Modeling and Analysis of Urban Traffic Lights Control Systems Using Timed CP-nets*

YI-SHENG HUANG[1] AND TA-HSIANG CHUNG[2]
[1]*Department of Electrical and Electronic Engineering*
[2]*School of Defense Science*
Chung Cheng Institute of Technology
National Defense University
Taoyuan, 335 Taiwan

Petri nets have been utilized as a visual formalism for the modeling of complex system. It illuminates the features on describing properties of causality, concurrency and synchronization. This paper focuses on the use of Timed Coloured Petri nets (Timed CP-nets) to model a basic traffic control system. A module of basic traffic control system of Timed CP-nets model is successfully constructed. The advantage of the proposed approach is that any complicated traffic signal models will be easily obtained based on the module. Moreover, a real-world supervisor of the urban traffic light system, which consists of three intersections with multiple phases, is implemented by the new methodology. The analysis method of place invariant is verified. And the *Occurrence Graphs* (OG) is performed to demonstrate how the model enforces the signal lights' transitions. To our knowledge, this is the first work that proposes the new modeling methodology to design the traffic light control systems by Timed CP-nets.

Keywords: timed coloured Petri nets, traffic signal, phase transition, intelligent transportation system, traffic control

1. INTRODUCTION

With the growing number of vehicles, the traffic congestion and transportation delay on urban arterials are increasing worldwide; hence it is imperative to improve the safety and efficiency of transportation. Subsequently, several research teams focus their attention on the area of Intelligent Transportation System (ITS) [1, 2]. In addition, they applied advanced communication, information, and electronics technology to solve transportation problems such as traffic congestion, safety, transportation efficiency [3]. It is an important issue in traffic light control systems for two main reasons: (1) the traffic light control systems exhibit a high degree of concurrency (2) the systems might make the shared resources (*i.e.*, intersection area) conflict and cause a tendency to deadlock and overload. The control measures and strategies that utilize traffic-light controls are classified into two categories [4]: (1) Fixed-time strategies and (2) Traffic-response strategies. However the urban traffic control of fixed-time strategies have been using in most industrialized countries. Hence, the authors focus the traffic control systems on fixed-time in this paper. And the purpose of this paper is to model a sophisticated phase that has not discussed in the pioneer literatures.

Received November 6, 2006; revised March 2, 2007; accepted April 14, 2007.
Communicated by Chin-Teng Lin.
* This paper was partially supported by the National Science Council of Taiwan, R.O.C. under grand No. NSC 94-2218-E-014-001. The earlier version of this paper was presented at 2004 IEEE International Conference on System, Man and Cybernetics, Hawaii, October 10-12, 2005.

Simulation has most successfully used to analyze and design new control strategies [5]. Petri nets (PN) have been proven to be a powerful modeling tool for various kinds of discrete event systems [6, 7]. The increasing interest in Petri net is stimulated by their analysis of the modeled systems. Petri nets formalism provides a clear means for presenting simulation and control logic. It also expedites the generation of control code from the PN graph [6, 7]. Traffic signal control has been accomplished by using Petri nets. For example, Wang *et al*. proposed a PN model that encompasses both traffic signal control logic and traffic flow [8], Tolba *et al*. submitted continuous Petri nets with variable speed (VCPN) and timed Petri nets (TPN) that are possible to study the traffic behavior at crossroad in a microscopic point of view and a macroscopic one for a single crossroad [9], and Wang *et al*. employed stochastic timed Petri nets (STPN) to presented a traffic control and vehicle movement behavior of an intersection [10], Moreover, Lin *et al*. approached a control methodology of traffic lights based on PN whose places represent vehicle streams [11], Basile *et al*. proposed a traffic control to model intersection and road link by way of Coloured timed PN model and Hybrid stochastic model [12], Dotoli *et al.* submitted Colored Timed Petri Net (CTPN) to model a signalized area, and a traffic light was modeled by Timed Petri Nets [13]. Recently, a development involving traffic light system models is reported in [14].

The above models, however, describe a traffic light to simplify and interpret the intersection to complicate. For example, there is only one green light describing a traffic light, yet a traffic flow includes left turn, go straight and right turn contained the model of intersection. It is important to notice that a fixed time traffic light system is not suitable for a real-time traffic since it provides only a guidance of traffic. The Timed CP-nets presented here can overcome this drawback and has following advantages:

(1) The proposed Timed CP-nets models are included three kinds of green lights (*i.e.*, a left turn arrow on green, a right turn arrow on green and a straight arrow on green) which support the designers are able to model a more complex phase of traffic light.
(2) A traffic light control system model is dramatically simplified by Timed CP-nets.
(3) The duration and the variation of the traffic signal lights can be directly observed from Timed CP-nets models.
(4) The traffic light models using Timed CP-nets could be regard as modularity. This feature shows the traffic light model is easy extended and is readable.

The advantages of the Timed CP-nets are pointed out as the mention above. It is hardly obtained the same results by other kinds of PN models for traffic light control systems. Especially, a supervisor of an urban traffic light system, which consists of three intersections with multiple phases, is implemented by Timed CP-nets.

The rest of the paper is organized as follows: Section 2 presents basic definitions of Timed CP-nets. Section 3 presents traffic light systems. Section 4 depicts how to model the traffic lights using Timed CP-nets. And section 5 presents the analysis of the Timed CP-nets models. Conclusions are presented in section 6.

2. THE BASIC DEFINITIONS OF TIMED CP-NETS

In this section, we first introduce the definitions of Timed CP-nets. The definitions of the later are useful in establishing a Timed CP-nets model for a traffic signal control logical. The definitions of Timed CP-nets are going to be presented here in a compact way. In this paper, the author follows the original definitions of Timed CP-nets by Jensen [15, 16].

