A Literature Overview of Fuzzy Database Models*

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Fuzzy set theory has been extensively applied to extend various database models and resulted in numerous contributions, mainly with respect to the popular relational model or to some related form of it. To satisfy the need of modeling complex objects with imprecision and uncertainty, recently many researches have been concentrated on fuzzy object-oriented database models. This paper reviews fuzzy database models, in which fuzzy relational and object-oriented databases are discussed.

Keywords: database models, fuzzy set, possibility distribution, fuzzy relational databases, fuzzy object-oriented databases

1. INTRODUCTION

Classical data models often suffer from their incapability of representing and manipulating imprecise and uncertain information that may occur in many real world applications. Since the early 1980's, Zadeh's fuzzy logic [71] has been used to extend various data models. The purpose of introducing fuzzy logic in databases is to enhance the classical models such that uncertain and imprecise information can be represented and manipulated. This resulted in numerous contributions, mainly with respect to the popular relational model or to some related form of it.

Also rapid advances in computing power have brought opportunities for databases in emerging applications (*e.g.*, CAD/CAM, multimedia and GIS). These applications characteristically require the modeling and manipulation of complex objects and semantic relationships. It has been proved that the object-oriented paradigm lends itself extremely well to the requirements. Since classical relational database model and its extension of fuzziness do not satisfy the need of modeling complex objects with imprecision and uncertainty, currently many researches have been concentrated on fuzzy object-oriented database models in order to deal with complex objects and uncertain data together.

A significant body of research in the area of fuzzy database modeling has been developed over the past thirty years and tremendous gain is hereby accomplished in this area. Various fuzzy database models (*e.g.*, relational and object-oriented databases) have been proposed, and some major issues related to these models have been investigated.

There have been a lot of fuzzy database papers published. But ones only find few comprehensive review papers of fuzzy database modeling [70, 35]. It has been nearly 10 years since a latest comprehensive overview paper has appeared in this area [34], where

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only fuzzy ER (entity-relationship) model and fuzzy relational databases (exactly data representation, queries, and design) are discussed. Since then, some new research results in, for example, fuzzy object-oriented databases come out. To investigate these issues and more importantly serve as identifying the direction of fuzzy database study, this paper aims to provide a comprehensive literature overview of fuzzy database models to satisfy the obvious need for an updating. Notice that, however, it does not means that this paper covers all publications in the research area and gives complete descriptions.

The remainder of this paper is organized as follows. Section 2 gives the basic knowledge about imperfect information and fuzzy sets theory. Issues about fuzzy relational database models are described in section 3. Section 4 investigates issues about fuzzy object-oriented databases. The last section concludes this paper.

2. IMPERFECT INFORMATION AND FUZZY SETS THEORY

2.1 Imprecise and Uncertain Information

Inconsistency, *imprecision*, *vagueness*, *uncertainty*, and *ambiguity* are five basic kinds of imperfect information in database systems.

- Inconsistency is a kind of semantic conflict, meaning the same aspect of the real world is irreconcilably represented more than once in a database or in several different databases. For example, the *age* of *George* is stored as 34 and 37 simultaneously. Information inconsistency usually comes from information integration.
- Intuitively, the imprecision and vagueness are relevant to the content of an attribute value, and it means that a choice must be made from a given range (interval or set) of values but we do not know exactly which one to choose at present. In general, vague information is represented by linguistic values. For example, the *age* of *Michael* is a set {18, 19, 20, 21}, a piece of imprecise information, and the *age* of *John* is a linguistic "*old*", a piece of vague information.
- The uncertainty is related to the degree of truth of its attribute value, and it means that we can apportion some, but not all, of our belief to a given value or a group of values. For example, the possibility that the *age* of *Chris* is 35 right now may be 98%. The random uncertainty described with probability theory is not considered here.
- The ambiguity means that some elements of the model lack complete semantics leading to several possible interpretations.

Generally, several different kinds of imperfection can co-exist with respect to the same piece of information. For example, the *age* of *Michael* is a set {18, 19, 20, 21} and their possibilities are 70%, 95%, 98%, and 85%, respectively. Imprecision, uncertainty, and vagueness are three major types of imperfect information.

