# Similarity Measurement of Rule-based Knowledge Using Conditional Probability*

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This study proposes the *RO-RA-RV* structure for rule-based knowledge and integrates methods of conditional probability, vector matrices, and artificial intelligence to establish a conditional probability knowledge similarity algorithm and calculation system. This calculation system can quickly and accurately calculate rule-based knowledge similarity matrices and determine the relationship among knowledge items, this relationship can function as the knowledge source for treatments that increase the added-value of the knowledge, through the inference of value-added treatment such as merging, integration, deletion, innovation and additions, the accuracy of the knowledge itself can be securely ensured and wrong decisions be avoided. According to the knowledge similarity matrices, the knowledge case most similar to the testing case can be quickly retrieved, and used for all types of with Knowledge-Based Reasoning or Case-Based Reasoning to help decision making and prediction.

***Keywords:*** rule-based knowledge, conditional probability, vector matrices, artificial intelligence, similarity

## 1. INTRODUCTION

In the process of rule-based deterministic knowledge accumulation, different sources of data and different opinions held by experts, may cause specific knowledge elements in the knowledge base to become duplicated, conflicting, or inconsistent. The requirements of various rules make this process more prone to error [6, 8, 19]. Additionally, as environments in time and space change and new technologies, new laws, new methods and new pieces of evidence appear, the knowledge base may grow to include material no longer suitable for use. As a result, it becomes impossible to accurately express overall implications and relationships which need to be derived from the knowledge base. It becomes possible that the inaccurate application of such erroneous conclusions may lead to wrong decisions.

Current knowledge applications mainly focus on the discovery, creation, preservation, sharing and direct use of information. However the accuracy of the knowledge is not adequately certified in the following necessary ways:

- By considering the treatment of conflicting or overlapping knowledge;
- Correct handling of corresponding data of inconsistent size;
- Proper implementation of requirements for the knowledge implication to be consistent;

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- Checking the merging and integration of the inferences of the knowledge;
- Identifying the relationship between knowledge items of identical meaning;
- Reliable discernment governing the innovation or deletion of knowledge.

The authors hope to expand and promote the scope of the knowledge application and the value-added functions [4], so that the knowledge management application will retain more of its original meaning, and therefore will be more reliable, accurate and efficient. Knowledge value enhancement should be done according to the relationships among the knowledge items, which are determined on the basis of their similarities. So, the most basic and necessary task is to accurately calculate the similarity.

The present research proposes to integrate the *RO-RA-RV* structure, conditional probability, vector matrices, and artificial intelligence rule base reasoning to build a Conditional Probability Knowledge Similarity Algorithm (CPKSA) and a Rule-Based Knowledge Similarity Calculation System (RBKSCS) for rule-based knowledge, which together can quickly and accurately calculate the similarity matrix and then further determine the relationship between two sets of knowledge.

## 2. LITERATURE REVIEW

Methods frequently used for knowledge representation include Rule-based Knowledge, Frame-based Knowledge, Semantic Network, Logic theory and Ontology theory. These approaches are explained as follows:

(1) Rule-based Knowledge is the most usual use expressive method [12]. Its syntax is

IF (condition), THEN (conclusion).

(2) Frame-based Knowledge: In 1975, Minsky proposed the Frame-based concept [11]. A Frame can be represented as a fact or as a body. The frame structure is a composed set of knowledge slots, and each slot is composed of several facets and facet-values.
(3) Semantic Network [13] is a representation method using directed arcs to connect each node composed of nets. Each node expresses a general concept or special body or value *etc*. Those directed arcs represent an interrelation of those nodes.
(4) Ontology is a meaning of existing or boding theory [18], mainly applied in natural language processing, knowledge sharing, knowledge engineering *etc*., to describe the relationships of key elements among the knowledge.
(5) Logic knowledge: The method uses logic to do knowledge representation. Logic methods include proposition logic, predicate logic and fuzzy logic *etc*., and are used to describe a fact.

According to statistics done by McGill, Koll and Noreault in 1979, then-current methods for measuring similarity were continually growing and already numbered more than sixty types, including inner product, Dice coefficient, cosine coefficient, Jaccard coefficient, overlap coefficient, *etc*. [9]. However, the most popular method today remains one based on the distance between the two end-points of two vectors and the angle between the two vectors. Generally, the two-vector similarity measure is represented by the distance between the two end-points of the two vectors or the angle between the two vectors. The less the distance or the angle is, the higher the similarity is. Distance in a geometric distance model is usually represented by Euclidean distance; and the angle by dot product [1].

Chen *et al*. observed that comparison of the similarity measures of two fuzzy values mainly adopts the geometric distance model, the union and intersection operations in the set-theoretic approach, along with the sum and difference of the grades of membership and matching function [5, 16, 17]. Frank proposed that the retrieval of different information may require different known similarity measuring methods; for example, after retrieval, the file similarity measure is transformed to the vector of numeric value and then the similarity between two vectors is calculated [7].

In 1999, Zhang put forward the similarity measure method integrating distance and angle. In this method, distance similarity uses an exponential function, with the bottom between 0.7-0.97, and angle similarity a cosine function [9, 10]. In 2005, Liu proposed the use of a sequence alignment and description, together with the Lempel-Ziva Algorithm, to calculate the similarity of DNA gene sequences [15]. In 2004, Yang *et al*. proposed the optimal classification method which mainly uses an exponential function as the similarity goal function [14].

In 2003, Deng *et al*. proposed a new similarity measurement of generalized fuzzy numbers which use the radius of gyration point to instead of center-of-gravity [20], and Liang *et al*. proposed the similarity measures can deal with more effectively and reasonably on intuitionistic fuzzy set [21]. Recently, Huang integrated Conditional Probability and Vector Matrices to compute the relative similarity between two sets of knowledge after comparison, but that algorithm is not yet complete [2, 3].

Moreover, seldom have studies aimed at the transformation of two sets of knowledge to two knowledge vectors respectively or discussed the simpler similarity measuring method which integrates distance and angle. The present study proposes to integrate the *RO-RA-RV* structure, with conditional probability, vector matrices, and an artificial intelligence rule base inference to simplify the measure integrating distance and angle similarity and to build RBKSCS, which can quickly and accurately calculate the similarity matrix and then further determine the relationship between two knowledge sets. This relationship can function as a knowledge source for the treatment of increasing the added value of the knowledge.

## 3. MEASURING RULE-BASED KNOWLEDGE SIMILARITY

To comply with the orientation of practical applications, the present research observes the following limiting conditions:

(1) It aims at single rule-based deterministic domain knowledge to discuss the similarity between two sets of knowledge; the similarity of knowledge of different domains is not included in this study.
(2) The domain knowledge involved in this study has been arranged by knowledge engineers as the representation of rule-based knowledge, characterized by the logic: “IF <antecedent>, THEN <consequent>”.