Definition A timed non-hierarchical CP-nets is a tuple TCPN = (CPN, R, r_0) such that:

1. $CPN = (\Sigma, P, T, A, N, C, G, E, I)$ satisfying the requirements below:
   (1) Σ is a finite set of non empty types, called colour sets.
   (2) P is a finite set of places.
   (3) T is a finite set of transitions.
   (4) A is a finite set of arcs such that: $P \cap T = P \cap A = T \cap A = \emptyset$.
   (5) N is a node function. It is defined from A into $P \times T \cup T \times P$.
   (6) C is a colour function. It is defined from P into Σ.
   (7) G is a guard function. It is defined from T into expressions.
   (8) E is an arc expression function. It is defined from A into expressions.
   (9) I is an initialization function. It is defined from P into closed expressions that an expression is without variables.
2. R is a set of time values, also called *time stamps*. It is closed under + and including 0.
3. r_0 is an element of R called the *start time*.

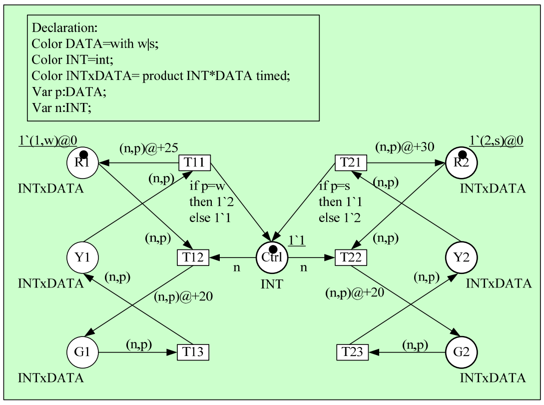


Fig. 1. A Timed CP-net model for a traffic light system.

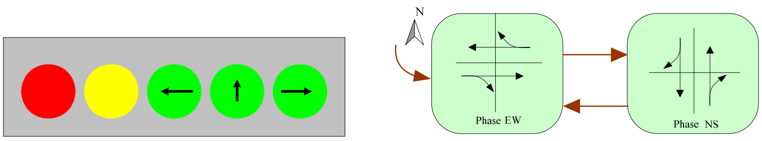
For convenience, we employ a Timed CP-net model for a traffic light system (*i.e.*, Fig. 1) as an example to illustrate the related definitions. In the following, we can easily to derive the equations form the Timed CP-net model. And the equations are listed below:

1. $tm_1 = 1'(1, w)@0 + 1'(2, s)@0 + 1'1$
2. $tm_1[1, w] = 0$, $tm_1[2, s] = 0$
3. $tm_{1U} = 1'(1, w) + 1'(2, s) + 1'1$
4. $\Sigma = \{INT, DATA, INT \times DATA\}$

5. $P = \{R1, Y1, G1, R2, Y2, G2, Ctrl\}$
6. $T = \{T11, T12, T13, T21, T22, T23\}$
7. $A = \{R1 \text{ to } T12, T12 \text{ to } G1, \ldots\}$
8. $N(a) = (SOURCE, DEST)$ if a is in the form $SOURCE$ to $DEST$
9. $$C(p) = \begin{cases} INT & \text{if } p = \{Ctrl\} \\ INT \times DATA & \text{otherwise} \end{cases}$$
10. $G(t) = \phi$
11. $$E(a) = \begin{cases} (n, p) & \text{otherwise} \\ (n, p)\,@{+}\,25 & \text{if } a = T11 \text{ to } R1 \\ (n, p)\,@{+}\,30 & \text{if } a = T21 \text{ to } R2 \\ \text{if } p = w \text{ then } 1`2 \text{ else } 1`1 & \text{if } a = T11 \text{ to } Ctrl \\ \text{if } p = s \text{ then } 1`1 \text{ else } 1`2 & \text{if } a = T21 \text{ to } Ctrl \end{cases}$$
12. $$I(p) = \begin{cases} 1`(1, w)\,@{+}\,0 & \text{if } p = R1 \\ 1`(2, s)\,@{+}\,0 & \text{if } p = R2 \\ 1`1 & \text{if } p = Ctrl \\ \phi & \text{otherwise} \end{cases}$$

3. THE TRAFFIC LIGHT CONTROL SYSTEMS

In this paper, a traffic light which has five signal lights shown in Fig. 2 (a) is employed. It consists of the following five signal lights: a red light signal (*R*), a yellow (*Y*), a left turn arrow on green (*GL*), a right turn arrow on green (*GR*), and a straight arrow on



(a) (b)

Fig. 2. (a) The five signal lights of a traffic light; (b) The phase transition of the basic traffic control system.

green (*GS*). Regularly, there are four traffic lights, *i.e.*, north, south, east and west traffic lights, placed at each intersection. It is usually encountered in an intersection that the vehicle moves go-straight and right turn in two directions: either north/south (denoted by *NS*) or east/west (denoted by *EW*). And such a system is called a basic traffic light control system in this paper. The phase transition of the basic traffic light control system shown in Fig. 2 (b) consists of phase_*EW* and phase_*NS*. And the operations are described as follows. In phase_*EW*, the westbound and eastbound vehicles are allowed to go straight and turn right. In phase_*NS*, the northbound and southbound vehicles are allowed to go straight and turn right.

Based on the basic traffic light control system, three types of extended system which involve a left turn are obtained. The first one describes that the vehicle turn left with the right turn and go straight at the same time. In other words, the three green lights (*i.e.*, *GL*, *GS* and *GR*) are turned on concurrently. The second one indicates that there is only one of the three green lights going on. It means that vehicles left turn is coming after right turn and go-straight. The third one states a sophisticated traffic light control system. It shows a green light is going on which allows the vehicles going straight, turning left and turning right in an intersection. At this moment, the vehicles are allowed to turn right in its perpendicular direction. It is worthy to notice that this situation of traffic light has not discussed in the pioneer literatures. However, this situation of traffic light is considered in this article and is constructed by Timed CP-nets model. For example, an east traffic light *GR* is turning on. At the moment, the north traffic light *G* is possibly to turn on. Usually, the first one has two phases (*i.e.*, shown in Fig. 3 (a)), the second one has four phases (*i.e.*, shown in Fig. 3 (b)), and the last one has three phases (*i.e.*, shown in Fig. 3 (a)). For convenience, a schedule of traffic light signal (*i.e.*, Table 1) is given for discussion of the traffic light control. Note that the others phase duration involve yellow duration except τ_{y2} and τ_{y4}. The yellow duration is assumed as 3 seconds. For insuring traffic safety, all traffic lights are all red in the phase transition completed, called short duration phase. This phase duration is neglected in this paper.

