Short Paper

Fuzzy Production Inventory Based on Signed Distance*

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In crisp production inventory problem, let q, a, b, d, R and r be the quantity produced per cycle, the holding cost, production cost, production quantity per day, the total demand quantity and the demand quantity per day, respectively. We consider three fuzzification methods. Firstly, we fuzzify q, a, b, d, R and r to triangular fuzzy numbers. Secondly, we fuzzify d and r to triangular fuzzy number with q, a, b and R staying in crisp case. Thirdly, we fuzzify q to triangular fuzzy number with a, b, d, R and r staying in crisp case. In three cases, we find the total costs in the fuzzy sense by signed distance and get the optimal solutions.

Keywords: fuzzy sets, fuzzy production inventory, fuzzy total cost, triangular fuzzy number, signed distance

1. INTRODUCTION

In a series of papers [3, 5-7, 10-14], Yao *et al*. considered the fuzzified problems for the inventory with or without backorder models, and the production inventory model. In [3], Chiang *et al*. fuzzified the storing cost, backorder cost, cost of placing an order, total demand, order quantity and shortage quantity as the triangular fuzzy number in the total cost and applied the signed distance method to defuzzify. Lee and Yao [5] fuzzified the production quantity per day and demand quantity per day as the triangular fuzzy numbers and obtained the fuzzy total cost. In [7], Lin and Yao fuzzified the production quantity per cycle as the trapezoid fuzzy numbers and obtained the fuzzy total cost. They applied the extension principle to find the membership functions of the fuzzy total cost, and applied the centroid method to estimate the total cost in the fuzzy sense and obtained the optimal production quantity per cycle. But, it is very hard and complex to derive them. Lee and Yao [6, 11] fuzzified the order quantity as the triangular and trapezoid fuzzy number, respectively, for the inventory without backorder and obtained the fuzzy total cost. Yao and Chiang [10] fuzzified the total demand, the cost of storing one unit per day to the triangular fuzzy numbers in a plan period and get the fuzzy total cost. Then they defuzzified by the centroid, the signed distance methods and compared these two results.

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In [12], Yao *et al*. fuzzified the order quantity and the total demand quantity as the triangular fuzzy numbers for the inventory without backorder and obtained the fuzzy total cost. In [2], Chang *et al*. fuzzified the shortage quantity as the triangular fuzzy number and regarded the order quantity as the crisp variable for the inventory with backorder and obtained the fuzzy total cost. In [11, 13], they fuzzified the order quantity as the triangular fuzzy number and trapezoid fuzzy number respectively, and regarded the shortage quantity as the crisp variable for the inventory with backorder and obtained the fuzzy total cost. In [14], they fuzzified the total demand quantity as the interval-valued fuzzy set for the inventory with backorder and obtained the fuzzy total cost. In papers [2, 5-7, 11-14], they applied the extension principle to find the membership functions of the fuzzy total cost. Then they applied the centroid method to estimate the total cost in the fuzzy sense and obtained the optimization problems. The derivations are also hard. In this paper, we fuzzify the quantity produced per cycle q, the holding cost a, production cost b, production quantity per day d, the total demand quantity R and the demand quantity per day r as the triangular fuzzy numbers. Then we can obtain the fuzzy total cost. We will apply the signed distance method instead of the extension principle and centroid method to solve the estimated total cost in the fuzzy sense to obtain the optimal quantity produced per cycle.

Section 2 is the preliminaries, in which we consider the definition of the signed distance. In section 3.1, we fuzzify the q, a, b, d, r and R in the total cost, as described in Abstract, to the triangular fuzzy numbers, and obtain the fuzzy total cost. In section 3.2, we fuzzify the d, r in the total cost as the triangular fuzzy numbers, and obtain the fuzzy total cost. In section 3.3, we fuzzify the q in the total cost as the triangular fuzzy number, and obtain the fuzzy total cost. Then we use the signed distance method to estimate the total cost in the fuzzy sense. In section 4, we apply the numerical analysis method to solve the optimal quantity produced per cycle, and we take three examples to compare with the crisp case and [5, 7]. In section 5, we have a discussion, and we give a conclusion in section 6.

2. PRELIMINARIES

Definition 1 *Fuzzy Point*: Let $\tilde{a}$ be a fuzzy set on $R = (-\infty, \infty)$. It is called a fuzzy point if its membership function is

$$\mu_{\tilde{a}}(x) = \begin{cases} 1, & \text{if } x = a \\ 0, & \text{if } x \neq a \end{cases}. \tag{1}$$

Definition 2 *Level α Fuzzy Interval*: Let $[a_\alpha, b_\alpha]$ be a fuzzy set on R. It is called a level α fuzzy interval, $0 \leq \alpha \leq 1$, $a < b$, if its membership function is

$$\mu_{[a_\alpha, b_\alpha]}(x) = \begin{cases} \alpha, & \text{if } a \leq x \leq b \\ 0, & \text{otherwise} \end{cases}. \tag{2}$$

If $a = b$, we call $[a_\alpha, b_\alpha]$ is a level α fuzzy point at a.

Definition 3 *Triangular Fuzzy Numbers*: Let $\tilde{C} = (p, q, r)$, $p < q < r$, be a fuzzy set on

R. It is called a triangular fuzzy number, if its membership function is

$$\mu_{\tilde{C}}(x)=\begin{cases}\dfrac{x-p}{q-p}, & \text{if } p\le x\le q\\ \dfrac{r-x}{r-q}, & \text{if } q\le x\le r\ .\\ 0, & \text{otherwise}\end{cases} \tag{3}$$

If $r = q$, $p = q$, then $\tilde{C}$ is (q, q, q). We call it the fuzzy point $\tilde{q}$ at q.

Let $\tilde{D}$ be a fuzzy set on R. Denote $D(\alpha) = \{x \mid \mu_D(x) \ge \alpha\} = [D_L(\alpha), D_U(\alpha)]$ as the α-cut of $\tilde{D}$, where $0 \le \alpha \le 1$, $D_L(\alpha)$ and $D_U(\alpha)$ are the left and right hand endpoints of $D(\alpha)$. Furthermore, $D_L(\alpha)$ and $D_U(\alpha)$ exist and are integrable for $\alpha \in [0, 1]$. In addition, we let F be the family of all these fuzzy sets $\tilde{D}$ on R.

