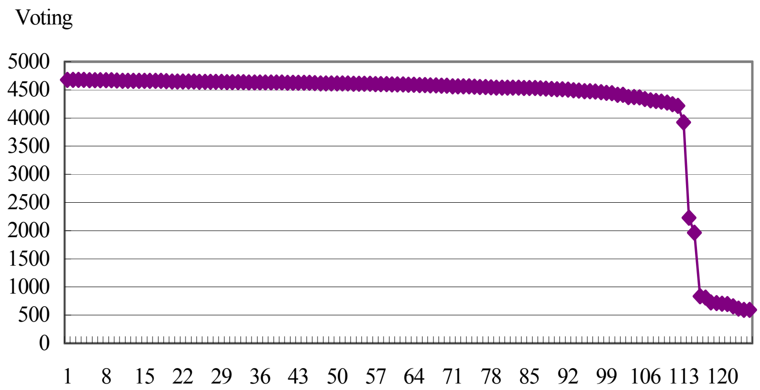


Fig. 2. An example of a sorted voting array.

F_i with these pairs of inliers and classify each pair of match points to be an inlier or an outlier based on their geometric distance and the standard deviation for the current fundamental matrix estimate F_i. Note that we use M-estimator here for computing the fundamental matrix F_i to alleviate the problem of mistakes in the sorted voting array. Then we store the total number of points being classified as inliers into the array *InlierNum*[*i*] and the mean geometric distance for all inliners into the array *D*[*i*]. If the hypothesis is true, then the total number of inliers *i* should confirm to the total number of the classified inliers *InlierNum*[*i*]. We define a relative difference between the hypothesis and the testing as follows

$$relativeDiff[i] = \frac{|i - InlierNum[i]|}{i}. \quad (10)$$

If *relativeDiff*[*i*] is small enough, we mark it as a possible cut-point. From all the possible cut-points, we select the one with a minimal mean geometric distance *D*[*i*] to be the final cut-point. Finally, the fundamental matrix **F** is determined by using the M-estimator on the inliers whose locations are before the cut-point in the sorted voting array.

Note that the above new voting strategy is significantly different from the voting scheme in the traditional RANSAC method. Our new voting strategy collects votes for each correspondence pair by checking the consistency between the point correspondence pair and the corresponding fundamental matrices determined from all the random subset samples. On the contrary, traditional RANSAC collects votes for each random subset sample by computing the total number of votes for the corresponding fundamental matrix from all the point correspondence pairs. The traditional RANSAC method selects the subset sample with the maximal votes to determine the outlier, while our modified voting strategy make use of all the random sampling and voting information to identify the outliers. Obviously, our modified RANSAC algorithm has the advantage of fully exploiting the information from random sampling and voting to make the final decision of outliers. Thus, it should lead to a more robust fundamental matrix estimation algorithm.

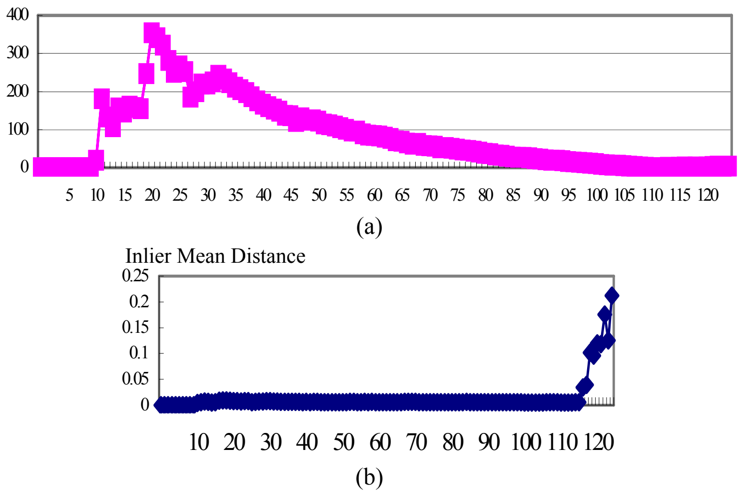


Fig. 3. An example of (a) a relative difference array *relativeDiff*[*i*] and (b) a mean geometric distance array *D*[*i*].