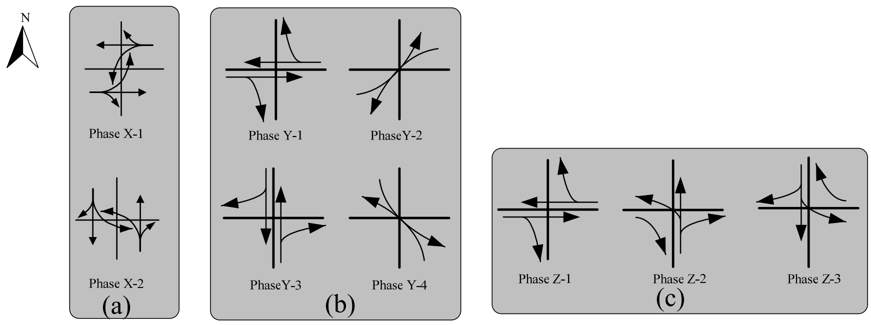


Fig. 3. (a) Two phase transitions; (b) Four phase transitions; (c) Three phase transitions.

Table 1. A signal timing plan.

Phase	*EW*	*NS*	*X*-1	*X*-2	*Y*-1	*Y*-2	*Y*-3	*Y*-4	*Z*-1	*Z*-12	*Z*-3
Phase duration	τ_{ew}	τ_{ns}	τ_{x1}	τ_{x2}	τ_{y1}	τ_{y12}	τ_{y3}	τ_{y4}	τ_{z1}	τ_{z12}	τ_{z3}

4. MODELING OF TRAFFIC LIGHTS CONTROL SYSTEM USING TIMED CP-NETS

It is more important to model the temporal behavior that includes delay and duration in a traffic light system. The Timed CP-nets provides a probability to interpret the properties of a traffic light system in time space. Considering the new methodology is based on a global clock. The global clock values represent the system model time where they may either be discrete time (*i.e.*, all clock values are integer numbers) or continuous time (*i.e.*, all clock values are real numbers). More precisely, each token has a time stamp on it. The time stamp describes the earliest model time at which the token can be moved by a binding element. The current state of a net will change when an enabled binding element occurs. And then the next enabled binding element will generate when the global clock is greater than or equal to the time stamp of the token. After a binding element occurs, the time stamp of the token of the new marking is obtained, which is the time of the current model added to a time delay. The delay time means that the token should stay in the current place for the time being. Note here that one can define the time stamp unit as seconds, microseconds, or millennia and so on. It depends completely on the designer. The authors assume one time unit is equal to one millisecond in this paper.

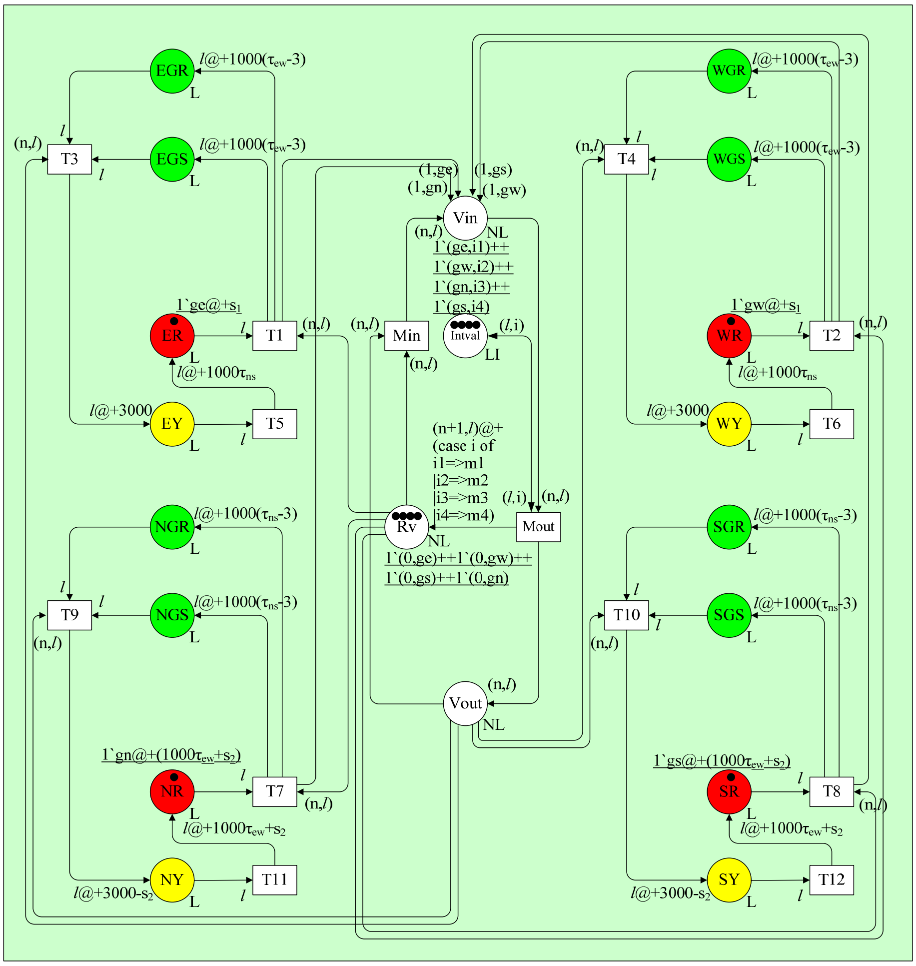


Fig. 4. The timed CP-nets model of basic traffic light control system.