Decomposition Theory [15]: Let $\tilde{D} \in F$ then

$$\tilde{D}=\bigcup_{0\le\alpha\le1}\alpha D(\alpha)=\bigcup_{0\le\alpha\le1}[D_L(\alpha)_\alpha, D_U(\alpha)_\alpha] \tag{4}$$

or

$$\mu_{\tilde{D}}(x)=\bigvee_{0\le\alpha\le1}\alpha C_{D(\alpha)}(x)=\bigvee_{0\le\alpha\le1}\mu_{[D_L(\alpha)_\alpha, D_U(\alpha)_\alpha]}(x). \tag{5}$$

Definition 4 *The Signed Distance*: We define $d_0(a, 0) = a$, for $a, 0 \in R$ [15].

Definition 5 Let $\tilde{D} \in F$, we define the signed distance of $\tilde{D}$ measured from $\tilde{0}$ as $d(\tilde{D}, \tilde{0}) = \frac{1}{2}\int_0^1 [D_L(\alpha)+D_L(\alpha)]d\alpha$ [3].

From Definition 5, we have

$$d(\tilde{C},\tilde{0})=\frac{1}{2}\int_0^1[p+r+(2q-p-r)\alpha]d\alpha=\frac{1}{4}(2q+p+r). \tag{6}$$

Let $\tilde{D}, \tilde{E} \in F$. For $\alpha \in [0, 1]$, we have the following operations for the level α fuzzy intervals:

$$\mu_{[D_L(\alpha)_\alpha, D_U(\alpha)_\alpha]\oplus[E_L(\alpha)_\alpha, E_U(\alpha)_\alpha]}(z)=\mu_{[(D_L(\alpha)+E_L(\alpha))_\alpha, (D_U(\alpha)+E_U(\alpha))_\alpha]}(z)$$

$$\tilde{D}\oplus\tilde{E}=\bigcup_{0\le\alpha\le1}[(D_L(\alpha)+E_L(\alpha))_\alpha, (E_U(\alpha)+E_U(\alpha))_\alpha]. \tag{7}$$

By the same way, we have

$$\tilde{D}\ominus\tilde{E}=\bigcup_{0\le\alpha\le1}[(D_L(\alpha)-E_U(\alpha))_\alpha, (D_U(\alpha)-E_L(\alpha))_\alpha]. \tag{8}$$

If $0 \le D_L(\alpha) \le D_U(\alpha)$ and $0 < E_L(\alpha) < E_U(\alpha)$ for $\alpha \in [0, 1]$, then we have

$$\tilde{D} \otimes \tilde{E} = \bigcup_{0 \le \alpha \le 1} [(D_L(\alpha) \cdot E_L(\alpha))_\alpha, (D_U(\alpha) \cdot E_U(\alpha))_\alpha], \tag{9}$$

$$\tilde{D} \oslash \tilde{E} = \bigcup_{0 \le \alpha \le 1} \left[\left(\frac{D_L(\alpha)}{E_U(\alpha)} \right)_\alpha, \left(\frac{D_U(\alpha)}{E_L(\alpha)} \right)_\alpha \right]. \tag{10}$$

3. FUZZY PRODUCTION INVENTORY BASED ON SIGNED DISTANCE

We introduce the following variables for the production inventory model.

T: The whole period for the plan (days)
q: Quantity produced per cycle
a: Holding cost per unit per day
b: Production cost per cycle
d: Production quantity per day
r: Demand quantity per day
R: The total demand quantity of whole plan period
s: The maximal stock quantity
t_s: Time for production in each cycle
t_q: Time for each cycle

Since $d \cdot t_S = q$, $r \cdot t_S = r \cdot q/d$, $s = q - r \cdot t_S = q \cdot (1 - \frac{r}{d})$, $q/t_q = R/T$ [9], the total cost is

$$F(q) = \frac{T}{2} a \cdot q - \frac{T}{2} \frac{a \cdot r}{d} \cdot q + \frac{b \cdot R}{q}, \; 0 < q. \tag{11}$$

The crisp optimal solutions are:

optimal production quantity per cycle $q_* = \sqrt{\dfrac{2a \cdot R}{c \cdot T}}$, (12)

minimal total cost $F(q_*) = \sqrt{2a \cdot c \cdot R \cdot T}$, where $c = a(1 - \frac{r}{d})$. (13)

Since the production quantity for each cycle may not be always the same in the real situation, q could have some little variation. Hence we fuzzify the quantity produced per cycle as the following triangular fuzzy number

$$\tilde{q} = (q_1, q, q_2), \text{ where } 0 < q_1 < q < q_2. \tag{14}$$

It is more difficult to determine the demand quantity per day r than the demand quantity per day in the interval $[r - \Delta_7, r + \Delta_8]$ for the decision maker, where $0 < \Delta_7 < r$, $0 < \Delta_8$, and Δ_7, Δ_8 are determined by the decision maker. Since $[r - \Delta_7, r + \Delta_8]$ is an interval and is not a value, the decision maker could select a suitable value in $[r - \Delta_7, r + \Delta_8]$ as the demand quantity per day. If the decision maker takes r as this value which is the same as the crisp case r, then the error is zero. If the value is farther from r, the error is larger. If the value is at the endpoint $r - \Delta_7$ or $r + \Delta_8$, then the error is the largest. We can express the error in the fuzzy sense by the confidence level. If the error is zero, then the confidence level is one. If the value is farther from r, the confidence level is smaller. If the demand quantity per day is at the endpoint $r - \Delta_7$ or $r + \Delta_8$, then the confidence level is zero. Corresponding to the interval $[r - \Delta_7, r + \Delta_8]$, we express the demand quantity per day by the triangular fuzzy number

$$\tilde{r} = (r - \Delta_7, r, r + \Delta_8). \tag{15}$$

The membership grade of $\tilde{r}$ at r is one. The membership grade of $\tilde{r}$ at endpoints $r - \Delta_7$ or $r + \Delta_8$ is zero. From Eq. (6), we have that the signed distance of $\tilde{r}$ is

$$r^* \equiv d(\tilde{r}, \tilde{0}) = r + \frac{1}{4}(\Delta_8 - \Delta_7) = \frac{3}{4}r + \frac{1}{4}\Delta_8 + \frac{1}{4}(r - \Delta_7) > 0,$$

where r^* expresses the demand quantity per day in the fuzzy sense.