2.2 Fuzzy Sets and Possibility Distributions

Many of the existing approaches dealing with imprecision and uncertainty are based on the theory of fuzzy sets [71] and possibility distribution theory [72]. A fuzzy set, say {0.7/18, 0.95/19, 0.98/20, 0.85/21} for the age of Michael, is more informative because it contains information imprecision (the age may be 18, 19, 20, or 21 and we do not know which one is true) and uncertainty (the degrees of truth of all possible age values are respectively 0.7, 0.95, 0.98, and 0.85) simultaneously.

Let U be a universe of discourse. A fuzzy value on U is characterized by a fuzzy set F in U. A membership function

$$\mu_F: U \rightarrow [0, 1]$$

is defined for the fuzzy set F, where $\mu_F(u)$, for each $u \in U$, denotes the degree of membership of u in the fuzzy set F. Thus the fuzzy set F is described as follows:

$$F = \{\mu_F(u_1)/u_1, \mu_F(u_2)/u_2, \ldots, \mu_F(u_n)/u_n\}.$$

When U is an infinite set, then the fuzzy set F can be represented by

$$F = \int_{u \in U} \mu_F(u)/u.$$

When the membership function $\mu_F(u)$ above is explained to be a measure of the possibility that a variable X has the value u, where X takes values in U, a fuzzy value is described by a possibility distribution π_X [71].

$$\pi_X = \{\pi_X(u_1)/u_1, \pi_X(u_2)/u_2, \ldots, \pi_X(u_n)/u_n\}.$$

Here, $\pi_X(u_i)$, $u_i \in U$ denotes the possibility that u_i is true. Let π_X and F be the possibility distribution representation and the fuzzy set representation for a fuzzy value, respectively. It is clear that $\pi_X = F$ is true [56].

For more concepts and operations about fuzzy sets, one can refer to [37].

3. FUZZY RELATIONAL DATABASES

Some major questions have been discussed and answered in the literature of the fuzzy relational databases (FRDBs), including representations and models, semantic measures and data redundancies, query and data processing, data dependencies and normalizations, implementation, and *etc*. For a comprehensive review of what has been done in the development of fuzzy relational databases, please refer to [16, 41, 54, 68].

3.1 Representations and Models

Several approaches have been taken to incorporate fuzzy data into relational databases. One of FRDB models is based on fuzzy relation [56] and similarity relation [13]. The other one is based on possibility distribution [55], which can further be classified into two categories: tuples associated with possibilities and attribute values represented by possibility distributions. The possibility-based FRDB model can be further extended into extended possibility-based FRDB model (see Table 1).

Table 1. Fuzzy data representation and fuzzy relational models.

	Fuzziness in Attribute Value	Fuzziness in Tuple
Fuzzy relation-based model	[56]	[56]
Similarity-based model	[13]	
Possibility-based model	[55]	[64]
Extended Possibility-based model	[19, 45]	

Definition 1 [45] A fuzzy relation r on a relational schema $R(A_1, A_2, \ldots, A_n)$ is a subset of the Cartesian product of $Dom(A_1) \times Dom(A_2) \times \ldots \times Dom(A_n)$, where $Dom(A_i)$ may be a fuzzy subset or even a set of fuzzy subset and there is the resemblance relation on the $Dom(A_i)$. A resemblance relation Res on $Dom(A_i)$ is a mapping: $Dom(A_i) \times Dom(A_i) \to [0, 1]$ such that

(i) for all x in $Dom(A_i)$, $Res(x, x) = 1$. (reflexivity)
(ii) for all x, y in $Dom(A_i)$, $Res(x, y) = Res(y, x)$. (symmetry)

The form of an n-tuple in each of the above-mentioned fuzzy relational models can be expressed, respectively, as

$$t = \langle p_1, p_2, \ldots, p_i, \ldots, p_n \rangle,$$

where $p_i \subseteq D_i$ with D_i being the domain of attribute A_i, $a_i \in D_i$. For each D_i, there exists a resemblance relation denoted Res_{D_i}, and

$$t = \langle a_1, a_2, \ldots, a_i, \ldots, a_n, d \rangle \text{ and } t = \langle \pi_{A_1}, \pi_{A_2}, \ldots, \pi_{A_i}, \ldots, \pi_{A_n} \rangle,$$

where $d \in (0, 1]$, π_{A_i} is the possibility distribution of attribute A_i on its domain D_i, and $\pi_{A_i}(x)$, $x \in D_i$, denotes the possibility that x is the actual value of $t[A_i]$.