Fig. 4 shows the Timed CP-nets model of the basic traffic light control system. The model consists of two parts: (1) the intersection model and (2) the traffic light model.

The former one models the vehicles passing through the intersection. The latter one models the changing rule of traffic lights which is also called signal indication display model. Based on the rules of Timed CP-nets, the token with time stamp consists of one (*i.e.*, (*l*)) or two (*i.e.*, (*l*, *i*), (*n*, *l*)) elements. The element *l* represents the incoming/outgoing vehicles (*e.g.* westbound vehicles, right turn vehicles from northbound direction). The element *i* represents the average time of vehicle interval. The element *n* stands for the number of incoming cars in the intersection. The more detail information is described as follows.

4.1 An Intersection of Timed CP-nets Model

As mention above, a signal timing plan controls traffic flow in an intersection of predetermining time. Therefore, the basic idea behind the part of intersection model is to allow the cars entering the intersection when the green lights are turned on or to inhibit the cars entering the intersection when the green lights are turned off. Consequently, the intersection model is dramatically simplified by way of a complete display of signal indication.

The places *Intval*, R_v, V_{in}, and V_{out} shown in Fig. 4 are belong to this part. A token in place *Intval* represents the average time of vehicle interval. Considering the firing sequence, a token in place R_v means a car ready for entering the intersection. And then place V_{in} with a token means a car passing through the intersection. Next, a token in place V_{out} represents an outgoing car departing the intersection. To continue, the transitions are discussing as follows. The transition M_{in} represents moving an incoming car into the intersection. The transition M_{out} represents moving an outgoing car departing the intersection. It is note that the values m_i (*i.e.*, $m1$, $m2$, $m3$, …) in the arc expression between place R_v and transition M_{out} are measuring from the real intersections (*i.e.*, introduced in 5.3).

4.2 The Traffic Lights of Timed CP-nets Models

The traffic lights of a generic traffic networks are defined according to a signal timing plan. Especially, a signal timing plan controls vehicle stream (*i.e.*, phase) in an intersection of predetermining time. It is important to construct a precise traffic light whose model has four signal lights, even five signal lights in order to display a complex phase. Hence, the model of traffic light is the kernel of the system model in this paper.

4.2.1 The basic traffic light model

The signal indication display model (*i.e.*, shown in Fig. 4) could be divided into four sub-models. Those are called north, south, east and west traffic light models. The four sub-models represent a set of traffic light facing four directions of an intersection. It is interesting that no arcs are connected with any of the sub-model in the signal indication display model. This main reason is that the firing sequences of the model are decided by the global reached time. As a result, the model is readable. And this is one illuminative feature of the Timed CP-nets model.

To explain this model, a sub-model of east traffic light is utilized as an example to interpret as follows. There are four places *EGR*, *EGS*, *EY*, *ER* which are called signal indication places and three transitions *T*1, *T*3, *T*5 in the sub-model. A token in place *EGR* (*EGS*) means the *GR* (*GS*) turns on in the east traffic light. And then place *EY* (*ER*) with a token represents the *Y* (*R*) turns on. The action of *T*1 turns on *G* and turns off *R*. The occurrence of *T*3 (*T*5) turns on *Y* (*R*) and turns off *G* (*Y*) currently. Note that $s_i = \{s_1, s_2\}$ where the number of s_i consists two bits in the basic traffic light model. The first bit represents which intersection is. The following bit means the trigger sequence of green light in a set of traffic light. For example, the numbers s_1 and s_2 in the places of *ER* and *NR* at the second intersection are 21 and 22. It hints one small time (*i.e.*, 0.021/0.022 seconds) is added in the system model. As a result, the number of s_i does not seem to effect the duration. Nevertheless, it can offer a priority condition to the system model.

4.2.2 The extended traffic light models

Based on the basic traffic light model, three cases of extended model whose phase transition are shown in Figs. 3 (a), (b) and (c) are obtained. These models are also shown in Figs. 5, 6 and 7, respectively. The case I (*i.e.*, shown in Fig. 5) describes that the three green lights (*i.e.*, *GR*, *GS* and *GL*) turn on at the same time. In this extended model, the authors add four places *EGL*, *WGL*, *NGL* and *SGL* on the basic traffic light model. And these places represent the left turn green lights in a set of traffic light. Usually, such a set of traffic light is placed in mirror streets. For simplified, the three green lights could be merged. And then there is only one green light displaying in the traffic light.

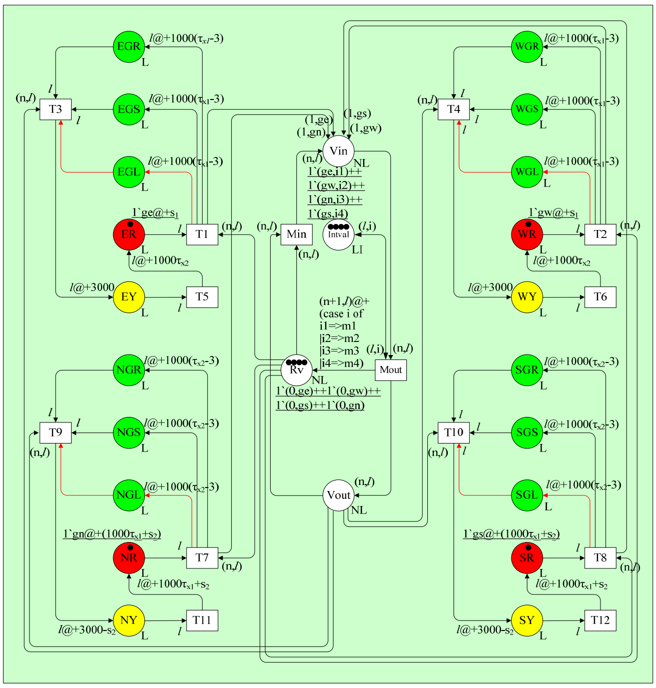


Fig. 5. The case I extended traffic light model.