For the given positive a, b, d, and R, by the same argument above, corresponding to the interval $[a - \Delta_1, a + \Delta_2]$, $[b - \Delta_3, b + \Delta_4]$, $[d - \Delta_5, d + \Delta_6]$, and $[R - \Delta_9, R + \Delta_{10}]$ we may define the fuzzy number of the holding cost per unit per day a, production cost per cycle b, production quantity per day d, the total demand quantity of whole plane period R as

$$\tilde{a} = (a - \Delta_1, a, a + \Delta_2),\ 0 < \Delta_1 < a,\ 0 < \Delta_2, \tag{16}$$
$$\tilde{b} = (b - \Delta_3, b, b + \Delta_4),\ 0 < \Delta_3 < b,\ 0 < \Delta_4, \tag{17}$$
$$\tilde{d} = (d - \Delta_5, d, d + \Delta_6),\ 0 < \Delta_5 < d,\ 0 < \Delta_6, \tag{18}$$
$$\tilde{R} = (R - \Delta_9, R, R + \Delta_{10}),\ 0 < \Delta_9 < R,\ 0 < \Delta_{10}, \tag{19}$$

respectively, where Δ_j, $j = 1, 2, \ldots, 10$ are determined by the decision maker.

$$\left.\begin{array}{l} q_L(\alpha) = q_1 + (q - q_1)\alpha > 0,\ q_U(\alpha) = q_2 - (q_2 - q)\alpha = q_2(1-\alpha) + q\alpha > 0 \\ a_L(\alpha) = a - \Delta_1 + \alpha\Delta_1 > 0,\ a_U(\alpha) = a + \Delta_2 - \alpha\Delta_2 > 0 \\ b_L(\alpha) = b - \Delta_3 + \alpha\Delta_3 > 0,\ b_U(\alpha) = b + \Delta_4 - \alpha\Delta_4 > 0 \\ d_L(\alpha) = d - \Delta_5 + \alpha\Delta_5 > 0,\ d_U(\alpha) = d + \Delta_6 - \alpha\Delta_6 > 0 \\ r_L(\alpha) = r - \Delta_7 + \alpha\Delta_7 > 0,\ r_U(\alpha) = r + \Delta_8 - \alpha\Delta_8 > 0 \\ R_L(\alpha) = R - \Delta_9 + \alpha\Delta_9 > 0,\ R_U(\alpha) = R + \Delta_{10} - \alpha\Delta_{10} > 0 \end{array}\right\}, \tag{20}$$

respectively. Let $\left(\frac{\tilde{T}}{2}\right) = \left(\frac{T}{2}, \frac{T}{2}, \frac{T}{2}\right)$ be the fuzzy point.

3.1 Fuzzify *q*, *a*, *b*, *d*, *r*, *R* in Total Cost Eq. (11) as the Triangular Fuzzy Numbers

We fuzzify q, a, b, d, r, R in total cost Eq. (11) as the triangular fuzzy numbers shown in Eqs. (14)-(19). Then we obtain the fuzzy total cost

$$H(\tilde{q}, \tilde{a}, \tilde{b}, \tilde{d}, \tilde{r}, \tilde{R}) = \tilde{P} \ominus \tilde{Q} \oplus \tilde{S}, \tag{21}$$

where $\tilde{P} = \left(\frac{\tilde{T}}{2}\right) \otimes \tilde{a} \otimes \tilde{q}$, $\tilde{Q} = \left(\frac{\tilde{T}}{2}\right) \otimes \tilde{a} \otimes \tilde{r} \otimes \tilde{q} \oslash \tilde{d}$, $\tilde{S} = \tilde{b} \otimes \tilde{R} \oslash \tilde{q}$. From Eq. (20), and Eqs. (7)-(10),

$$P_L(\alpha) = \frac{T}{2}[a - \Delta_1 + \Delta_1\alpha][q_1 + (q - q_1)\alpha],$$

$$P_U(\alpha) = \frac{T}{2}[a + \Delta_2 - \Delta_2\alpha][q_2 - (q_2 - q)\alpha],$$

$$Q_L(\alpha) = \frac{T}{2}[a - \Delta_1 + \Delta_1\alpha][r - \Delta_7 + \Delta_7\alpha][q_1 + (q - q_1)\alpha]/[d + \Delta_6 - \Delta_6\alpha],$$

$$Q_U(\alpha) = \frac{T}{2}[a + \Delta_2 - \Delta_2\alpha][r + \Delta_8 - \Delta_8\alpha][q_2 - (q_2 - q)\alpha]/[d - \Delta_5 + \Delta_5\alpha],$$

$$S_L(\alpha) = [b - \Delta_3 + \Delta_3\alpha][R - \Delta_9 + \Delta_9\alpha]/[q_2 - (q_2 - q)\alpha],$$

$$S_U(\alpha) = [b + \Delta_4 - \Delta_4\alpha][R + \Delta_{10} - \Delta_{10}\alpha]/[q_1 + (q - q_1)\alpha]. \tag{22}$$

Therefore,

$$H(\tilde{q}, \tilde{a}, \tilde{b}, \tilde{d}, \tilde{r}, \tilde{R})_L(\alpha) = P_L(\alpha) - Q_U(\alpha) + S_L(\alpha),$$

$$H(\tilde{q}, \tilde{a}, \tilde{b}, \tilde{d}, \tilde{r}, \tilde{R})_U(\alpha) = P_U(\alpha) - Q_L(\alpha) + S_U(\alpha), \tag{23}$$

respectively. We let

$$W_1(e_1, e_2, e_3) = \int_0^1 [e_1 + e_2\alpha + e_3\alpha^2]d\alpha = e_1 + \frac{1}{2}e_2 + \frac{1}{3}e_3, \tag{24}$$

$$W_2(e_1, e_2, e_3, e_4, e_5) = \int_0^1 \frac{e_3\alpha^2 + e_2\alpha + e_1}{e_4 + e_5\alpha}d\alpha = \frac{e_3}{2e_5} + \frac{e_2e_5 - e_3e_4}{e_5^2} + \frac{e_1e_5^2 - e_2e_4e_5 + e_3e_4^2}{e_5^3}\ln\left|\frac{e_4 + e_5}{e_4}\right|, \tag{25}$$