4. EXPERIMENTAL RESULTS

In this section, we show some experimental results of our robust algorithm. We compare it with the results in the survey by Armangue and Salvi [1]. The test data and matlab code are provided by them on the Internet (http://eia.udg.es/~armangue/research). We took the test data of the author to be the input to our algorithm. The test data includes simulation data and real image data. In addition, we include experiments with increasing percentages of outliers, which were not covered in the original paper.

First of all, we show the results of the experiments on the simulation data in [1] in Table 1. Note the vector (a, b) means the random noise added to the match points is a Gaussian distribution $N(0, a)$ and the percentage of the outlier is b. We can see from the table that only LMedS, MAPSAC and our proposed algorithm can provide accurate fundamental matrix estimation under different combinations of (a, b) in this experiments.

In the second experiment, we test the robustness of these algorithms with different outlier percentages. We vary the outlier percentages from 0% to 30% in the experiments. The accuracy comparison of all the methods on this experiment is summarized in Table 2. The percentage of the outlers usually needs to be known in some robust methods, such as RANSAC. Unfortunately, this information is normally not available. The proposed robust algorithm does not require this information. In fact, our algorithm not only estimates the outlier percentage but also identify the outliers from all pairs of point correspondences. When outlier percentage is up to 30%, only our algorithm and MAPSAC can work well. Our algorithm can identify outliers from the correspondence pairs very precisely and the corresponding geometric distance is the smallest in this experiment. It is obvious that the proposed robust algorithm outperforms the other methods in this experiment of varying outlier percentages.

The third experiment consists of four real image pairs also from the survey paper [1]. They are (1) urban scene, (2) underwater scene, (3) road scene and (4) kitchen scene.

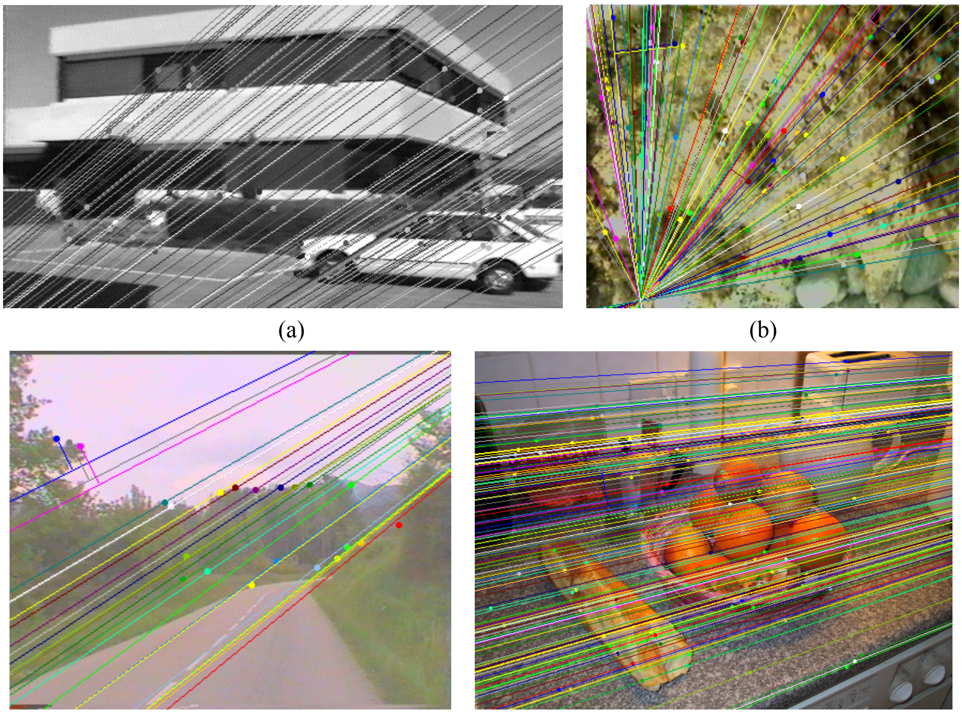
Table 1. Average distance errors (in pixels) of different fundamental matrix estimation methods under different noise levels *a* and different percentages of outliers *b*.