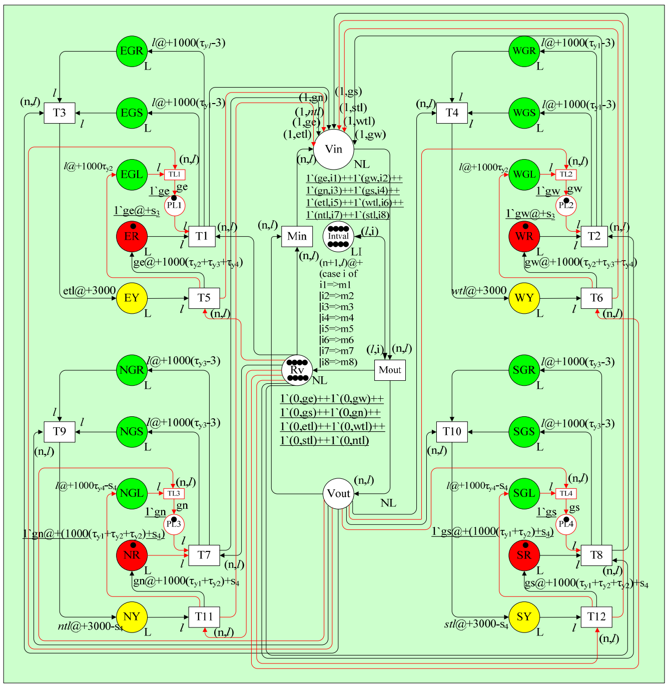


Fig. 6. The case II extended traffic light model.

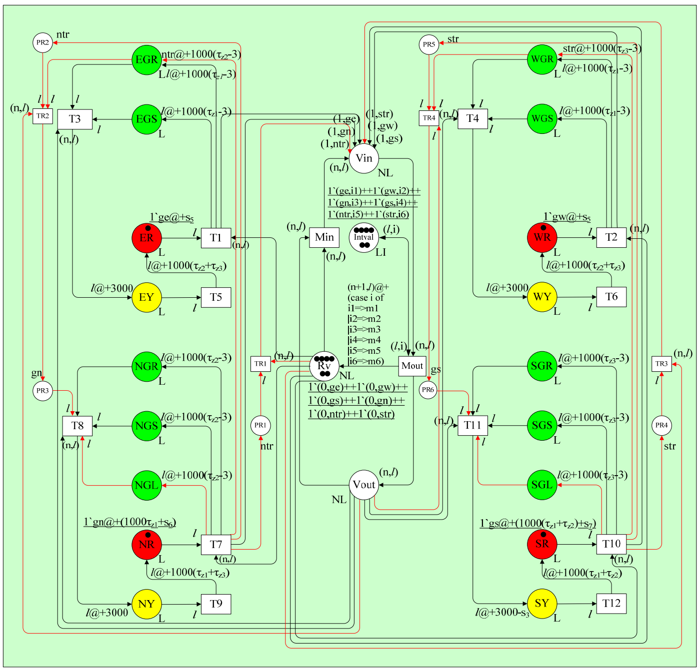


Fig. 7. The case III extended traffic light model.

The case II (*i.e.*, shown in Fig. 6) indicates that the *GL* is turned on after both *GR* and *GS* are termination. This extended model is not only added four places *EGL*, *WGL*, *NGL* and *SGL* on the basic model but also added one additional place (*i.e.*, *PL*1, *PL*2, *PL*3, or *PL*4) and added one transition (*i.e.*, *TL*1, *TL*2, *TL*3, or *TL*4) for each traffic light. The additional places and transitions are used to model a more complex traffic. Note that the number of s_i presents in this model where $s_i = \{s_3, s_4\}$.

The case III shown in Fig. 7 is the complex intersection, yet it is easily constructed by the Timed CP-nets. The solution of the dilemma is that connects the place *EGR* (*WGR*) with the transition *T*7 (*T*8). As a result, the three green light (*i.e.*, in north (south) traffic light) turn on with *GR* (*i.e.*, in east (west) traffic light). In essence, the reason for the sophisticated intersection could be easily constructed is that there are three separate green lights (*i.e.*, *GS*, *GL* and *GR*). Depending on the separate green lights in a set of traffic light, a more complex phase could be derived. In this case, there are a few places and transitions are added. The two places *NGL* and *SGL* represent signal indications. Right turn vehicles are allowed to enter an intersection which means a token is depositing in places *PR*1 and *PR*4. And then the additional places (*i.e.*, *PR*2, *PR*3, *PR*5, *PL*6) and transition (*i.e.*, *TR*1, *TR*2, *TR*3, *TR*4) are used to model a more complex traffic. Note that the number of s_i presents in this model where $s_i = \{s_5, s_6, s_7\}$.

4.3 The Operations of the Timed CP-nets Models

After the elements of the basic Timed CP-nets model are introduced, the operations of the Timed CP-nets model are described as follows. The system model is beginning at places *NR*, *SR*, *ER* and *WR*. And their tokens are with s_1, s_1, $1000\tau_{ew} + s_2$ and $1000\tau_{ew} + s_2$ time stamps, respectively. At this moment, called initial state, the red lights go on at the four traffic lights. Note that the time stamps of *NR* and *SR* are the same. And the time stamps of *WR* and *ER* are the same, too. This situation might cause an uncertain fire sequence (*i.e.*, both *T*1 and *T*2 are enabled concurrently) during the phase alternating. However, the result has no concern with the transition sequence. The reason states as follows.

Once the global time reaches s_1, both transitions *T*1 and *T*2 with the same time stamp (*i.e.*, @ + s_1) could be occurred. Then the token should be moved to places *EGR*, *EGL* and *WGR*, *WGL* from the initial marking. At this time, the time stamps are with $1000(\tau_{ew} - 3) + s_1$ time units. It hints that the tokens will stayed in both places *EGR*, *EGL* and *WGR*, *WGL* for $\tau_{ew} - 3$ seconds. And now the *GR* and *GS* signal is going on and the *R* signal is going off in the east and west traffic lights concurrently. In the meantime, the *phase*_1 is derived.