(3) The assessment of the knowledge similarity in question refers to the relative similarity between two sets of knowledge after comparison.
(4) The minimal distance similarity is zero where occur the maximal of whole distance.

### 3.1 Knowledge Representation in the *RO-RA-RV* Format

The "IF <antecedent> THEN <consequent>", logic sequence of both antecedent and consequent can be represented as a sentence or several sentences. A sentence can be expressed in the *RO-RA-RV* format, comprised of the following four components: the relationship operator (*R*), the object (*O*), the attribute (*A*) and the linguistic value (*V*) , and here symbols of relationship operator (*R*) may be include the terms >, =, <, ≥ and ≤. Examples are shown in Table 1, which can be represented in vector form after proper transformation and mapping into numerical types.

**Table 1. Representation of the *RO-RA-RV* structure of a sentence.**

| Sentence | *RO* | *RA* | *RV* |
|---|---|---|---|
| The temperature of the engine is more than 100 °c. | = Engine | = temperature | > 100 °c |
| The vehicle's color is red. | = Vehicle | = color | = red |
| The span of the bridge is less than 50m. | = Bridge | = span | < 50m |

If the antecedent or consequent sentences contain the processing of logic operators like AND or OR, they are described as follows:

Described by the logic, "IF ($a_1$ AND $a_2$), THEN $c_1$", the antecedent vector is formed with six components, written as, $[RO_{a_1}\ RA_{a_1}\ RV_{a_1}\ RO_{a_2}\ RA_{a_2}\ RV_{a_2}]$.

IF ($a_3$ OR $a_4$) THEN $c_2$ is first divided into two knowledge operations, IF $a_3$ THEN $c_2$ and IF $a_4$ THEN $c_2$, with their antecedent vectors represented as $[RO_{a_3}\ RA_{a_3}\ RV_{a_3}]$, and $[RO_{a_4}\ RA_{a_4}\ RV_{a_4}]$, respectively; after the calculations are completed, they are then merged into the original knowledge form IF ($a_3$ OR $a_4$) THEN $c_2$.

If the antecedent and consequent vectors are of different dimensions, then they are represented by the maximal dimension between them, with the augmented dimensions reduced to zero, let the antecedent and consequent vectors are with same dimensions.

### 3.2 Transforming Mapping of *RO-RA-RV* Components

The possible data types of *O-A-V* components in a knowledge sentence are nominal, ordinal, or interval and ratio, as shown in Table 2. And the transforming mapping methods of *RO-RA-RV* Components are same.

#### 3.2.1 When the data type of any components is nominal

Then all the relationship operators (*R*) of the components are equal symbol, its transformational mapping is 0 or 1 according to Table 3.

**Table 2. Data types of *O-A-V* components.**

| Data Type | Operation Property | Example |
|---|---|---|
| Nominal | Unable to compare magnitude.<br>Unable to do arithmetic calculations. | Car color |
| Ordinal | Finite, with order relationships, able to compare its magnitude, unable to do arithmetic calculations. | Clothes size:<br>XS < S < M < L < XL |
| Interval and ratio | Numerical, able to compare its magnitude and do arithmetic calculations | Body weight, Test score, Rainfall |

**Table 3. Transform mapping of *RO-RA-RV* components.**

| RO-RA-RV Components | Mapping Value |
|---|---|
| Existing identical character | 1 |
| No identical character | 0 |

**Table 4. Assignment of specific transforming mapping values for ordinal data.**

| Grade | Description of $V$ | Transforming Mapping Values of $RV$ |
|---|---|---|
| 1 | Super-large, very large, very fast, super-fast | 1 |
| 2 | Large, fast | 0.75 |
| 3 | Medium, common speed | 0.5 |
| 4 | Small, slow | 0.25 |
| 5 | Super-small, very small, very slow, super-slow | 0 |

### 3.2.2 When the data type of any components is ordinal

Then all the relationship operators ($R$) of *O-A-V* components are equal signs, they are directly assigned a specific transform mapping value between zero and one. The highest grade element in component $V$ is assigned a mapping value of one, the lowest grade element in component $V$ is assigned a mapping value of zero, and the elements between the highest and the lowest grade in component $V$ are assigned mapping by Eq. (1). An example of transforming mapping values of $RV$ is shown in Table 4.

$$V_i = \frac{m-i}{m-1},\ i = 1, 2, \ldots, m \qquad (1)$$

$m$: total series to distinguish weight $V$;
$V_i$: the mapping value of $i$th grade in component $V$, when $i = 1$ is the highest grade.

### 3.2.3 When the data type of any components is interval and ratio

(1) It keeps its original numerical value. Furthermore, if all the relationship operators ($R$) of the component are equal signs, the effect is to normalize Eq. (2), so as to map the value of $RV$ into the range from 0 to 1.

$$V_{norm} = \frac{V - V_{\min}}{V_{\max} - V_{\min}} \qquad (2)$$

$V_{norm}$: the normalized value of $V$, ranging between 0 and 1;
$V$: the value of $V$ before normalization;
$V_{\max}$: the maximal element in component $V$;
$V_{\min}$: the minimal element in component $V$.

(2) If not all the relationship operators ($R$) are the equal sign, the transforming mapping of the $RV$ component is determined by the conditional probability theory.

Conditional probability is the probability that the event $B$ occurs, under the condition that the event $A$ occurs too. Mathematically, this is written as

$$P(B \mid A) = \frac{P(B \cap A)}{P(A)}$$

where $P(B \cap A)$ is the probability that both the events $A$ and $B$ occur, and $P(A)$ is the probability of the event $A$.

When conducting transform mapping for two knowledge weights, the testing case weight has been set for the occurred conditions, in order to compute the change of having the same weight for any knowledge base case. Hence, the probability for the testing case is 1, then it means that the transform mapping is 1. The probability for the same weight in any knowledge base case is the transform mapping of the same weight.

Because weight $A$ in the testing case indicates that it has occurred, and the probability of weight $A$ is $P(A) = 1$, thus the conditional probability $P(B|A)$ can be expressed by Eq. (3).

$$P(B \mid A) = P(B \cap A) \tag{3}$$

**Definition 1**
$V_{\max}$: the maximal element of the component $V$;
$V_{\min}$: the minimal element of the component $V$;
$r$: the distributed range of the component $V$, $r = V_{\max} - V_{\min}$;
$\max = V_{\max} + r$
$\min = V_{\min} - r$
$x$: the value of the component $V$ in testing cases ($T$);
$y$: the values of the component $V$ in knowledge base cases ($K$).

"max" is the maximal element of the component $V$ with a value $r$ further added, and "min" is the minimal element of the component $V$ with a value $r$ further subtracted, as shown in Fig. 1.