$$W_3(e_1, e_2, e_3, e_4, e_5, e_6) = \int_0^1 \frac{e_4\alpha^3 + e_3\alpha^2 + e_2\alpha + e_1}{e_5 + e_6\alpha}d\alpha = \frac{e_4}{3e_6} + \frac{e_3e_6 - e_4e_5}{2e_6^2} + \frac{e_2e_6^2 - e_3e_5e_6 + e_4e_5^2}{e_6^3} + \frac{e_1e_6^3 - e_2e_5e_6^2 + e_3e_5^2e_6 - e_4e_5^3}{e_6^4}\ln\left|\frac{e_5 + e_6}{e_5}\right| \tag{26}$$

$$\begin{aligned}
&d(H(\tilde{q}, \tilde{a}, \tilde{b}, \tilde{d}, \tilde{r}, \tilde{R}), \tilde{0}) \\
&= \frac{T}{4}W_1[(a - \Delta_1)q_1, (a - \Delta_1)(q - q_1) + \Delta_1 q_1, \Delta_1(q - q_1)] \\
&\quad + \frac{T}{4}W_1[(a + \Delta_2)q_2, -(a + \Delta_2)(q_2 - q) - \Delta_2 q_2, \Delta_2(q_2 - q)] \\
&\quad + \frac{1}{2}W_2[(b - \Delta_3)(R - \Delta_9), (b - \Delta_3)\Delta_9 + (R - \Delta_9)\Delta_3, \Delta_3\Delta_9, q_2, -q_2 + q] \\
&\quad + \frac{1}{2}W_2[(b + \Delta_4)(R + \Delta_{10}), -(b + \Delta_4)\Delta_{10} - (R + \Delta_{10})\Delta_4, \Delta_4\Delta_{10}, q_1, q - q_1] \\
&\quad - \frac{T}{4}W_3[(a + \Delta_2)(r + \Delta_8)q_2, -(a + \Delta_2)(r + \Delta_8)(q_2 - q) - (a + \Delta_2)q_2\Delta_8 - \Delta_2(r + \Delta_8)q_2, \\
&\qquad (a + \Delta_2)\Delta_8(q_2 - q) + \Delta_2(r + \Delta_8)(q_2 - q) + \Delta_2\Delta_8 q_2, -\Delta_2\Delta_8(q_2 - q), d - \Delta_5, \Delta_5]
\end{aligned}$$

$$-\frac{T}{4}W_3[(a-\Delta_1)(r-\Delta_7)q_1, (a-\Delta_1)(r-\Delta_7)(q-q_1)+(a-\Delta_1)q_1\Delta_7+\Delta_1(r-\Delta_7)q_1,$$
$$(a-\Delta_1)\Delta_7(q-q_1)+\Delta_1(r-\Delta_7)(q-q_1)+\Delta_1\Delta_7 q_1, \Delta_1\Delta_7(q-q_1), d+\Delta_6, -\Delta_6]$$
$$\equiv K_1(q_1, q, q_2, \Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5, \Delta_6, \Delta_7, \Delta_8, \Delta_9, \Delta_{10}). \quad (27)$$

Property 1: If we fuzzify the q, r, a, b, d, R in the total cost Eq. (11) for the production inventory model to be the triangular fuzzy numbers $\tilde{q}, \tilde{r}, \tilde{a}, \tilde{b}, \tilde{d}, \tilde{R}$ (in Eqs. (14)-(19)) respectively, then we have

(1.1) the fuzzy total cost $H(\tilde{q}, \tilde{a}, \tilde{b}, \tilde{d}, \tilde{r}, \tilde{R})$ (in Eq. (21)),
(1.2) the estimate total cost in the fuzzy sense $K_1(q_1, q, q_2, \Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5, \Delta_6, \Delta_7, \Delta_8, \Delta_9, \Delta_{10})$ (in Eq. (27)).

3.2 Fuzzify *d*, *r* in Total Cost Eq. (11) as the Triangular Fuzzy Numbers

If we let $q_1 = q_2 = q$ in Eq. (14), $\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = \Delta_9 = \Delta_{10} = 0$ in Eqs. (16), (17), (19), then we have the following fuzzy points and triangular fuzzy numbers

$$\tilde{q} = (q, q, q),\ \tilde{a} = (a, a, a),\ \tilde{b} = (b, b, b),\ \tilde{R} = (R, R, R),$$
$$\tilde{d} = (d-\Delta_5, d, d+\Delta_6),\ \tilde{r} = (r-\Delta_7, r, r+\Delta_8). \quad (28)$$

From Eqs. (22) and (28), we have

$$P_L(\alpha) = P_U(\alpha) = \frac{T}{2}a\cdot q,\ S_L(\alpha) = S_U(\alpha) = \frac{b\cdot R}{q},$$
$$Q_L(\alpha) = \frac{T}{2}a\cdot q[r-\Delta_7+\Delta_7\alpha]/[d+\Delta_6-\Delta_6\alpha],$$
$$Q_U(\alpha) = \frac{T}{2}a\cdot q[r+\Delta_8-\Delta_8\alpha]/[d-\Delta_5+\Delta_5\alpha], \quad (29)$$

$$d(H(\tilde{q}, \tilde{a}, \tilde{b}, \tilde{d}, \tilde{r}, \tilde{R}), \tilde{0})$$
$$= \frac{T}{2}a\cdot q + \frac{b\cdot R}{q} - \frac{T\cdot a}{4}q\cdot[-\frac{\Delta_7}{\Delta_6} - \frac{\Delta_6\cdot r+\Delta_7\cdot d}{\Delta_6^2}\ln\left|\frac{d}{d+\Delta_6}\right| \quad (30)$$
$$-\frac{\Delta_8}{\Delta_5} + \frac{\Delta_5\cdot r+\Delta_8\cdot d}{\Delta_5^2}\ln\left|\frac{d}{d-\Delta_5}\right|] \equiv K_2(q; \Delta_5, \Delta_6, \Delta_7, \Delta_8).$$

For fixed $\Delta_5, \Delta_6, \Delta_7, \Delta_8$, let

$$\frac{d}{dq}K_2(q; \Delta_5, \Delta_6, \Delta_7, \Delta_8) \equiv -\frac{b\cdot R}{q^2} + M.$$

Property 2: If we fuzzify d, r in the production inventory model as the triangular fuzzy numbers $\tilde{d}$ (in Eq. (18)), $\tilde{r}$ (in Eq. (15)), we have the fact that if $q = \left(\frac{b\cdot R}{M}\right)^{1/2}$ $(\equiv q^{(0)})$ then $K_2(q^{(0)}; \Delta_5, \Delta_6, \Delta_7, \Delta_8)$ is the minimum.