It is worthy to notice that there are two tokens (*i.e.*, (1, *ge*) and (1, *gw*)) in place *Vin* which are also removed from places *ER* and *WR*. Then the tokens are entering the intersection model. The two tokens with the same time stamp (*i.e.*, @ + s_1) represents that an eastbound vehicle (*i.e.*, (1, *ge*)) and a westbound vehicle (*i.e.*, (1, *gw*)) are entering the intersection when the green lights (*i.e.*, *GS* and *GR*) are turned on. The intersection model is to continue discussing. The enable binding elements (M_{out}, $\langle n = 1, l = ge\rangle$) and ($M_{out}$, $\langle n = 1, l = gw\rangle$) occur because the global time still remain s_1. As a result, the tokens are removed to places V_{out} and R_v. Notice that the average time of interval vehicles are given by place *Intval* in terms of incoming car. The token in place R_v means that the next vehicle is ready for entering the intersection. The tokens in place V_{out} represent vehicles are departure. Next, these tokens in places R_v and V_{out} are removed to place V_{in}. And the occurring sequence will repeat. The circulations are terminated until the green duration is over.

To go back to the traffic light model, the token should be moved to place *EY* (*WY*) from place *EGL* (*WGL*) *and EGS* (*WGS*) when the global time reaches $1000(\tau_{ew} - 3) + s_1$ (*i.e.*, *T*3 and *T*4 firing). It means the *Y* signal turns on for 3 seconds. The token should be moved to place *ER* (*WR*) from place *EY* (*WY*) when the global time reaches $1000\tau_{ew} + s_1$ time units. Likewise, once the global time reaches $1000\tau_{ew} + s_2$ time units, the *phases*_2 is obtained. It implies the marking is coming back to the initial marking. It is note that the signal variation and phase duration could be directly observed from the Timed CP-nets model. These are also the features of the model.

5. ANALYSIS OF THE URBAN TRAFFIC CONTROL SYSTEM

A traffic light control system model must have certain features for proper and save operation. For example, the controller should not lock up (deadlock) due to some unexpected combination of actions, should not allow conflicting movements to have right of way simultaneously, should be able to serve all signal phases and return to some initial state. A major strength of Timed CP-nets is the availability of methods for analyzing the properties of the model. Those properties of Timed CP-nets model reveal whether the model is reliable or not.

There are three methods to analyze a Timed CP-nets model [14]: (1) occurrence graphs method; (2) simulation method and (3) invariant method. In this section the basic traffic light control system model is analyzed by the occurrence graphs method and invariant method. In addition, the extended models proposed in section 4 are validated by simulating a real urban control system.

5.1 Occurrence Graphs Method

The basic idea behind occurrence graphs (OG) is to construct a graph containing a node for each reachable marking and an arc for each occurring binding element. Also, it is intuitive in this approach to see that there is no possibility of deadlock in the network.

The occurrence graph is shown in the Fig. 8. Note that each node represents a marking, and the content of the marking is described by the text inscription of the node. And each arc represents the occurrence of a binding element, and the content of this binding element is described by the text attached to the arc. For example, the text *EGS*, *EGR*, *WR*, *SR* and *NR* are used to describe the node *N*1. This means that the signals *GS* and *GR* turn on in the east traffic light. And the *R*'s turn on in the west, south and north traffic lights. After a binding element *T*2 occurs, the marking will be changed to node *N*3. It is worthy to note that the initial marking (*i.e.*, node *N*0) shows all the *R*'s turn on in a set of traffic lights. Especially, the initial marking will change in a negligible time. Notice that the text inscriptions of the nodes in the occurrence graph describe only the places of the signal lights and vehicle in/out. And the other places (*i.e.*, *Intval* and R_v) are omitted.

From this analysis, it can be concluded that: (1) there are no dead-end node in the OG, so therefore the net is live and (2) the net is reversible since one can always find a occurring sequence that bring the system back to the initial marking.

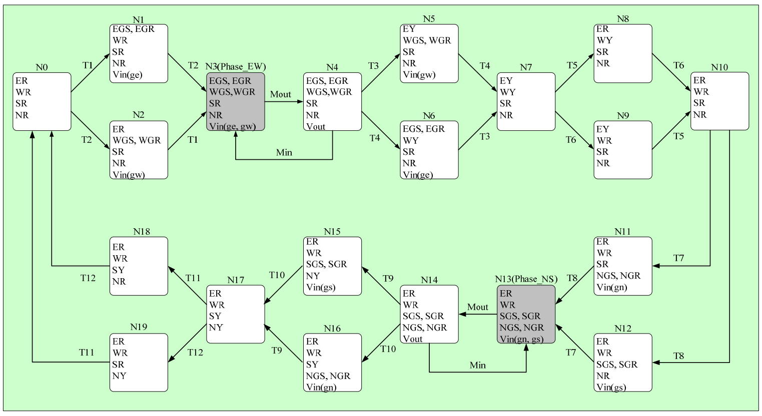


Fig. 8. The basic traffic lights control system model OG.

5.2 Place Invariants Method

The basic idea behind place invariants creates equations that are satisfied in all reachable marking. In Timed CP-nets model, the sets of removed tokens are not fully determined by the binding elements. Based on the rules of Timed CP-nets, a transition can be fired if the global time is great than or equal to the time stamp. It hints the system's actions are according to the time stamps of the binding elements. In fact, the system models require only the time stamps to be small enough, instead of requiring them to have some exact time values. This means that linearity of weight functions is insufficient to guarantee that each flow determines an invariant. However, our traffic light control system model is predetermining time. Therefore, it is certainly to use invariants in analysis of Timed CP-nets models.