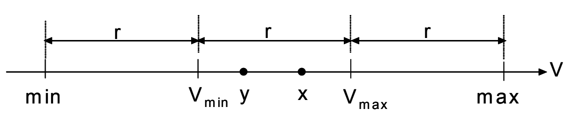

Fig. 1. The distributed range of the component $V$.

For example, the calculations of the transform mapping values of $RV$ component for conditions $V > x$, $V > y$ and $x > y$ are shown in Fig. 2 and Table 5.

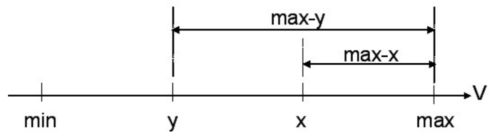


Fig. 2. Transform mapping for $x > y$, $V > x$ and $V > y$.

**Table 5. The *RV* component for conditions *V* > *x*, *V* > *y* and *x* > *y*.**

| Category | Value of *RV* | Value of *x* or *y* |
|---|---|---|
| Testing case (*T*) | $V > x$ | $x$ |
| Knowledge base cases (*K*) | $V > y$ | $y$ |

**Table 6. *RV* component transforming mapping for conditions *V* > *x*, *V* > *y* and *x* > *y*.**

| Category | Value of *RV* | Mapping Value of *RV* | Remark |
|---|---|---|---|
| *T* | $V > x$ | 1 | The event $V > x$ has occurred, so $P(V > x) = 1$. |
| *K* | $V > y$ | $\frac{\max - x}{\max - y}$ | The event $V > y$ occurring given that the event $V > x$ has occurred, then $P(V > y \mid V > x) = \frac{\max - x}{\max - y}$. |

Because the probability for weight $V > x$ in t*he testing case* is $P(V > x) = 1$, the transform mapping for weight $V > x$ is also 1. It indicates that weight $V$ between $x$ and max can satisfy all $V > x$ conditions. The linear length formed by $V$ values that satisfy $V > x$ conditions is (max-$x$). As for the same weight $V > y$ in *the knowledge base*, the range of weight $V$ is from $y$ to max, and the linear length form by $V$ values that satisfy $V > y$ conditions is (max-$y$), as shown in Fig. 2.

In the linear length (max-$y$) formed by all $V$ values that satisfy the same weight $V > y$ in a knowledge base, due to $x > y$, the linear length formed by all $V$ values that can satisfy $V > x$ conditions in the testing case is (max-$x$). According to Eq. (3), the ratio between the two linear lengths is $\frac{\max - x}{\max - y}$, which is also the probability of the same weight $V > y$ in a knowledge base, in other words, the transform mapping of the same weight $V > y$, as shown in Table 6.

The calculations of the transforming mapping values of *RV* component, under the different situations are then shown in Tables 7 to 9.

For the testing cases of $x_1 < V < x_2$, and the knowledge base cases $y_1 < V < y_2$, which fall within a certain range respectively, the calculations of the transformational mapping value of the *RV* component are shown in Fig. 3 as well as Tables 10 to 12, respectively.

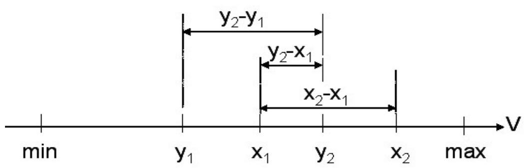


Fig. 3. Transforming mapping for $V$ that falls within $y_1 < x_1 < y_2$ and $y_2 < x_2$.

**Table 7. *RV* component transforming mapping for condition $x > y$.**

| Value K / T | $V > y$ | $V = y$ | $V < y$ |
|---|---|---|---|
| $V > x$ | $\frac{\max - x}{\max - y}$ | 0 | 0 |
| $V = x$ | $\frac{1}{\max - y}$ | 0 | 0 |
| $V < x$ | $\frac{x - y}{\max - y}$ | 1 | 0 |

**Table 8. *RV* component transforming mapping for condition $x = y$.**

| Value K / T | $V > y$ | $V = y$ | $V < y$ |
|---|---|---|---|
| $V > x$ | 1 | 0 | 0 |
| $V = x$ | 0 | 1 | 0 |
| $V < x$ | 0 | 0 | 1 |

**Table 9. *RV* component transforming mapping for condition $x < y$.**

| Value K / T | $V > y$ | $V = y$ | $V < y$ |
|---|---|---|---|
| $V > x$ | 1 | 1 | $\frac{x - y}{y - \min}$ |
| $V = x$ | 0 | 0 | $\frac{1}{y - \min}$ |
| $V < x$ | 0 | 0 | $\frac{x - \min}{y - \min}$ |

**Table 10. Transforming mapping for *RV* that falls within a range.**

| Value K / T | $x_2 > y_2$ | $y_1 < x_2 < y_2$ | $x_2 < y_1$ |
|---|---|---|---|
| $x_1 > y_2$ | 0 | No ($x_2 > x_1$) | No ($x_2 > x_1$) |
| $y_1 < x_1 < y_2$ | $\frac{y_2 - x_1}{y_2 - y_1}$ | $\frac{x_2 - x_1}{y_2 - y_1}$ | No ($x_2 > x_1$) |
| $x_1 < y_1$ | 1 | $\frac{x_2 - y_1}{y_2 - y_1}$ | 0 |

**Table 11. Transforming mapping for *RV* when $y_1$ = min.**

| Value K / T | $x_2 > y_2$ | $y_1 < x_2 < y_2$ | $x_2 < y_1$ |
|---|---|---|---|
| $x_1 > y_2$ | 0 | No ($x_2 > x_1$) | No ($x_2 > x_1$) |
| $y_1 < x_1 < y_2$ | $\frac{y_2 - x_1}{y_2 - \min}$ | $\frac{x_2 - x_1}{y_2 - \min}$ | No ($x_2 > x_1$) |
| $x_1 < y_1$ | 1 | $\frac{x_2 - \min}{y_2 - \min}$ | 0 |

**Table 12. Transforming mapping for *RV* when $x_2$ = max and $y_2$ = max.**

| Value K / T | $x_1 > y_1$ | $x_1 < y_1$ |
|---|---|---|
| | $\frac{\max - x_1}{\max - y_1}$ | 1 |

For weights with the same meaning and attributes, a content comparison may deliver more meanings. Therefore, the transform should show similar characteristics. On the contrary, weights that do not share similarities are worthless for comparison, thus, the fact that they have completely different characteristics should be revealed after the transform mapping. The rules of NULL value transformation are described as below:

(a) If the *RV* component is not Null in the testing case, then all *RV* components that are "Null" in the knowledge base cases are transformed with the mapping value of zero.
(b) If the *RV* component is Null in testing case, then all *RV* components that are "Null"

are transformed with the mapping value of one, and all RV components that are NOT "Null" are transformed with the mapping value of zero, in the knowledge base cases.