3.3 Fuzzify *q* in Total Cost Eq. (11) as the Triangular Fuzzy Number

If we let $\Delta_j = 0$, $j = 1, 2, 3, \ldots, 10$ in Eq. (20), then we have the following fuzzy points $\tilde{a}, \tilde{b}, \tilde{R}, \tilde{d}, \tilde{r}$ and triangular fuzzy numbers $\tilde{q} = (q_1, q, q_2)$

$$d(H(\tilde{q}, \tilde{a}, \tilde{b}, \tilde{d}, \tilde{r}, \tilde{R}), \tilde{0}) = \frac{T}{8} a \cdot [q_1 + 2q + q_2] - \frac{T \cdot a}{8d} r[q_1 + 2q + q_2] + \frac{b \cdot R}{2}[\frac{1}{q_2 - q}(\ln q_2 - \ln q) + \frac{1}{q - q_1}(\ln q - \ln q_1)] \equiv K_3(q_1, q, q_2) \text{ (say)}. \quad (31)$$

Property 3: If we fuzzify q in the production inventory model as the triangular fuzzy number $\tilde{q}$ (in Eq. (14)), then we have the estimate of total cost in the fuzzy sense $K_3(q_1, q, q_2)$ (in Eq. (30)).

4. OPTIMAL PRODUCTION QUANTITY PER CYCLE

For given Δ_j, $j = 1, 2, \ldots, 10$, there are three variables q_1, q, q_2 which satisfy $0 < q_1 < q < q_2$. We want to find q_1, q, q_2 such that $K_1(q_1, q, q_2; \Delta_j, j = 1, 2, \ldots, 10)$ is the minimum. There are three variables q_1, q, q_2 which satisfy $0 < q_1 < q < q_2$. We want to find q_1, q, q_2 such that $K_3(q_1, q, q_2)$ is the minimum. We apply the Nelder-Mead method [8]. For more details, please see Chiang, Yao and Lee[2].

When we find q_1^*, q^*, q_2^* such that $K_1(q_1^*, q^*, q_2^*; \Delta_j, j = 1, 2, \ldots, 10)$ or $K_3(q_1^*, q^*, q_2^*)$ is the local minimal value, we substitute these q_1^*, q^*, q_2^* into Eq. (6) and get the best economic production quantity in cycle $q^{**} = \frac{1}{4}(q_1^* + 2q^* + q_2^*)$, and the minimal total cost $K_1(q_1^*, q^*, q_2^*; \Delta_j, j = 1, 2, \ldots, 10)$ or $K_3(q_1^*, q^*, q_2^*)$ in the fuzzy sense.

Example 1: We apply Property 1 to some examples to find the optimal production quantity q^{**} and the minimal total cost $K_1(q_1, q, q_2; \Delta_j, j = 1, 2, \ldots, 10)$ in the fuzzy sense. Let q_1, q, q_2 be the initial points, q_1^*, q^* and q_2^* the coordinates of local minimum, $q^{**} = \frac{1}{4}(q_1^* + 2q^* + q_2^*)$ the best economic production quantity, and the minimal total cost $K_1(q_1^*, q^*, q_2^*; \Delta_j, j = 1, 2, \ldots, 10)$ in the fuzzy sense, $q_* = \sqrt{\frac{2a \cdot R}{c \cdot T}}$ the optimal order for crisp order, $F(q_*) = \sqrt{2a \cdot c \cdot R \cdot T}$ the total cost for order quantity q_*.

When we run the program to solve the optimal solution for $K_1(q_1, q, q_2; \Delta_j, j = 1, 2, \ldots, 10)$, we should assign a set of four initial points for q_1, q, q_2 which satisfies $0 < q_1 < q < q_2$.

Given $a = 5$, $b = 10$, $T = 80$, $R = 100$, $r = 2$, $d = 10$, we get the optimal order quantity $q_* = 2.5$, the minimal total cost $F(q_*) = 800$ in the crisp case.

Let $FC = K_1(q_1^*, q^*, q_2^*; \Delta_j, j = 1, 2, \ldots, 10)$, $q_r^* = \frac{q^{**} - q_*}{q_*} \times 100\%$, $F_r = \frac{FC - F(q_*)}{F(q_*)} \times 100\%$.

Case 1: $(q_1, q, q_2) = (2.2, 2.4, 2.8), (2.3, 2.55, 3.1), (2.45, 2.65, 3.15), (2.0, 2.45, 3.5)$. We have the result as shown in Table 1.

Case 2: $(q_1, q, q_2) = (2.4, 2.6, 3.0), (1.9, 2.65, 3.05), (2.35, 2.85, 3.15), (2.1, 2.45, 3.0)$. We have the result as shown in Table 2.

Table 1. Optimal solution for case 1.

Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	Δ_6	Δ_7	Δ_8	Δ_9	Δ_{10}	q_1^*	q^*	q_2^*	q^{**}	FC	$q_r^*(\%)$	$F_r(\%)$
0.2	0.2	0.1	0.1	0.1	0.1	0.2	0.2	0.3	0.3	2.25210	2.44977	2.86633	2.50449	802.25225	0.18	0.28
0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	2.25201	2.44968	2.86622	2.50440	802.21952	0.18	0.28
0.1	0.2	0.1	0.2	0.1	0.2	0.1	0.2	0.1	0.2	2.25267	2.45032	2.86708	2.50510	804.37499	0.20	0.55
0.1	0.2	0.2	0.1	0.1	0.2	0.2	0.1	0.1	0.2	2.23530	2.43317	2.84320	2.48621	804.89642	– 0.55	0.61
0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	2.25269	2.45035	2.86711	2.50512	802.29996	0.21	0.29
0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	2.27453	2.46834	2.88567	2.52422	801.75985	0.97	0.22
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	2.27453	2.46834	2.88567	2.52422	798.46616	0.97	– 0.19
2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.34104	2.53187	2.96958	2.59359	781.82253	3.74	– 2.27

Table 2. Optimal solution for case 2.

Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	Δ_6	Δ_7	Δ_8	Δ_9	Δ_{10}	q_1^*	q^*	q_2^*	q^{**}	FC	$q_r^*(\%)$	$F_r(\%)$
0.2	0.2	0.1	0.1	0.1	0.1	0.2	0.2	0.3	0.3	2.34391	2.56819	2.99972	2.62000	803.11551	4.8	0.39
0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	2.33685	2.56406	2.99957	2.61614	803.08083	4.65	0.39
0.1	0.2	0.1	0.2	0.1	0.2	0.1	0.2	0.1	0.2	2.34336	2.56786	2.99970	2.61970	805.23173	4.76	0.65
0.1	0.2	0.2	0.1	0.1	0.2	0.2	0.1	0.1	0.2	2.32528	2.55854	2.99748	2.60996	805.95485	4.40	0.74
0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	2.35134	2.57254	2.99989	2.62408	803.15399	4.96	0.39
0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	2.39882	2.59937	3.00003	2.64940	802.57588	5.98	0.32
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	2.4	2.6	3	2.65	798.95501	6	– 0.13
2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.4	2.6	3	2.65	781.51088	6	– 2.31

Table 3. Optimal solution for case 3.

Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	Δ_6	Δ_7	Δ_8	Δ_9	Δ_{10}	q_1^*	q^*	q_2^*	q^{**}	FC	$q_r^*(\%)$	$F_r(\%)$
0.2	0.2	0.1	0.1	0.1	0.1	0.2	0.2	0.3	0.3	2.18360	2.50540	3.05944	2.56346	804.44425	2.54	0.56
0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	2.18360	2.50540	3.05944	2.56346	804.35031	2.54	0.54
0.1	0.2	0.1	0.2	0.1	0.2	0.1	0.2	0.1	0.2	2.18360	2.50540	3.05944	2.56346	806.55847	2.54	0.83
0.1	0.2	0.2	0.1	0.1	0.2	0.2	0.1	0.1	0.2	2.16225	2.49557	3.04763	2.55026	807.23240	2.01	0.90
0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	2.18743	2.51127	3.05891	2.56722	804.49776	2.69	0.56
0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	2.19084	2.51625	3.06120	2.57113	804.27908	2.85	0.54
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	2.24752	2.56240	3.09977	2.61803	801.31020	4.72	0.16
2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.31022	2.60266	3.14395	2.66487	784.67477	6.60	– 1.92

Case 3: $(q_1, q, q_2) = (2.45, 2.7, 3.25), (1.9, 2.65, 3.05), (2.25, 2.85, 3.15), (2.1, 2.45, 3.0)$. We have the result as shown in Table 3.

In general, if the Δ_j, $j = 1, 2, \ldots, 10$ is small then the result of fuzzy case will approach the crisp case.

Example 2: For given Δ_5, Δ_6, Δ_7, Δ_8, we may have the optimal solution from $K_2(q^{(0)}; \Delta_5, \Delta_6, \Delta_7, \Delta_8)$ of Property 2 in section 3.2.

Given $a = 10$, $b = 500$, $T = 40$, $R = 80$, $r = 2$ $(= r_0)$, $d = 10$ $(= d_0)$, we get the optimal production quantity $q* = 15.811388$, the minimal total cost $F(q*) = 5059.644256$ in the crisp case.

Table 4. Comparing with Property 2, the crisp case and [5].

Property 2						Comparing with the crisp case		Comparing with [5]	
Δ_5	Δ_6	Δ_7	Δ_8	$q^{(0)}$	$K2$	$q_r^{(0)}$ (%)	K_c(%)	$q_r^{(*)}$ (%)	$K_c^{(*)}$ (%)
0.05	0.05	0.05	0.05	15.811487	5059.612632	0.001	− 0.001	− 1.0782	− 0.00062
0.1	0.1	0.1	0.1	15.811784	5059.517756	0.003	− 0.003	− 1.07638	− 0.0025
0.15	0.15	0.15	0.15	15.812278	5059.359605	0.006	− 0.006	− 1.17329	− 0.00562
0.05	0.1	0.05	0.1	15.821485	5056.415231	0.064	− 0.064	− 1.11574	− 0.06381
0.1	0.2	0.1	0.2	15.832012	5053.053186	0.13	− 0.13	− 1.04995	− 0.13026
0.2	0.1	0.2	0.1	15.792486	5065.700277	− 0.12	0.12	− 1.29699	0.12
0.1	0.2	0.1	0.4	15.882187	5037.089612	0.448	− 0.446	− 0.78515	− 0.44577
0.2	0.1	0.4	.01	15.743804	5081.364046	− 0.427	0.429	− 1.60125	0.42927
0.5	0.5	0.5	0.5	15.821295	5056.476235	0.063	− 0.063	− 1.11693	− 0.06261
1	1	1	1	15.851306	5046.902661	0.252	− 0.252	− 0.92936	− 0.25182
1	2	1	2	16.095866	4970.220449	1.799	− 1.767	0.59913	− 1.76739
2	1	2	1	15.702953	5094.583186	− 0.686	0.691	− 1.85657	0.69054
2	2	1	1	15.906756	5029.309490	0.603	− 0.6	− 0.5828	− 0.59954
2	2	2	2	15.975942	5007.529293	1.041	− 1.03	− 0.15039	− 1.03001

Lee and Yao [5] fuzzified r and d in total cost Eq. (11) as the triangular fuzzy numbers $\tilde{r} = (r_1, r_0, r_2)$, $\tilde{d} = (d_1, d_0, d_2)$, respectively, and r_0, d_0 were known. Then they applied the extension principle to find the membership functions and defuzzified by the centroid method to obtain the estimate of total cost in the fuzzy sense $M(r_1, r_2, d_1, d_2, q)$ where $0 < r_1 < r_2 < d_1 < d_2 < q$. They solve the optimal solution as $q^* = 16.000005$, $M(1.899999, 2.100000, 9.899999, 10.100002, 16.000005) = 5059.644256$.

$$r_d = \frac{q^* - q_*}{q_*} \times 100\% = 1.193\%,\ r_c = \frac{M(r_1^*, r_2^*, d_1^*, d_2^*, q^*) - F(q_*)}{F(q_*)} \times 100\% = 0.003\%.$$

Let $K_2 = K_2(q^{(0)}; \Delta_5, \Delta_6, \Delta_7, \Delta_8)$, $M(r_1^*, r_2^*, d_1^*, d_2^*, q^*)$, $q_r^{(0)} = \frac{q^{(0)} - q_*}{q_*} \times 100\%$, $q_r^{(*)} = \frac{q^{(0)} - q^*}{q^*} \times 100\%$, $K_c^* = \frac{K_2 - M}{M} \times 100\%$, $K_c = \frac{K_2 - F(q_*)}{F(q_*)} \times 100\%$. Then, we have the result as shown in Table 4.