Methods \ (a, b)	(0.0, 0%)	(0.0, 10%)	(0.5, 0%)	(0.5, 10%)	(1.0, 0%)	(1.0, 10%)
Seven Points	14.250	25.370	163.839	140.932	65.121	128.919
Eigenvalue Minimization	0.000	17.124	0.538	19.262	1.065	21.264
Iterative Newton	0.000	20.445	0.538	31.740	1.068	37.480
Gradient eigen	0.000	18.224	0.554	19.409	1.071	18.730
FNS	0.000	17.124	0.538	22.302	1.065	18.374
CFNS	0.000	16.978	0.543	22.262	1.066	19.683
M-estimator proposed by Torr (IJCV97) Torr (1997IJCB)	0.000	4.714	0.367	3.147	0.814	4.089
LmedS	0.000	0.000	0.538	0.586	1.065	1.052
RANSAC	0.000	16.457	0.538	18.942	1.065	14.076
MLESAC	0.100	19.375	0.550	23.859	1.089	19.298
MAPSAC	0.011	0.115	0.762	0.629	1.072	1.041
Our Method	0.000	0.000	0.528	0.580	1.047	1.046

Table 2. Comparison of average geometric distances of various fundamental matrix estimation methods under different levels of outlier percentages. (unit: pixels)

Methods \ (a, b)	(1.0, 0%)	(1.0, 10%)	(1.0, 20%)	(1.0, 30%)
Seven Points	65.121	128.919	137.984	114.912
Eigenvaluse Minimization	1.065	21.264	30.971	42.531
Iterative Newton	1.068	37.480	53.544	116.438
Gradient-eigenvalue	1.071	18.730	23.999	40.152
FNS	1.065	18.374	27.365	33.325
CFNS	1.066	19.683	24.647	33.544
M-estimator Torr(1997IJCB)	0.814	4.089	4.564	4.528
LMedS	1.065	1.052	1.166	8.0959
RANSAC	1.065	14.076	30.971	35.392
MLESAC	1.089	19.298	35.882	45.698
MAPSAC	1.072	1.041	1.939	3.550
Our Method	1.047	1.046	1.178	1.157

A set of correspondence points for each pair of images are given [1]. We apply the proposed robust algorithm on these data sets. The results of fundamental matrix estimation are given by the corresponding epipolar lines and the associated error distances shown in Fig. 4. The average geometric distances of the inliers of different estimation methods on these four data sets are summarized in Table 3. It is obvious from the table that our robust algorithm outperforms the other robust methods, including RANSAC, LMedS,



(a) (b) (c) (d)

Fig. 4. The epipolar lines of the corresponding points computed from the fundamental matrix estimated by the proposed robust algorithm for (a) urban scene, (b) underwater scene, (c) road scene and (d) kitchen scene.

Table 3. Average geometric distances of various fundamental matrix estimation methods on the four pairs of real images. (unit: pixels)

Methods \ Scenes	Urban	Underwater	Road	Kitchen
Seven Points	51.633	97.977	27.668	16.956
Eigenvalue Minimization	0.440	1.725	0.609	2.623
Iterative Newton	0.468	1.752	0.559	2.966
Gradient-eigen	0.446	1.581	0.809	1.901
FNS	0.437	1.599	0.466	2.623
CFNS	0.437	1.609	0.505	1.892
M-estimator	0.279	0.475	0.310	0.263
LMedS	0.319	0.847	0.609	0.545
RANSAC	0.440	1.725	0.609	2.623
MLESAC	0.449	3.678	0.427	0.864
MAPSAC	0.440	1.000	0.471	0.582
Our Method	0.299	0.635	0.168	0.515

MLESAC and MAPSAC [1, 4, 7, 8]. It is noted that M-estimator has very high accuracy on these four examples. This is because the outlier percentage is much lower in this experiment. It is well-known that M-estimator performs very well under low outlier percentages. Since our algorithm is adaptive to the outlier percentage, we can see from these experiments that it consistently provides very accurate estimation under different levels of outliers and noises.

5. CONCLUSIONS

In this paper, we proposed a modified RANSAC algorithm for robustly estimating the fundamental matrix. Compared with the traditional RANSAC-like algorithms, our algorithm collects votes for each pair of point correspondence from all the random subset sampling with fundamental matrix estimation for consistency check, thus providing evidence for distinguishing inliers from outliers in the noisy data. Subsequently, a hypothesis-and-testing scheme is employed to precisely identify outliers from the sorted voting array. The proposed algorithm is very different from the traditional RANSAC algorithm mainly in the different voting processes. The RANSAC method collects votes for each random subset sampling by checking the consistency between the corresponding fundamental matrix and all pairs of point correspondences. In contrast, the proposed robust algorithm collects votes for each pair of point correspondence from the consistency checks of all the random sampling. Experimental results demonstrated the proposed algorithm in general outperforms other best-performed methods to date under different levels of outliers.