For example, if in knowledge base there are 3 knowledge sets with same antecedent $a_1$, as follows.

**Rule 1** IF $a_1$ THEN $V > 190$.
**Rule 2** IF $a_1$ THEN $V = 195$.
**Rule 3** IF $a_1$ THEN $V > 165$.

The values of the component $V$ have {190, 195, 165} in knowledge base, then $V_{max} = 195$, $V_{min} = 165$, $r = 30$, max = 225 and min = 135. The calculations of the transform mapping value of the *RV* component are shown from Tables 13 to 17.

**Table 13. The *RV* component for Rule 2 and Rule 3 subject to Rule 1.**

| Category | Rule no. | Value of *RV* | Value of *x* or *y* |
|---|---|---|---|
| Testing case (*T*) | Rule 1 | > 190 | $x = 190$ |
| Knowledge base cases (*K*) | Rule 2 | = 195 | $y = 195$ |
| | Rule 3 | > 165 | $y = 165$ |

**Table 14. *RV* component transforming mapping for Rule 2 and Rule 3 subject to Rule 1.**

| Category | Rule no. | Mapping Value of *RV* |
|---|---|---|
| *T* | Rule 1 | 1 |
| *K* | Rule 2 | 1 |
| | Rule 3 | 0.583 |

**Table 15. *RV* component transforming mapping for Rule 1 and Rule 3 subject to Rule 2.**

| Category | Rule no. | Mapping Value of *RV* |
|---|---|---|
| *T* | Rule 2 | 1 |
| *K* | Rule 1 | 0.0286 |
| | Rule 3 | 0.0167 |

**Table 16. *RV* component transforming mapping for Rule 1 and Rule 2 subject to Rule 3.**

| Category | Rule no. | Mapping Value of *RV* |
|---|---|---|
| *T* | Rule 3 | 1 |
| *K* | Rule 1 | 1 |
| | Rule 2 | 1 |

**Table 17. The transforming matrix of *RV* component of 3 knowledge sets.**

| Rule no. | Rule 1 | Rule 2 | Rule 3 |
|---|---|---|---|
| Rule 1 | 1 | 1 | 0.583 |
| Rule 2 | 0.0286 | 1 | 0.0167 |
| Rule 3 | 1 | 1 | 1 |

### 3.3 Knowledge Similarity Calculation

After the proper transformational mapping and normalization for the antecedent and consequent knowledge, they may be written in the *RO-RA-RV* format, and represented with vectors.

*m* rules represented with an *n*-dimensional antecedent and a 1-dimensional consequent, are combined into a single knowledge matrix as shown in Eq. (4):

$K$ (Knowledge Matrix): $K = [k_{ij}]_{m*(n+l)}$, with $i = 1\ldots m$, and $j = 1\ldots(l + m)$

$$K = \left[\begin{array}{ccccc|ccccc} a_{11} & a_{12} & \cdots & \cdots & a_{1n} & c_{11} & c_{12} & \cdots & \cdots & c_{1l} \\ a_{21} & a_{22} & \cdots & \cdots & a_{2n} & c_{21} & c_{22} & \cdots & \cdots & c_{2l} \\ \vdots & \vdots & & & & \vdots & \vdots & & & \\ \vdots & \vdots & & & & \vdots & \vdots & & & \\ a_{m1} & a_{m2} & \cdots & \cdots & a_{mn} & c_{m1} & c_{m2} & \cdots & \cdots & c_{ml} \end{array}\right] \tag{4}$$

where $\vec{k}_m = [k_{m1} \quad k_{m2} \quad k_{m3} \quad \ldots \quad k_{m(n+l)}]$, which represents the knowledge row vectors of the $m$th knowledge representation.

When rule-based knowledge is represented as knowledge vectors, such as $\vec{k}_i = (k_{i1}, k_{i2}, k_{i3}, \ldots, k_{i(n+l)})$ and $\vec{k}_j = (k_{j1}, k_{j2}, k_{j3}, \ldots, k_{j(n+l)})$, their Euclidean distance, length, and inner product are defined in Eqs. (5)-(7) respectively.

$$\text{Euclidean Distance} = D(\vec{k}_i, \vec{k}_j) = \|\vec{k}_i - \vec{k}_j\| = \sqrt{\sum_{v=1}^{n+l} |k_{iv} - k_{jv}|^2} \tag{5}$$

$$\text{Length} = \|\vec{k}_i\| = \sqrt{\sum_{v=1}^{n+l} |k_{iv}|^2} \tag{6}$$

$$\text{Inner Product} = \langle \vec{k}_i, \vec{k}_j \rangle = \sum_{v=1}^{n+l} k_{iv} k_{jv} \tag{7}$$

The present research proposes a knowledge similarity (*KKS*), which is simpler than the one in [10] and much easier to understand. It is the multiplicity between the Distance Similarity (*DS*) and Angle Similarity (*θS*). KKS between two knowledge representations, $i$ and $j$, with $m$ rules, will have $m^2$ number of pairwise calculations, which are described in Eqs. (8)-(10).

$$\text{Distance Similarity} = DS(\vec{k}_i, \vec{k}_j) = 1 - \left[ \frac{D(\vec{k}_i, \vec{k}_j)}{\max_{\forall i,j} D(\vec{k}_i, \vec{k}_j)} \right] \tag{8}$$

$$\text{Angle Similarity} = \theta S(\vec{k}_i, \vec{k}_j) = \frac{<\vec{k}_i, \vec{k}_j>}{\|\vec{k}_i\| \, \|\vec{k}_j\|} = \cos\theta \tag{9}$$

$$\text{Similarity of Knowledge} = KKS_{ij} = DS(\vec{k}_i, \vec{k}_j) \cdot \theta S(\vec{k}_i, \vec{k}_j) \tag{10}$$

The *KKS* value should be in the range from 0 to 1 since both the Distance Similarity (*DS*) and Angle Similarity (*θS*) range from 0 to 1. In the same way, the pairwise knowledge similarity can be described by Eqs. (11)-(14).

$$\text{Antecedent Similarity} = AAS_{ij} = DS(\vec{a}_i, \vec{a}_j) \cdot \theta S(\vec{a}_i, \vec{a}_j) \tag{11}$$

$$\text{Consequent Similarity} = CCS_{ij} = DS(\vec{c}_i, \vec{c}_j) \cdot \theta S(\vec{c}_i, \vec{c}_j) \tag{12}$$

$$\text{Antecedent-Consequent Similarity} = ACS_{ij} = DS(\vec{a}_i, \vec{c}_j) \cdot \theta S(\vec{a}_i, \vec{c}_j) \tag{13}$$

$$\text{Consequent-Antecedent Similarity} = CAS_{ij} = DS(\vec{c}_i, \vec{a}_j) \cdot \theta S(\vec{c}_i, \vec{a}_j) \tag{14}$$