Based on the above-mentioned basic FRDB models, there are several extended FRDB models. It is clear that one can combine two kinds of fuzziness in possibility-based FRDBs, where attribute values may be possibility distributions and tuples are connected with membership degrees. Such FRDBs are called *possibility-distribution-fuzzy relational models* in [64]. Another possible extension is to combine possibility distribution and similarity (proximity or resemblance) relation, and the *extended possibility-based fuzzy relational databases* are hereby proposed in [19, 45], where possibility distribution and resemblance relation arise in a relational database simultaneously.

3.2 Semantic Measures

To measure the semantic relationship between fuzzy data, some investigation results for assessing data redundancy can be found in literature, which are the closeness measure based on resemblance [11].

(a) The notion of nearness measure is proposed in [57]. Two fuzzy data π_A and π_B are considered α-β redundant if and only if the following inequality equations hold true:

$$\min_{x, y \in \mathrm{supp}(\pi_A) \cup \mathrm{supp}(\pi_B)}(Res(x, y)) \geq \alpha \text{ and } \min_{z \in U}(1 - |\pi_A(z) - \pi_B(z)|) \geq \beta,$$

where α and β are the given thresholds, $Res(x, y)$ denotes the resemblance relation on the attribute domain, and $\mathrm{supp}(\pi_A)$ denotes the support of π_A. It is clear that a twofold condition is applied in their study: the resemblance criterion and the matching criterion.

(b) For two data π_A and π_B, the following approach is defined in [19] to assess the possibility and impossibility that $\pi_A = \pi_B$.

$$E_c(\pi_A, \pi_B)(T) = \sup_{x, y \in U, c(x,y) \geq \alpha}(\min(\pi_A(x), \pi_B(y))) \text{ and}$$
$$E_c(\pi_A, \pi_B)(F) = \sup_{x, y \in U, c(x,y) < \alpha}(\min(\pi_A(x), \pi_B(y))).$$

Here $c(x, y)$ denotes a closeness relation (being the same as the resemblance relation). Classical equality is extended by means of a function E_c: $\Pi(D) \times \Pi(D) \rightarrow \Pi(\{T, F\})$ where T denotes *True* and F denotes *False*. The key idea is to extend the operations to be performed not only upon the identical elements, but also upon the close elements.

(c) In [27], the notions of weak resemblance and strong resemblance are proposed for representing the possibility and the necessity that two fuzzy values π_A and π_B are approximately equal, respectively. Weak resemblance and strong resemblance can be expressed as follows.

$$\Pi(\pi_A \approx \pi_B) = \sup_{x, y \in U}(\min(Res(x, y), \pi_A(x), \pi_B(y))) \text{ and}$$
$$N(\pi_A \approx \pi_B) = \inf_{x, y \in U}(\max(Res(x, y), 1 - \pi_A(x), 1 - \pi_B(y))).$$

The weak resemblance gives the extent to which some crisp element in an imprecise values $A(x)$ is resemblant to some crisp element in another imprecise values $A(y)$. The strong resemblance gives the extent to which all the crisp elements in $A(x)$ are resemblant to all the crisp elements in $A(y)$.

(d) The following function is given in [11] to measure the interchangeability that fuzzy value π_A can be replaced with another fuzzy data π_B, *i.e.*, the possibility that π_A is close to π_B from the left-hand side:

$$\mu_{repl}(\pi_A, \pi_B) = \inf_{x \in \mathrm{supp}(\pi_A)}(\max(1 - \pi_A(x), \mu_S(\pi_A, \pi_B)(x))),$$

where $\mu_S(\pi_A, \pi_B)(x)$ is defined as

$$\mu_S(\pi_A, \pi_B)(x) = \sup_{y \in \mathrm{supp}(\pi_B)}(\min(Res(x, y), 1 - |\pi_A(x) - \pi_B(y)|)).$$

$\mu_S(\pi_A, \pi_B)(x)$ can measure the extent to which there exists a representative $<y, \pi_B(y)>$ in π_B which can be substituted for x.