REFERENCES

1. X. Armangué and J. Salvi, "Overall view regarding fundamental matrix estimation," *Image and Vision Computing*, Vol. 21, 2003, pp. 205-220.
2. Z. Zhang, "Determining the epipolar geometry and its uncertainty: a review," *International Journal of Computer Vision*, Vol. 27, 1998, pp. 161-198.
3. Q. T. Luong and O. D. Faugeras, "The fundamental matrix: theory, algorithms, and stability analysis," *International Journal of Computer Vision*, Vol. 17, 1996, pp. 43-75.
4. P. H. S. Torr and D. W. Murray, "The development and comparison of robust methods for estimating the fundamental matrix," *International Journal of Computer Vision*, Vol. 24, 1997, pp. 271-300.
5. R. Hartley and A. Zisserman, *Multiple View Geometry in Computer Vision*, Cambridge University Press, London, 2000.
6. O. Faugeras and Q. T. Luong, *The Geometry of Multiple Images*, MIT Press, Cambridge, Massachusetts, 2001.
7. P. H. S. Torr and A. Zisserman, "MLESAC: a new robust estimator with application to estimating image geometry," *Computer Vision and Image Understanding*, Vol. 78, 2000, pp. 138-156.
8. P. H. S. Torr, "Bayesian model estimation and selection for epipolar geometry and generic manifold fitting," *International Journal of Computer Vision*, Vol. 50, 2002,

pp. 35-61.
9. H. C. Longuet-Higgins, "A computer algorithm for reconstructing a scene from two projections," *Nature*, Vol. 293, 1981, pp. 133-135.
10. R. Hartley, "In defense of the 8-point algorithm," in *Proceedings of the 8th International Conference on Computer Vision*, 1995, pp. 1064-1070.
11. C. V. Stewart, "Robust parameter estimation in computer vision," *SIAM Review*, Vol. 41, 1999, pp. 513-537.
12. G. Li, "Robust regression," in *Exploring Data Tables*, *Trends*, *and Shapes*, in D. C. Hoaglin, F. Mosteller, and J. W. Tukey, ed., Wiley, New York, 1985, pp. 281-343.
13. M. Fischler and R. Bolles, "Random sample consensus: a paradigm for model fitting with applications to image analysis and automated cartography," *Communications of the ACM*, Vol. 24, 1981, pp. 381-385.
14. E. Mosteller and J. Turkey, *Data and Analysis and Regression*, Addison-Wesley, Reading, MA, 1977.
15. P. Rousseeuw and A. Leroy, *Robust Regression and Outlier Detection*, John Wiley & Sons, New York, 1987.

Jing-Fu Huang (黃俊富) received his B.S. degree in Computer Science and M.S. degree in Information System and Application from National Tsing Hua University, Hsinchu, Taiwan in 2001 and 2003, respectively. He joined 精品科技 as a software engineer in 2004. His research interests include computer vision and image processing

Shang-Hong Lai (賴尚宏) received the B.S. and M.S. degrees in Electrical Engineering from National Tsing Hua University, Hsinchu, Taiwan, and the Ph.D. degree in Electrical and Computer Engineering from University of Florida, Gainesville, in 1986, 1988 and 1995, respectively. He joined Siemens Corporate Research in Princeton, New Jersey, as a member of technical staff in 1995. Since 1999, he returned to Taiwan as a faculty member in the Department of Computer Science, National Tsing Hua University. He is currently an associate professor in the same department. In 2004, he was a visiting scholar with the Department of Electrical Engineering, Princeton University. Dr. Lai's research interests include computer vision, visual computing, pattern recognition, medical imaging, and multimedia signal processing. He has authored more than 80 papers published in the related international journals and conferences. He holds ten US patents for inventions related to computer vision and medical image analysis. He has been a member of program committee of several international conferences, including CVPR, ICCV, ECCV, ACCV and ICME.

Chia-Ming Cheng (鄭嘉珉) received his B.S. degree in Engineering and System Science and M.S. degree in Computer Science from National Tsing Hua University, Hsinchu, Taiwan, in 1999 and 2001, respectively. He is currently a Ph.D. candidate in the Department of Computer Science at National Tsing Hua University. His research interests include computer vision and computer graphics, especially the 3D-related issues.