The Knowledge Similarity Matrix (*KSM*) is constructed as follows.

$$KSM = [KKS_{ij}]_{m*m} = \begin{bmatrix} 1 & KKS_{12} & \ldots & \ldots & KKS_{1m} \\ KKS_{21} & 1 & \ldots & \ldots & KKS_{2m} \\ : & : & & & \\ : & : & & & \\ KKS_{m1} & KKS_{m2} & \ldots & \ldots & 1 \end{bmatrix} \tag{15}$$

In the same way, the pairwise knowledge similarity matrice can be described by Eqs. (16)-(19).

$$\text{Antecedent Similarity Matrix} = ASM = [AAS_{ij}]_{m*m} \tag{16}$$

$$\text{Consequent Similarity Matrix} = CSM = [CCS_{ij}]_{m*m} \tag{17}$$

$$\text{Antecedent Consequent Similarity Matrix} = ACSM = [ACS_{ij}]_{m*m} \tag{18}$$

$$\text{Consequent Antecedent Similarity Matrix} = CASM = [CAS_{ij}]_{m*m} \tag{19}$$

Based on the above discussions, the conditional probability is $P(B \mid A) = \dfrac{P(B \cap A)}{P(A)}$ and $P(A \mid B) = \dfrac{P(B \cap A)}{P(B)}$, $P(B \cap A)$ under $P(A) = 1$ may not equal to $P(B \cap A)$ under $P(B) = 1$. Therefore, the knowledge matrix formed by the knowledge vectors after transform mapping of two equal knowledge weights could be asymmetrical. The similarity matrix obtained by using asymmetrical knowledge matrix may not be symmetrical.

### 3.4 Conditional Probability Knowledge Similarity Algorithm (CPKSA) and Rule-based Knowledge Similarity Calculation System (RBKSCS)

To sum up, the present research proposes, as an integration of knowledge engineering, an improved *RO-RA-RV* knowledge representation compared with the traditional *O-A-V* one. It does so by employing conditional probability, vectors and matrices, and artificial intelligent rule-based reasoning, and by building Conditional Probability knowledge Similarity Algorithm (CPKSA).

**Algorithm** Conditional Probability Knowledge Similarity Algorithm
**Input:**
*RO*, *RA* and *RV* // components of m rules
*m* // total numbers of rule
*l* // the maximal dimension of the antecedent vectors
*n* // the maximal dimension of the consequent vectors
**Output:**
*ASM*, *CSM* and *KSM* //Antecedent Similarity Matrix (ASM), Consequent Similarity

Matrix (CSM), Knowledge Similarity Matrix (ASM)

**Step 1: If** the antecedent and consequent vectors are in different dimensions **then** they are represented by the maximal dimension between them, with the augmented dimensions reduced to zero.
**Step 2: According to** the data type of components *O-A-V* and the symbol of relationship operators (*R*) to **do** the testing case and knowledge base cases pairwise transform mapping that the values are between 0 and 1.
**Step 3: To calculate** the length of each knowledge vector, the distance and inner product of any two knowledge vectors, and decide the maximum value of the whole distance among.
**Step 4: To compute** the similarities of the distance, the angle, the antecedent, the consequent and the knowledge for pairwise knowledge vectors.
**Step 5: Output** ASM, CSM and KSM.
**Step 6: End**.

For two vectors $\vec{a} = (0.8, 0.9, 0.7)$ and $\vec{b} = (0.7, 0.8, 0.6)$ from paper [16], A comparison of the calculation results for the proposed similarity measure with existing methods is shown in Table 18.

**Table 18. A comparison of the calculation results for the proposed similarity measure with existing methods.**

| Index | Formula of Similarity | Similarity | Remark |
|---|---|---|---|
| 1 | $S(\vec{a}, \vec{b}) = \left[1 - \frac{D}{D_{\max}}\right] \cdot \left(\frac{\vec{a} \cdot \vec{b}}{\lVert\vec{a}\rVert \cdot \lVert\vec{b}\rVert}\right)$ | 0.8755 | From this paper |
| 2 | $S(\vec{a}, \vec{b}) = a^{D} \cdot \left(\frac{\vec{a} \cdot \vec{b}}{\lVert\vec{a}\rVert \cdot \lVert\vec{b}\rVert}\right), 0.7 \le a \le 0.97$ | 0.9399 ($a = 0.7$) | From paper [8] |
| 3 | $S(\vec{a}, \vec{b}) = a^{-D} \cdot \left(\frac{\vec{a} \cdot \vec{b}}{\lVert\vec{a}\rVert \cdot \lVert\vec{b}\rVert}\right), a > 1$ | 0.9320 ($a = 1.5$) | From paper [9] |
| 4 | $S(\vec{a}, \vec{b}) = \frac{\vec{a} \cdot \vec{b}}{\max(\vec{a} \cdot \vec{a}, \vec{b} \cdot \vec{b})}$ | 0.8763 | From paper [16] |
| 5 | $S(\vec{a}, \vec{b}) = 1 - \sqrt{\frac{\sum_{i}^{n} [a(x_i) - b(x_i)]^2}{n}}$ | 0.9 | From paper [5] |

Based on CPKSA, a Rule-Based Knowledge Similarity Calculation System (RBKSCS) in the format of Web pages using PHP is thus built. It can be used to quickly calculate ASM, CSM, and KSM, thereby characterizing the relationship between two knowledge instances. 20 instances of rule-based knowledge are shown in Table 19. Through executed by RBKSCS, the antecedent, consequent and knowledge similarity matrices of the 20 rules are given in the Figs. 5 to 7, respectively.

**Table 19. Representation of knowledge instances.**

| Knowledge index | Antecedent | | | Consequent | | |
|---|---|---|---|---|---|---|
| | Gender | Height | Weight | Gender | Height_type | Weight_type |
| Rule 1 | = male | > 190 | = Null | = male | = Very_large | = Null |
| Rule 2 | = male | = Null | = 45 | = male | = Null | = Very_small |
| Rule 3 | = male | = Null | < 55 | = male | = Null | = Small |
| Rule 4 | = female | > 180 | = Null | = female | = Very_large | = Null |
| Rule 5 | = female | = Null | > 70 | = female | = Null | = Very_large |
| Rule 6 | = male | = 173 | = Null | = male | = Medium | = Null |
| Rule 7 | = female | = Null | = 50 | = female | = Null | = Small |
| Rule 8 | = male | = Null | = 65 | = male | = Null | = Medium |
| Rule 9 | = male | = 195 | = Null | = male | = Very_large | = Null |
| Rule 10 | = female | = 160 | = Null | = female | = Medium | = Null |
| Rule 11 | = female | = Null | < 40 | = female | = Null | = Very_small |
| Rule 12 | = female | < 150 | = Null | = female | = Small | = Null |
| Rule 13 | = male | > 180 | = Null | = male | = Large | = Null |
| Rule 14 | = male | < 162 | = Null | = male | = Small | = Null |
| Rule 15 | = female | = 160 | = Null | = female | = Small | = Null |
| Rule 16 | = female | = Null | = 50 | = female | = Null | = Small |
| Rule 17 | = female | = 160 | = Null | = female | = Small | = Null |
| Rule 18 | = male | > 180 | = Null | = male | = Large | = Null |
| Rule 19 | = female | = Null | = 50 | = female | = Null | = Small |
| Rule 20 | = female | = Null | < 48 | = female | = Null | = Small |