The treatment of (a) sets two criteria separately for redundancy evaluation and counterintuitive results are produced [11, 19]. The approaches of (b) and (d), in which the approach in (d) is actually an extension of the approach of (a), tried to set two criteria together for the redundancy evaluation. But the counterintuitive problem in (a) still exists in the approach in (d) [45]. For the approach in (b), there also exist some inconsistencies for assessing the redundancy of fuzzy data represented possibility distribution [11, 45]. As to the approach in (c), the weak resemblance, however, appears to be too “optimistic” and strong resemblance is too severe for the semantic assessment of fuzzy data. The approach in (b) is somewhat similar to the weak resemblance measure except that the degree of resemblance between crisp values is no longer incorporated into the *min* but is used to calibrate the set of comparable values [11].

(e) In [45], two notions semantic inclusion degree SID(π_A, π_B) and semantic equivalence degree SED(π_A, π_B) are introduced for the semantic measure of two fuzzy data π_A and π_B. Based on possibility distribution and resemblance relation, the definitions of calculating SID(π_A, π_B) and SED(π_A, π_B) are given as follows.

$$\mathrm{SID}_\alpha(\pi_A, \pi_B) = \sum_{i=1}^{n} \min_{u_i,\, u_j \in U \text{ and } \mathrm{Res}_U(u_i,\, u_j) \geq \alpha} (\pi_B(u_i),\ \pi_A(u_j)) \Big/ \sum_{i=1}^{n} \pi_B(u_i)$$

and

$$\mathrm{SED}_\alpha(\pi_A, \pi_B) = \min(\mathrm{SID}_\alpha(\pi_A, \pi_B), \mathrm{SID}_\alpha(\pi_B, \pi_A)).$$

Here SID$_\alpha$(π_A, π_B) means that the degree that π_A semantically includes π_B and SED(π_A, π_B) means that the degree that π_A and π_B are equivalent to each other.

3.3 Query and Data Processing

Classical relational databases suffer from a lack of flexibility in query. The given selection condition and the contents of the relations are all crisp. A query is flexible if the following conditions can be satisfied [9]:

- A qualitative distinction between the selected tuples is allowed.
- Imprecise conditions inside queries are introduced when the user cannot define his/her needs in a definite way, or when a prespecified number of responses are desired and therefore a margin is allowed to interpret the query.

Here typically, the former case occurs when the queried relational databases contain incomplete information and the query conditions are crisp and the later case occurs when the query conditions are imprecise even if the queried relational databases do not contain imperfect information [8].

In [33], a “human-consistent” database querying system based on fuzzy logic with linguistic quantifiers is presented. Using clustering techniques, a fuzzy query processing method is presented in [34]. Takahashi presents a fuzzy query language for relational databases [61] and discusses the theoretical foundation of query languages to fuzzy databases in [62]. Two fuzzy database query languages are proposed, which are a fuzzy calculus query language and a fuzzy algebra query language. In [7], the concepts of fuzzy integrals and database flexible querying are presented. In [10], a relational database language called SQLf for fuzzy querying is presented. Selection, join, and projection operations are extended to handle fuzzy conditions.

Also fuzzy query translation techniques for relational database systems and techniques of fuzzy query processing for fuzzy database systems are presented in [20, 43] and [21], respectively. In addition, based on matching strengths of answers in FRDBs, a method for fuzzy query processing is presented in [22]. In [67], nested fuzzy SQL queries in a FRDB are discussed.

In addition to query processing in FRDBs, there are also few studies focusing on the operations of relational algebra in FRDBs [42, 64]. In [73], a type of fuzzy equi-join is defined using fuzzy equality indicators. Updating FRDBs is investigated in [44].

3.4 Data Dependencies and Normalizations

Integrity constraints play a critical role in a logical database design. Among these constraints, data dependencies are of more interest. Based on various FRDB models, some attempts have been taken to express the data dependencies, mainly including fuzzy functional dependency (*FFD*) and fuzzy multivalued dependency (*FMVD*).