There are two equations from Fig. 4 obtained. And the performance of the system is verified by the equations. The detail information is given as follows.

$$EGS/EGR + EY + ER = 1 \tag{1}$$

This invariant states that there is only one token in any one of the places involved in Eq. (1). And it indicates that the firing sequence of the binding elements should be in order at east traffic light. For example, the variations of traffic lights are red, green and yellow in turn. Similarly, the other invariant can be obtained, *i.e.*, $WGS/WGR + WY + ER = 1$, $NGS/NGR + NY + NR = 1$ and $SGS/EGR + SY + SR = 1$.

$$EGS/EGR + EY + ER = NR/SR \text{ and } WGS/WGR + WY + ER = SR/NR, \\ \text{where } NR = SR = 1 \tag{2}$$

This invariant depicts that if the places *NR* and *SR* deposit separately a token. Then there must be the tokens in the signal indication places of east and west traffic light. It means that once a red signal goes on in the *NS* traffic lights, then *G*, *Y* or *R* turns on in the *EW* traffic lights.

In the other way, the invariant $NGS/NGR + NY + NR = ER/WR$ and $SGS/SGR + SY +$

SR = *WR*/*ER* asserts that if there are two tokens deposited in separate place *ER* and *WR*, then there must be the tokens in the signal indication places of north and south traffic light. It shows that once *R* is going on in the *EW* traffic lights, then *G*, *Y* or *R* turns on in the *NS* traffic lights.

The two Eqs. (1) and (2) show the key invariants in the Timed CP-nets model. It proves that the control logic modeled by Timed CP-nets model is accurate and it does not allow any accident to happen.

5.3 Simulation Results

The Timed CP-nets model is implemented and simulated in the CPN tools [16]. In this section, the authors show how to calculate the number of vehicles that pass through intersections. This simulation work is done by three real intersections which are located at Taipei city in Taiwan (*i.e.*, shown in Fig. 9). In fact, the traffic is currently ruled by a fix time control strategy with a given signal timing plan. The fixed durations of the real traffic light control system are listed in Table 2. For this simulation work, the average time of vehicle intervals (*i.e.*, i_j) of incoming cars into the intersections are needed by the CPN tools. Therefore, the authors measure the data at the real intersections. The data is collected five cycles of the green duration in one day. Hence, the authors approximate the average time of vehicle interval for each cycle and the information is shown in Table 3.

For convenience, the authors construct Table 4 which is obtained from Tables 2 and 3. Table 4 is able to show the number of vehicles passing through the intersections. For example, there are four columns in cycle 1 of the phase X-1. The first/third column is the eastbound/westbound vehicle number (*i.e.*, 20/22) which is simulated by the case I model. And the model is with average time of vehicle interval (*i.e.*, 2.47/2.14 seconds). The second/forth column shows the real measurement data from the same intersection (*i.e.*, eastbound/westbound). It is important to point that the simulation results are almost consistent with the real traffic one.

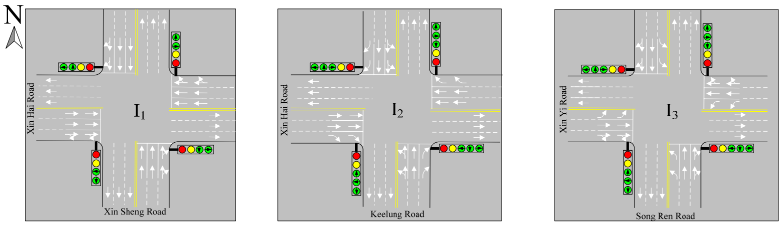


Fig. 9. The three real intersections located in Taipei, Taiwan.

Table 2. The phase duration of real traffic control system.

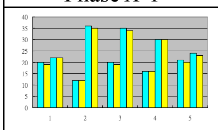
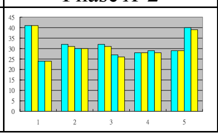
phase	*X*-1	*X*-12	*Y*-1	*Y*-12	*Y*-3	*Y*-4	*Z*-1	*Z*-12	*Z*-3
Phase duration	50	80	60	30	60	35	55	80	80

Table 3. The time of vehicle interval for the real intersections.

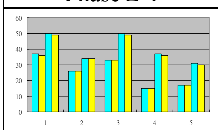
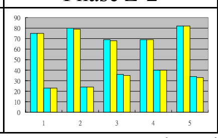
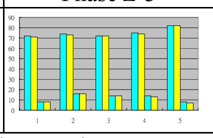
Two-phase intersection (Xin Hai Road-Xin Sheng Road)								
	Phase *X*-1		Phase *X*-2					
Cycle	East	West	North	South				
1	2.47	2.14	1.88	3.21				
2	3.92	1.34	2.48	2.57				
3	2.47	1.38	2.48	2.96				
4	2.94	1.57	2.75	2.75				
5	2.35	2.04	2.66	1.97				
Three-phase intersection (Xin Hai Road-Keelung Road Road)								
	Phase *Z*-1		Phase *Z*-2		Phase *Z*-3			
Cycle	East	West	North	Right turn	South	Right turn		
1	1.44	1.06	1.03	3.35	1.08	9.63		
2	2.00	1.53	0.97	3.21	1.05	4.81		
3	1.58	1.06	1.13	2.20	1.07	5.50		
4	3.47	1.44	1.12	1.93	1.04	5.92		
5	3.06	1.73	0.94	2.33	0.94	11.00		
Four-phase intersection (Xin Yi Road-Song Ren Road)								
	Phase *Y*-1		Phase *Y*-2		Phase *Y*-3		Phase *Y*-4	
Cycle	West	East	Left turn	Left turn	North	South	Left turn	Left turn
1	1.84	0.97	1.58	2.50	1.58	1.46	2.33	11.67
2	1.46	1.16	1.25	2.50	1.30	1.90	1.35	17.50
3	1.68	1.39	0.94	1.88	1.39	1.78	1.75	8.75
4	1.24	1.58	1.43	1.76	1.54	1.58	1.67	7.00
5	1.21	1.43	1.43	2.00	1.54	1.33	2.50	17.50

Table 4. Comparison of simulated and real traffic.