| | Rule1 | Rule2 | Rule3 | Rule4 | Rule5 | Rule6 | Rule7 | Rule8 | Rule9 | Rule10 | Rule11 | Rule12 | Rule13 | Rule14 | Rule15 | Rule16 | Rule17 | Rule18 | Rule19 | Rule20 | Count |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Rule1 | 1 | 0.106 | 0.106 | 0.337 | 0 | 0.345 | 0 | 0.106 | 1 | 0.106 | 0 | 0.106 | 0.901 | 0.345 | 0.106 | 0 | 0.106 | 0.901 | 0 | 0 | 2 |
| Rule2 | 0.106 | 1 | 0.36 | 0 | 0.106 | 0.106 | 0.106 | 0.345 | 0.106 | 0 | 0.106 | 0 | 0.106 | 0.106 | 0 | 0.106 | 0 | 0.106 | 0.106 | 0.115 | 1 |
| Rule3 | 0.106 | 1 | 1 | 0 | 0.106 | 0.106 | 0.345 | 0.345 | 0.106 | 0 | 0.345 | 0 | 0.106 | 0.106 | 0 | 0.345 | 0 | 0.106 | 0.345 | 0.345 | 2 |
| Rule4 | 0.345 | 0 | 0 | 1 | 0.106 | 0.106 | 0.106 | 0 | 0.345 | 0.345 | 0.106 | 0.345 | 0.345 | 0.106 | 0.345 | 0.106 | 0.345 | 0.345 | 0.106 | 0.106 | 1 |
| Rule5 | 0 | 0.106 | 0.106 | 0.106 | 1 | 0 | 0.345 | 0.106 | 0 | 0.106 | 0.345 | 0.106 | 0 | 0 | 0.106 | 0.345 | 0.106 | 0 | 0.345 | 0.345 | 1 |
| Rule6 | 0.345 | 0.106 | 0.106 | 0.106 | 0 | 1 | 0 | 0.106 | 0.345 | 0.106 | 0 | 0.106 | 0.345 | 0.345 | 0.106 | 0 | 0.106 | 0.345 | 0 | 0 | 1 |
| Rule7 | 0 | 0.106 | 0.113 | 0.106 | 0.345 | 0 | 1 | 0.106 | 0 | 0.106 | 0.345 | 0.106 | 0 | 0 | 0.106 | 1 | 0.106 | 0 | 1 | 0.345 | 3 |
| Rule8 | 0.106 | 0.345 | 0.345 | 0 | 0.106 | 0.106 | 0.106 | 1 | 0.106 | 0 | 0.106 | 0 | 0.106 | 0.106 | 0 | 0.106 | 0 | 0.106 | 0.106 | 0.106 | 1 |
| Rule9 | 0.358 | 0.106 | 0.106 | 0.112 | 0 | 0.345 | 0 | 0.106 | 1 | 0.106 | 0 | 0.106 | 0.356 | 0.345 | 0.106 | 0 | 0.106 | 0.356 | 0 | 0 | 1 |
| Rule10 | 0.106 | 0 | 0 | 0.345 | 0.106 | 0.106 | 0.106 | 0 | 0.106 | 1 | 0.106 | 0.345 | 0.106 | 0.112 | 1 | 0.106 | 1 | 0.106 | 0.106 | 0.106 | 3 |
| Rule11 | 0 | 0.106 | 0.314 | 0.106 | 0.345 | 0 | 0.345 | 0.106 | 0 | 0.106 | 1 | 0.106 | 0 | 0 | 0.106 | 0.345 | 0.106 | 0 | 0.345 | 0.874 | 1 |
| Rule12 | 0.106 | 0 | 0 | 0.345 | 0.106 | 0.106 | 0.106 | 0 | 0.106 | 0.345 | 0.106 | 1 | 0.106 | 0.332 | 0.345 | 0.106 | 0.345 | 0.106 | 0.106 | 0.106 | 1 |
| Rule13 | 1 | 0.106 | 0.106 | 0.345 | 0 | 0.345 | 0 | 0.106 | 1 | 0.106 | 0 | 0.106 | 1 | 0.345 | 0.106 | 0 | 0.106 | 1 | 0 | 0 | 4 |
| Rule14 | 0.345 | 0.106 | 0.106 | 0.106 | 0 | 0.345 | 0 | 0.106 | 0.345 | 0.345 | 0 | 0.345 | 0.345 | 1 | 0.345 | 0 | 0.345 | 0.345 | 0 | 0 | 1 |
| Rule15 | 0.106 | 0 | 0 | 0.345 | 0.106 | 0.106 | 0.106 | 0 | 0.106 | 1 | 0.106 | 0.345 | 0.106 | 0.112 | 1 | 0.106 | 1 | 0.106 | 0.106 | 0.106 | 3 |
| Rule16 | 0 | 0.106 | 0.113 | 0.106 | 0.345 | 0 | 1 | 0.106 | 0 | 0.106 | 0.345 | 0.106 | 0 | 0 | 0.106 | 1 | 0.106 | 0 | 1 | 0.345 | 3 |
| Rule17 | 0.106 | 0 | 0 | 0.345 | 0.106 | 0.106 | 0.106 | 0 | 0.106 | 1 | 0.106 | 0.345 | 0.106 | 0.112 | 1 | 0.106 | 1 | 0.106 | 0.106 | 0.106 | 3 |
| Rule18 | 1 | 0.106 | 0.106 | 0.345 | 0 | 0.345 | 0 | 0.106 | 1 | 0.106 | 0 | 0.106 | 1 | 0.345 | 0.106 | 0 | 0.106 | 1 | 0 | 0 | 4 |
| Rule19 | 0 | 0.106 | 0.113 | 0.106 | 0.345 | 0 | 1 | 0.106 | 0 | 0.106 | 0.345 | 0.106 | 0 | 0 | 0.106 | 1 | 0.106 | 0 | 1 | 0.345 | 3 |
| Rule20 | 0 | 0.345 | 0.338 | 0.106 | 0.345 | 0 | 0.345 | 0.106 | 0 | 0.106 | 1 | 0.106 | 0 | 0 | 0.106 | 0.345 | 0.106 | 0 | 0.345 | 1 | 2 |

Fig. 5. Antecedent similarity matrix (ASM).