Example 3: For $K_3(q_1, q, q_2)$ of Property 3, since there are three variables in $K_3(q_1, q, q_2)$, by the algorithm discussed above, when we run the program to solve the optimal solution for $K_3(q_1, q, q_2)$, we should assign a set of four initial points for q_1, q, q_2 which satisfies $0 < q_1 < q < q_2$.

Given $a = 5$, $b = 10$, $T = 80$, $R = 100$, $r = 2$, $d = 10$, we get the optimal production quantity per cycle $q_* = 2.5$, the minimal total cost $F(q_*) = 800$ in the crisp case.

Yao and Lin [7] fuzzified q in total cost Eq. (11) as the trapezoid fuzzy number $\tilde{q} = (q_1, q_2, q_3, q_4)$, where $0 < q_1 < q_2 < q_3 < q_4$. Then, they applied the extension principle to find the membership functions and defuzzified by the centroid method to obtain the estimate of total cost in the fuzzy sense $M(q_1, q_2, q_3, q_4)$. Since there are four variables in $M(q_1, q_2, q_3, q_4)$, we should assign a set of five initial points for q_1, q_2, q_3, q_4 which satisfies $0 < q_1 < q_2 < q_3 < q_4$. There are three cases, Cases 1-3, (Ref. Example 1 in [7]). They had the optimal solution: the coordinates of local minimum q_1^*, q_2^*, q_3^*, q_4^* and the economic production quantity $q^{**} = \frac{1}{4}(q_1^* + q_2^* + q_3^* + q_4^*)$. Let $M = M(q_1^*, q_2^*, q_3^*, q_4^*)$, $q_r = \frac{q^{**} - q_*}{q_*} \times 100\%$, $M_r = \frac{M(q_1^*, q_2^*, q_3^*, q_4^*) - F(q_*)}{F(q_*)} \times 100\%$.

In order to minimize $K_3(q_1, q, q_2)$ in this study, we should assign a set of four initial points q_1, q, q_2 which satisfy $0 < q_1 < q < q_2$. We take the first four sets of initial points of Cases 1-3 in [6] as our initial points, we assign q_4 in [7] as our q_2, and $\frac{1}{2}(q_2 + q_3)$ as our q.

Case 1: (q_1, q, q_2) = (1.5347, 2.4004, 3.3713), (1.9297, 2.4261, 3.0605), (1.6246, 2.8326, 2.9701).

Case 2: (q_1, q, q_2) = (1.6557, 2.6436, 3.1425), (1.6228, 3.2858, 3.4826), (1.6204, 1.8133, 3.4385), (2.2690, 3.1616, 3.4542).

Case 3: (q_1, q, q_2) = (2.0725, 3.0542, 3.4285), (1.7785, 2.1450, 2.4245), (1.8344, 2.8009, 3.4132), (1.5336, 1.9092, 3.4238).

Let $q_r^* = \frac{q^{(00)} - q_*}{q_*} \times 100\%$, $q_r^{**} = \frac{q^{(00)} - q^{**}}{q^{**}} \times 100\%$, $F_c = K_3(q_1^{(0)}, q^{(0)}, q_2^{(0)})$, $F_r^* = \frac{F_c - F(q_*)}{F(q_*)} \times 100\%$, $F_r^{**} = \frac{F_c - M(q_1^*, q_2^*, q_3^*, q_4^*)}{M(q_1^*, q_2^*, q_3^*, q_4^*)} \times 100\%$, $q^{(00)} = \frac{1}{4}(q_1^{(0)} + 2q^{(0)} + q_2^{(0)})$.

Then, we have the results shown in Table 6.

Using the initial points of the Cases 1-3 in Example 1, as the initial points and apply to $K_3(q_1, q, q_2)$, and comparing the results in [7], the crisp case, and Property 3 of this study, we have the results shown in Table 7.

For assigning the initial points as in Tables 6 and 7, there are some differences for the results with the crisp case and [7].

Table 5. The result of the Example 1 shown in [7].

Case	q_1^*	q_2^*	q_3^*	q_4^*	q^{**}	M	q_r	M_r
1	1.66134	2.75111	2.83765	2.799050	2.49218	805.06410	– 0.3128	0.663
2	1.97430	2.66790	2.67750	2.87330	2.50764	802.64816	0.3056	0.331
3	1.90898	2.60101	2.78063	3.12122	2.58123	806.57905	3.25	0.822

Table 6. Comparing Property 3, crisp case and [7].

Property 3						Comparing with crisp case		Comparing with [7]	
Case	$q_1^{(0)}$	$q^{(0)}$	$q_2^{(0)}$	$q^{(00)}$	Fc	q_r^*(%)	F_r^*(%)	q_r^{**}	F_r^{**} (%)
1	1.9292	2.4261	3.0605	2.460475	807.36648	– 1.581	0.921	– 1.272	0.28597
2	1.808246	2.788232	3.215380	2.650022	811.79914	6.001	1.475	5.678	1.14
3	1.881480	2.626838	2.970143	2.526325	806.97131	1.053	0.871	– 2.127	0.0486

Table 7. Comparing Property 3, crisp case and [7].

Property 3						Comparing with crisp case		Comparing with [7]	
Case	$q_1^{(0)}$	$q^{(0)}$	$q_2^{(0)}$	$q^{(00)}$	Fc	q_r^*(%)	F_r^*(%)	q_r^{**}	F_r^{**} (%)
1	2.251568	2.449262	2.865649	2.503965	802.01984	0.157	0.252	0.472	0.378
2	2.325285	2.558537	2.997479	2.609959	802.86404	4.398	0.359	4.080	0.027
3	2.45	2.55	2.65	2.55	800.35808	2.	0.045	1.210	0.797

5. DISCUSSION

5.1 The Explanations and Comparisons of Sections 3.1-3.3

In section 3.1, the fuzzy total cost in Eq. (21) can be represented as

$$H(\tilde{q}, \tilde{a}, \tilde{b}, \tilde{d}, \tilde{r}, \tilde{R}) = \frac{T}{2} \cdot (\tilde{a} \otimes \tilde{q}) \ominus \frac{T}{2} \cdot (\tilde{a} \otimes \tilde{r} \otimes \tilde{q} \oplus\!\!\!\!\div \tilde{d}) \oplus (\tilde{b} \otimes \tilde{R} \oplus\!\!\!\!\div \tilde{q}). \tag{32}$$

In section 3.1, if we set $q_1 = q_2 = q$, $\Delta_j = 0$, $j = 1, 2, 3, 4, 9, 10$ then they fuzzy total cost is

$$H(q, a, b, \tilde{d}, \tilde{r}, \tilde{R}) = \frac{T}{2} \cdot (a \cdot q) \ominus \frac{T}{2} \cdot (a \cdot q \cdot (\tilde{r} \oplus\!\!\!\!\div \tilde{d})) \oplus (b \cdot R \oplus\!\!\!\!\div \tilde{q}). \tag{33}$$

This is the result in section 3.2. So Eq. (33) is a special case of Eq. (32).