Some papers focus on *FFD*, in which we can classify two kinds of papers:

- the first one has a focus on the axiomatization of *FFD* [15, 17, 27, 39, 58].
- the second has a focus on the lossless join and decomposition [3, 12, 56], which is the basis to implement the normalization of fuzzy relational databases [18].

There are some papers that focus on *FMVD* [2, 32, 63]. Finally some papers focus both on *FFD* and *FMVD* and present the axiomatization of *FFD* and *FMVD* [48, 60].

Note that the fuzzy data dependencies can be applied in data handling. In [6], *FFD* is used for redundancy elimination. In [31], *FFD* is used for approximate data querying. In [39, 47], *FFD* is used for fuzzy data compression.

To solve the problems of update anomalies and data redundancies that may exist in FRDBs, the normalization theory of the classical RDB model must be extended so as to provide theoretical guideline for FRDB design. By employing equivalence classes from domain partitions, the functional dependencies and normal forms for FRDB model are defined in [59] and then the associated normalization issues are discussed. Based on the notion of *FFD*, some notions such as relation keys and normal forms are generalized in [18]. As a result, *q*-keys, Fuzzy First Normal Form, *q*-Fuzzy Second Normal Form, *q*-Fuzzy Third Normal Form, and *q*-Fuzzy Boyce-Codd Normal Form are formulated. Dependency-preserving and lossless-join decompositions into *q*-F3NFs are discussed.

Within the framework of the similarity-based fuzzy data representation, similarity, conformance of tuples, the concept of *FFD*s, and partial *FFD*s are discussed in [1]. On the basis, the fuzzy key notion, transitive closures, and fuzzy normal forms are defined for similarity-based FRDBs and the algorithms for dependency preserving and lossless join decompositions of fuzzy relations are given. Also it is shown how normalization, dependency preserving, and lossless join decomposition based on *FFD*s of fuzzy relation are done and applied to some real-life applications.

4. FUZZY OBJECT-ORIENTED DATABASES

In the fuzzy object-oriented databases (FOODBs), fuzziness is witnessed at the levels of object instances and class hierarchies (see Table 2). For most recent research and application issues about fuzzy object-oriented databases, ones can refer to [40].

Table 2. Fuzziness in object-oriented databases.

	Focus	Fuzziness in Object	Fuzziness in Class	Fuzziness in Object-Class	Fuzziness in Class-subclass	Operation
[4, 5]	imprecise data management	attribute type	yes	no	explicit	graph-based operations
[30]	uncertain in hierarchy	range of attribute value	yes	membership degree	weak and strong class hierarchy	no
[69]	semantic data model	imprecision in attribute	imprecision	fuzzy similarity	fuzzy similarity	fuzzy rules
[29]	uncertain in hierarchy	possibility distribution of attribute value	yes	fuzzy inclusion	fuzzy implication	no
[46]	imprecise data management	possibility distribution of attribute	yes	yes	yes	algebraic operations, SQL
[24, 25]	fuzzy classification	uncertainty in attribute	yes	fuzzy predicate	fuzzy predicate	fuzzy rules
[53]	fuzzy intelligent architecture	possibility distribution of attribute value	yes	fuzzy inclusion	fuzzy implication	fuzzy rules
[66]	modeling fuzziness and uncertainty	linguistic attribute	uncertain	uncertain	uncertain	no

4.1 Some Basic Fuzzy Object-Oriented Database Models

A FOODB model defined in [65] uses fuzzy attribute values with a certain factor and an SQL type data manipulation language. An UFO (uncertainty and fuzziness in an object-oriented) databases model is proposed in [66] to model fuzziness and uncertainty by means of conjunctive fuzzy sets and generalized fuzzy sets, respectively. That the behaviors and structure of the object are incompletely defined results in a gradual nature for the instantiation of an object. The partial inheritance, conditional inheritance, and multiple inheritances are permitted in fuzzy hierarchies.