## 4. CONCLUSIONS

In summary, the present research reaches the following five conclusions:

1. A new knowledge vector representation for rule-based deterministic knowledge can be proposed on the basis of the *RO-RA-RV* structure, which replaces the traditional *O-A-V*,

| | Rule1 | Rule2 | Rule3 | Rule4 | Rule5 | Rule6 | Rule7 | Rule8 | Rule9 | Rule10 | Rule11 | Rule12 | Rule13 | Rule14 | Rule15 | Rule16 | Rule17 | Rule18 | Rule19 | Rule20 | Count |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Rule1 | 1 | 0.106 | 0.106 | 0.345 | 0 | 0.684 | 0 | 0.106 | 1 | 0.275 | 0 | 0.195 | 0.849 | 0.513 | 0.195 | 0 | 0.195 | 0.849 | 0 | 0 | 2 |
| Rule2 | 0.299 | 1 | 0.843 | 0 | 0.092 | 0.299 | 0.278 | 0.671 | 0.299 | 0 | 0.299 | 0 | 0.299 | 0.299 | 0 | 0.278 | 0 | 0.299 | 0.278 | 0.278 | 1 |
| Rule3 | 0.282 | 0.843 | 1 | 0 | 0.171 | 0.282 | 0.303 | 0.845 | 0.282 | 0 | 0.282 | 0 | 0.282 | 0.282 | 0 | 0.303 | 0 | 0.282 | 0.303 | 0.303 | 1 |
| Rule4 | 0.345 | 0 | 0 | 1 | 0.106 | 0.275 | 0.106 | 0 | 0.345 | 0.684 | 0.106 | 0.513 | 0.327 | 0.195 | 0.513 | 0.106 | 0.513 | 0.327 | 0.106 | 0.106 | 1 |
| Rule5 | 0 | 0.106 | 0.195 | 0.106 | 1 | 0 | 0.513 | 0.275 | 0 | 0.106 | 0.345 | 0.106 | 0 | 0 | 0.106 | 0.513 | 0.106 | 0 | 0.513 | 0.513 | 1 |
| Rule6 | 0.684 | 0.236 | 0.236 | 0.251 | 0 | 1 | 0 | 0.236 | 0.684 | 0.315 | 0 | 0.294 | 0.846 | 0.845 | 0.294 | 0 | 0.294 | 0.846 | 0 | 0 | 1 |
| Rule7 | 0 | 0.282 | 0.303 | 0.282 | 0.513 | 0 | 1 | 0.284 | 0 | 0.282 | 0.843 | 0.282 | 0 | 0 | 0.282 | 1 | 0.282 | 0 | 1 | 1 | 4 |
| Rule8 | 0.236 | 0.671 | 0.845 | 0 | 0.251 | 0.236 | 0.294 | 1 | 0.236 | 0 | 0.236 | 0 | 0.236 | 0.236 | 0 | 0.294 | 0 | 0.236 | 0.294 | 0.294 | 1 |
| Rule9 | 1 | 0.106 | 0.106 | 0.345 | 0 | 0.684 | 0 | 0.106 | 1 | 0.275 | 0 | 0.195 | 0.849 | 0.513 | 0.195 | 0 | 0.195 | 0.849 | 0 | 0 | 2 |
| Rule10 | 0.251 | 0 | 0 | 0.684 | 0.236 | 0.315 | 0.236 | 0 | 0.251 | 1 | 0.236 | 0.845 | 0.297 | 0.294 | 0.845 | 0.236 | 0.845 | 0.297 | 0.236 | 0.236 | 1 |
| Rule11 | 0 | 0.299 | 0.278 | 0.299 | 0.345 | 0 | 0.843 | 0.224 | 0 | 0.299 | 1 | 0.299 | 0 | 0 | 0.299 | 0.843 | 0.299 | 0 | 0.843 | 0.843 | 1 |
| Rule12 | 0.171 | 0 | 0 | 0.513 | 0.282 | 0.284 | 0.282 | 0 | 0.171 | 0.845 | 0.282 | 1 | 0.235 | 0.303 | 1 | 0.282 | 1 | 0.235 | 0.282 | 0.282 | 3 |
| Rule13 | 0.849 | 0.174 | 0.174 | 0.313 | 0 | 0.846 | 0 | 0.174 | 0.849 | 0.311 | 0 | 0.255 | 1 | 0.676 | 0.255 | 0 | 0.255 | 1 | 0 | 0 | 2 |
| Rule14 | 0.513 | 0.282 | 0.282 | 0.171 | 0 | 0.845 | 0 | 0.282 | 0.513 | 0.284 | 0 | 0.303 | 0.676 | 1 | 0.303 | 0 | 0.303 | 0.676 | 0 | 0 | 1 |
| Rule15 | 0.171 | 0 | 0 | 0.513 | 0.282 | 0.284 | 0.282 | 0 | 0.171 | 0.845 | 0.282 | 1 | 0.235 | 0.303 | 1 | 0.282 | 1 | 0.235 | 0.282 | 0.282 | 3 |
| Rule16 | 0 | 0.282 | 0.303 | 0.282 | 0.513 | 0 | 1 | 0.284 | 0 | 0.282 | 0.843 | 0.282 | 0 | 0 | 0.282 | 1 | 0.282 | 0 | 1 | 1 | 4 |
| Rule17 | 0.171 | 0 | 0 | 0.513 | 0.282 | 0.284 | 0.282 | 0 | 0.171 | 0.845 | 0.282 | 1 | 0.235 | 0.303 | 1 | 0.282 | 1 | 0.235 | 0.282 | 0.282 | 3 |
| Rule18 | 0.849 | 0.174 | 0.174 | 0.313 | 0 | 0.846 | 0 | 0.174 | 0.849 | 0.311 | 0 | 0.255 | 1 | 0.676 | 0.255 | 0 | 0.255 | 1 | 0 | 0 | 2 |
| Rule19 | 0 | 0.282 | 0.303 | 0.282 | 0.513 | 0 | 1 | 0.284 | 0 | 0.282 | 0.843 | 0.282 | 0 | 0 | 0.282 | 1 | 0.282 | 0 | 1 | 1 | 4 |
| Rule20 | 0 | 0.282 | 0.303 | 0.282 | 0.513 | 0 | 1 | 0.284 | 0 | 0.282 | 0.843 | 0.282 | 0 | 0 | 0.282 | 1 | 0.282 | 0 | 1 | 1 | 4 |

Fig. 6. Consequent similarity matrix (CSM).