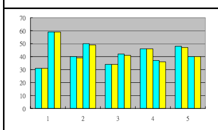
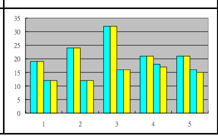
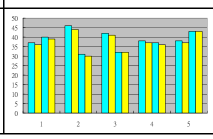
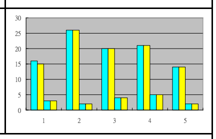
Two-phase intersection

Phase *X*-1	Phase *X*-2
	

Three-phase intersection

Phase *Z*-1	Phase *Z*-2	Phase *Z*-3
		

Four-phase intersection

Phase *Y*-1	Phase *Y*-2	Phase *Y*-3	Phase *Y*-4
			

:real data :simulation results

6. CONCLUSION

This article presents the modeling, analysis and implementation of an urban traffic lights system using Timed CP-nets models. Especially, this paper also proposed the module of basic traffic light system model which can assist in designing the extended models. The advantage of the proposed approach is the clear presentation of the system behavior and readiness for implementation. To summarize, this paper has the following contributions.

(1) This paper has demonstrated how to use Timed CP-nets to model the traffic lights of the urban network. And then the applications of Timed CP-nets to urban traffic lights have been realized.
(2) Structural analysis of Timed CP-nets models was performed.
(3) The complex phase transition is successful to convert Timed CP-nets models. It is helpful to obtain a Timed CP-nets model for a sophisticated urban traffic lights system.

To our knowledge, this is the first work that employs Timed CP-nets to construct the models of the urban traffic light systems.

REFERENCES

1. D. I. Robertson and R. D. Bretherton, "Optimizing networks of traffic signals in real time-the SCOOT method," *IEEE Transactions on Vehicular Technology*, Vol. 40, 1991, pp. 11-15.
2. J. H. Lee and H. Lee-Kwang, "Distributed and cooperative fuzzy controllers for traffic intersection group," *IEEE Transactions on Systems*, *Man and Cybernetics*, Vol. 29, 1999, pp. 263-271.
3. L. Figureiredo, I. Jesus, J. A. T. Machado, J. R. Ferreira, and J. L. M. de Carvalho, "Towards the development of intelligent transportation systems," in *Proceedings of IEEE Intelligent Transportation Systems*, 2001, pp. 1206-1211.
4. M. Papageorgiou, C. Diakaki, V. Dinopoulou, A. Kotsialos, and Y. Wang, "Review of road traffic control strategies," in *Proceedings of IEEE*, 2003, pp. 2043-2067.
5. F. D. Cesare, P. Kulp, K. Gile, and G. List, "The application of Petri nets to the modeling analysis and control of intelligent urban traffic networks," in *Proceedings of the 15th International Conference on Application and Theory of Petri Nets*, LNCS 815, 1994, pp. 2-15.
6. T. Murata, "Petri nets: properties, analysis and applications," in *Proceedings of IEEE*, 1989, pp. 541-580.
7. J. L. Peterson, *Petri Net Theory and the Modeling of Systems*, Prentice Hall, Taipei, 1981.
8. H. Wang, G. F. List, and F. DiCesare, "Modeling and evaluation of traffic signal control using timed Petri nets," in *Proceedings of IEEE Systems*, *Man and Cybernetics*, 1993, pp. 180-185.
9. C. Tolba, P. Thomas, A. ElMoudni, and D. Lefebvre, "Performances evaluation of

the traffic control in a single crossroad by Petri nets," in *Proceedings of IEEE Emerging Technologies and Factory Automation*, 2003, pp. 157-160.
10. J. Wang, C. Jin, and Y. Deng, "Performance analysis of traffic networks based on stochastic timed Petri net models," in *Proceedings of IEEE Engineering of Complex Computer Systems*, 1999, pp. 77-85.
11. L. Lin, T. Nan, M. Xiangyang, and S. Fubing, "Implementation of traffic lights control based on Petri nets," in *Proceedings of IEEE Intelligent Transportation Systems*, 2003, pp. 1087-1090.
12. F. Basile, C. Carbone, P. Chiacchio, R. K. Boel, and C. C. Avram, "A hybrid model for urban traffic control," *IEEE International Conference on Systems*, *Man and Cybernetics*, 2004, pp. 1795-1800.
13. M. Dotoli, M. P. Fanti, and G. Lacobellis, "Validation of an urban traffic network model using colored timed Petri nets," *IEEE International Conference on Systems*, *Man and Cybernetics*, 2005, pp. 1347-1352.
14. Y. S. Huang, "Design of traffic light control systems using statecharts," *The Computer Journal*, Vol. 49, 2007, pp. 634-649.
15. K. Jensen, *Coloured Petri Nets*, Vol. 1, Springer-Verlag, New York, 1997.
16. K. Jensen, *Coloured Petri Nets*, Vol. 2, Springer-Verlag, New York, 1995.
17. "Computer tool for coloured Petri," http://wiki.daimi.au.dk/cpntools/cpntools.wiki.

Yi-Sheng Huang (黃義盛) received the B.S. degree in Automatic Control Engineering from Feng Chia University, Taiwan, in 1989, the M.S. degree in Electronic Engineering from Chung Yuan Christian University, Taiwan, in 1991, and the Ph.D. degree in Electrical Engineering from National Taiwan University of Science and Technology (NTUST), Taiwan, in 2001. He was an Associate Professor in the Department of Aeronautical Engineering at Chung Cheng Institute of Technology (CCIT), National Defense University in Taiwan. He is presently a full professor in the Department of Electrical and Electronic Engineering at CCIT, National Defense University in Taiwan. His research interests include discrete event systems, Petri nets, computer integrated manufacturing, automation, reactive systems, air traffic control, and intelligent transport systems.

Ta-Hsiang Chung (鍾達翔) received the B.E. degree from Military Academy, Taiwan, in 1988, and the M.E. degree from Chung Cheng Institute of Technology, Taiwan, in 1994, both in Civil Engineering. He is currently a Ph.D. degree student in the School of Defense Science, National Defense University in Taiwan. His research interests include Petri nets, intelligent transportation system, and air traffic control.