Based on the extension of a graphs-based model object model, a fuzzy object-oriented data model is defined in [5]. The notion of strength expressed by linguistic qualifiers is proposed, which can be associated with the instance relationship as well as an object with a class. Fuzzy classes and fuzzy class hierarchies are thus modeled in the OODB. The definition of graph-based operations to select and browse such a FOODB that manages both crisp and fuzzy information is proposed in [4].

Based on similarity relationship, the range of attribute values is used to represent the set of allowed values for an attribute of a given class in [30]. Depending on the inclusion of the actual attribute values of the given object into the range of the attributes for the class, the membership degrees of an object to a class can be calculated. The weak and strong class hierarchies are defined based on monotone increase or decrease of the membership of a subclass in its superclass.

Based on possibility theory, vagueness and uncertainty are represented in class hierarchies in [29], where the fuzzy ranges of the subclass attributes defined restrictions on that of the superclass attributes and then the degree of inclusion of a subclass in the superclass is dependent on the inclusion between the fuzzy ranges of their attributes. Also based possibility distribution theory, in [46], some major notions in object-oriented databases such as objects, classes, objects-classes relationships, subclass/superclass, and multiple inheritances are extended under fuzzy information environment. A generic model for FOODBs and some operations are hereby developed.

4.2 ODMG-Based Fuzzy Object-Oriented Databases

Some efforts have been paid on the establishment of consistent framework for a fuzzy object-oriented model based on the standard for the Object Data Management Group (ODMG) object data model [26]. In [28], an object-oriented database modeling technique is presented based on the concept ‘level-2 fuzzy set’ to deals with a uniform and advantageous representation of both perfect and imperfect ‘real world’ information. It is illustrated and discussed how the ODMG data model can be generalized to handle ‘real world’ data in a more advantageous way.

4.3 Other Fuzzy Extension of Object-Oriented Databases

Based on two different strategies, fuzzy types are added into FOODBs to manage vague structures in [51, 52]. It is also presented how the typical classes of an OODB can be used to represent a fuzzy type and how the mechanisms of instantiation and inheritance can be modeled using this kind of new type in an OODB. In [50], complex object comparison in a fuzzy context is developed. In [24, 25], fuzzy relationships in object models are investigated.

In [53], a fuzzy intelligent architecture based on the uncertain object-oriented data model introduced initially in [29], is proposed. The classes include fuzzy IF-THEN rules to define knowledge and the possibility theory is used for representations of vagueness and uncertainty. In [38], an approach to OO modeling based on fuzzy logic is proposed to formulate imprecise requirements along four dimensions: *fuzzy class*, *fuzzy rules*, *fuzzy class relationships*, and *fuzzy associations between classes*. The fuzzy rules, *i.e.*, the rules with linguistic terms are used to describe the relationships between attributes.

4.4 Special Fuzzy Object-Oriented Databases

Some special fuzzy object-oriented databases, *e.g.*, fuzzy deductive object-oriented databases [36, 69], and fuzzy and probabilistic object bases [14], have been developed. In addition, fuzzy object-oriented database have been applied in some areas such as geographical information systems [23] and multimedia [49].

5. CONCLUSION

Incorporation of fuzzy information in database models has been an important topic of database research because such information extensively exists in data and knowledge intensive applications, where fuzzy data play an import role in nature. Research has been conducted into various approaches to represent and handle fuzzy data in the context of databases. Originally fuzzy database models are extensively investigated mainly with respect to the popular relational model. However, classical relational database model and its extension of fuzziness do not satisfy the need of modeling complex objects with imprecision and uncertainty. Object-oriented database model can represent complex object structures without fragmentation of aggregate data and model complex relationships among attributes. Current efforts have been concentrated on extending object-oriented databases to handle complex objects and imprecise and uncertain information together.

Various fuzzy database models, including relational and object-oriented databases, have been proposed over the past thirty years and tremendous gain is hereby accomplished in this area. Some major issues related to these models have been investigated, including query and data processing, data dependencies and normalization in FRDBs, index, design and implementation, *etc*. This paper elaborates on the issue of fuzziness management in the database models, in which FRDBs and FOODBs are discussed, respectively. The FRDBs model has been the subject of more thorough data presentation and models, query and data processing, and data dependencies and formalization.

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