In section 3.1, if we set $\Delta_j = 0$, $j = 1, 2, 3, \ldots, 10$ and $\tilde{q}$ is a triangular fuzzy number, then the fuzzy total cost can be written as

$$H(\tilde{q}, a, b, d, r, R) = \frac{T}{2} \cdot (\tilde{a} \otimes \tilde{q}) \ominus (\frac{T}{2} \cdot (a \cdot r \oplus\!\!\!\!\div d) \cdot \tilde{q}) \oplus (b \cdot R \oplus\!\!\!\!\div \tilde{q}). \tag{34}$$

This is the result in section 3.3. So Eq. (34) is a special case of Eq. (32).

5.2 We Make Some Comparisons with This Study, Crisp Case, [5] and [7] as Follows

(1) For the Properties 1-3 in this study, we defuzzify the fuzzy total cost by the signed distance and obtain the estimate of total cost in the fuzzy sense. This way is very easy and convenient for us to derive it. In [5, 7], they derived the membership functions of the fuzzy total cost by the extension principle. Then they defuzzified by the centroid method. But, it was too complex and difficult to derive them.

(2) From Example 1, in Property 1, we find that the computing result is dependent on the selecting initial points. If the fuzzy term Δ_j, $j = 1, 2, \ldots, 10$ is small then the difference of computing result with the crisp case is small. From Example 2, in Property 2, the difference of the computing result with the crisp case and [5] is small. From Example 3, in Property 3, we find that the computing result is also dependent on the selecting initial points and the difference of the computing result with the crisp case and [7] is small.

(3) In Property 1, since there are four operators $\oplus$, $\ominus$, $\otimes$, $\oplus\!\!\!\!\div$ for six fuzzy numbers $\tilde{q}$, $\tilde{a}$, $\tilde{b}$, $\tilde{d}$, $\tilde{r}$, $\tilde{R}$ in the fuzzy total cost (in Eq. (21)), therefore, if we want to solve

the membership functions of the fuzzy total cost by extension principle, it is too difficult and complex. Moreover, it is impossible to derive them. In this study, we use the signed distance and easy to obtain the estimate of the total cost in the fuzzy sense.

(4) From (1)-(3), we know that using the signed distance to defuzzify the fuzzy total cost is more convenient and easier than the extension principle and centroid method. Also, differences of the computing results are small. If the fuzzy term Δ_j, $j = 1, 2, \ldots, 10$ is small, then the difference of the computing result in this study with the crisp case is small.

6. CONCLUSION

The comparison of the fuzzification method used in this article and other fuzzification method ordinarily used.

6.1 Fuzzification Methods

6.1.1 Ordinarily used method

In crisp total cost Eq. (11), people fuzzify a, b, d, r, R and q directly to the following triangular fuzzy numbers

$$\begin{aligned}&\tilde{a}=(a-\Delta_1, a, a+\Delta_2),\ \tilde{b}=(b-\Delta_3, b, b+\Delta_4),\ \tilde{d}=(d-\Delta_5, d, d+\Delta_6),\\&\tilde{r}=(r-\Delta_7, r, r+\Delta_8),\ \tilde{R}=(R-\Delta_9, R, R+\Delta_{10}),\ q=(q_1, q_2, q_3).\end{aligned} \tag{35}$$

Then they obtain the fuzzy total cost (in Eq. (21)) as

$$H(\tilde{q}, \tilde{a}, \tilde{b}, \tilde{d}, \tilde{r}, \tilde{R}) = \tilde{P} \ominus \tilde{Q} \oplus \tilde{S} \tag{36}$$

where $\tilde{P}=\left(\frac{\tilde{T}}{2}\right)\otimes\tilde{a}\otimes\tilde{q}$, $\tilde{Q}=\left(\frac{\tilde{T}}{2}\right)\otimes\tilde{a}\otimes\tilde{q}\oslash\tilde{d}$ and $\tilde{S}=\tilde{b}\otimes\tilde{R}\oslash\tilde{q}$.

6.1.2 The fuzzification method we use

In crisp total cost Eq. (11), a, b, d, r, R are estimated values before the planning. These values may vary a little. We let them lie in the intervals Eq. (35) respectively. The decision maker can determine suitable $\Delta_j > 0$, $j = 1, 2, \ldots, 10$, through past statistical data. For example, in interval $[a - \Delta_1, a + \Delta_2]$, the grade of membership of a is set to 1. The farther from a, the lass the grade of membership. The grades of membership at $a - \Delta_1$ and $a + \Delta_2$ are zeros. Therefore, corresponding to the interval $[a - \Delta_1, a + \Delta_2]$, we can set a triangular fuzzy number $\tilde{a}=(a-\Delta_1, a, a+\Delta_2)$. $\tilde{b}$, $\tilde{d}$, $\tilde{r}$, $\tilde{R}$ can be treated similarly. And we can obtain Eq. (36).

From sections 6.1.1 and 6.1.2, the fuzzification method in this article is more realistic.

6.2 Defuzzification Method

6.2.1 Ordinarily used method

People often used the centroid method to defuzzify the fuzzy total cost Eq. (36). To use this method, we must find the membership function of Eq. (36). In general, we use extension principle. But it is very difficult to find the membership function of Eq. (36) by using the extension principle.

6.2.2 The method we used

We used the signed distance to defuzzify the fuzzy total cost as discussed in section 3.3. It is easier to find the total cost in the fuzzy sense through signed distance to defuzzify. Then we can find the optimal solution as discussed in section 4. This is the contribution of this article.

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