| | Rule1 | Rule2 | Rule3 | Rule4 | Rule5 | Rule6 | Rule7 | Rule8 | Rule9 | Rule10 | Rule11 | Rule12 | Rule13 | Rule14 | Rule15 | Rule16 | Rule17 | Rule18 | Rule19 | Rule20 | Count |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Rule1 | 1 | 0.106 | 0.106 | 0.341 | 0 | 0.484 | 0 | 0.106 | 1 | 0.18 | 0 | 0.147 | 0.872 | 0.422 | 0.147 | 0 | 0.147 | 0.872 | 0 | 0 | 2 |
| Rule2 | 0.185 | 1 | 0.524 | 0 | 0.095 | 0.185 | 0.178 | 0.472 | 0.185 | 0 | 0.185 | 0 | 0.185 | 0.185 | 0 | 0.178 | 0 | 0.185 | 0.178 | 0.184 | 1 |
| Rule3 | 0.18 | 0.893 | 1 | 0 | 0.132 | 0.18 | 0.329 | 0.515 | 0.18 | 0 | 0.319 | 0 | 0.18 | 0.18 | 0 | 0.329 | 0 | 0.18 | 0.329 | 0.329 | 1 |
| Rule4 | 0.345 | 0 | 0 | 1 | 0.106 | 0.18 | 0.106 | 0 | 0.345 | 0.484 | 0.106 | 0.422 | 0.335 | 0.147 | 0.422 | 0.106 | 0.422 | 0.335 | 0.106 | 0.106 | 1 |
| Rule5 | 0 | 0.106 | 0.147 | 0.106 | 1 | 0 | 0.422 | 0.18 | 0 | 0.106 | 0.345 | 0.106 | 0 | 0 | 0.106 | 0.422 | 0.106 | 0 | 0.422 | 0.422 | 1 |
| Rule6 | 0.477 | 0.163 | 0.163 | 0.166 | 0 | 1 | 0 | 0.163 | 0.477 | 0.192 | 0 | 0.184 | 0.518 | 0.517 | 0.184 | 0 | 0.184 | 0.518 | 0 | 0 | 1 |
| Rule7 | 0 | 0.18 | 0.192 | 0.18 | 0.414 | 0 | 1 | 0.18 | 0 | 0.18 | 0.515 | 0.18 | 0 | 0 | 0.18 | 1 | 0.18 | 0 | 1 | 0.53 | 3 |
| Rule8 | 0.163 | 0.474 | 0.517 | 0 | 0.166 | 0.163 | 0.184 | 1 | 0.163 | 0 | 0.163 | 0 | 0.163 | 0.163 | 0 | 0.184 | 0 | 0.163 | 0.184 | 0.184 | 1 |
| Rule9 | 0.55 | 0.106 | 0.106 | 0.211 | 0 | 0.484 | 0 | 0.106 | 1 | 0.18 | 0 | 0.147 | 0.534 | 0.422 | 0.147 | 0 | 0.147 | 0.534 | 0 | 0 | 1 |
| Rule10 | 0.166 | 0 | 0 | 0.477 | 0.163 | 0.192 | 0.163 | 0 | 0.166 | 1 | 0.163 | 0.517 | 0.185 | 0.188 | 0.893 | 0.163 | 0.893 | 0.185 | 0.163 | 0.163 | 1 |
| Rule11 | 0 | 0.185 | 0.3 | 0.185 | 0.338 | 0 | 0.514 | 0.157 | 0 | 0.185 | 1 | 0.185 | 0 | 0 | 0.185 | 0.514 | 0.185 | 0 | 0.514 | 0.858 | 1 |
| Rule12 | 0.132 | 0 | 0 | 0.414 | 0.18 | 0.18 | 0.18 | 0 | 0.132 | 0.515 | 0.18 | 1 | 0.16 | 0.322 | 0.53 | 0.18 | 0.53 | 0.16 | 0.18 | 0.18 | 1 |
| Rule13 | 0.894 | 0.138 | 0.138 | 0.329 | 0 | 0.521 | 0 | 0.138 | 0.894 | 0.192 | 0 | 0.17 | 1 | 0.479 | 0.17 | 0 | 0.17 | 1 | 0 | 0 | 2 |
| Rule14 | 0.414 | 0.18 | 0.18 | 0.132 | 0 | 0.515 | 0 | 0.18 | 0.414 | 0.319 | 0 | 0.329 | 0.473 | 1 | 0.329 | 0 | 0.329 | 0.473 | 0 | 0 | 1 |
| Rule15 | 0.132 | 0 | 0 | 0.414 | 0.18 | 0.18 | 0.18 | 0 | 0.132 | 0.893 | 0.18 | 0.53 | 0.16 | 0.191 | 1 | 0.18 | 1 | 0.16 | 0.18 | 0.18 | 2 |
| Rule16 | 0 | 0.18 | 0.192 | 0.18 | 0.414 | 0 | 1 | 0.18 | 0 | 0.18 | 0.515 | 0.18 | 0 | 0 | 0.18 | 1 | 0.18 | 0 | 1 | 0.53 | 3 |
| Rule17 | 0.132 | 0 | 0 | 0.414 | 0.18 | 0.18 | 0.18 | 0 | 0.132 | 0.893 | 0.18 | 0.53 | 0.16 | 0.191 | 1 | 0.18 | 1 | 0.16 | 0.18 | 0.18 | 2 |
| Rule18 | 0.894 | 0.138 | 0.138 | 0.329 | 0 | 0.521 | 0 | 0.138 | 0.894 | 0.192 | 0 | 0.17 | 1 | 0.479 | 0.17 | 0 | 0.17 | 1 | 0 | 0 | 2 |
| Rule19 | 0 | 0.18 | 0.192 | 0.18 | 0.414 | 0 | 1 | 0.18 | 0 | 0.18 | 0.515 | 0.18 | 0 | 0 | 0.18 | 1 | 0.18 | 0 | 1 | 0.53 | 3 |
| Rule20 | 0 | 0.319 | 0.325 | 0.18 | 0.414 | 0 | 0.53 | 0.18 | 0 | 0.18 | 0.893 | 0.18 | 0 | 0 | 0.18 | 0.53 | 0.18 | 0 | 0.53 | 1 | 1 |

Fig. 7. Knowledge similarity matrix (KSM).

and is comprised of four components: relationship operators ($R$), object ($O$), attribute ($A$), and linguistic values ($V$).

2. A calculation method for knowledge similarity can be proposed by integrating the distance and angle representations of rule-based deterministic knowledge. Also a conditional probability knowledge similarity algorithm (CPKSA) can be proposed.
3. Based on CPKSA, a rule-based knowledge similarity calculation system (RBKSCS) of Web page format can be built and used to quickly and accurately obtain knowledge similarity matrixes.
4. The knowledge case most similar to the testing case can be quickly retrieved from the knowledge base by applying CPKSA and RBKSCS, and used for all types of with Knowledge-Based Reasoning (KBR) or Case-Based Reasoning (CBR) to help decision making and prediction.

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