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Short Paper

Hooke-Jeeves' Method Applied to a New Economic Dispatch Problem Formulation

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The Economic Dispatch Problem (EDP) is one of the important optimization problem in a power system. Traditionally, in EDP, the cost function for each generator has been approximately represented by a single quadratic function. The main aim of EDP is to minimize the total cost of generating real power while satisfying the equality constraints of power balance and the inequality generator capacity constraints. In this paper a New Economic Dispatch Problem Formulation (NEDPF) has been proposed to solve EDP. This new formulation is based on the reduction of the number of variables (number of generators) and elimination of the equality and inequality constraints, thus the transformation of the constrained non linear programming problem to an unconstrained one. The new unconstrained objective function, is minimized by Hooke-Jeeves' method. The NEDPF was tested for different cases (2, 3 and 6 generator units) and the results are judged satisfactory.

Keywords: economic dispatch, fuel cost, trigonometric variable, Hooke-Jeeves'method, new formulation

1. INTRODUCTION

In an electrical power system, a continuous balance must be maintained between electrical generation and varying load demand, while system frequency, voltage levels and security also have to be maintained. Further more, it is desirable that the cost of such generation must be minimal [1].

So the basic objective of economic dispatch (ED) of electric power generation is to schedule the committed generating unit outputs so as to meet the load demand at minimum operating cost while satisfying all units and system equality and inequality constraints. This makes the EDP a large-scale highly non-linear constrained optimisation problem. Many algorithms using linear and non-linear programming were proposed to solve the EDP [1, 2].

In this paper, we propose a New Economic Dispatch Problem Formulation (NEDPF) based on the transformation of the constrained non linear programming problem of (*ng*) real variables to an unconstrained problem with (*ng* – 1) trigonometric variables [3]. The

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new trigonometric function (fuel cost) is minimized by Hooke-Jeeves' method [4, 5]. This latter is a technique used to minimize an unconstrained objective function and does not need calculation of the gradient and the Hessian as other optimisation methods.

Several simulations with NEDPF are carried out on the standard 14-bus, 9-bus and 30-bus test systems.

The comparison of the classical economic dispatch problem formulation with NEDPF proves that the new one gives results which are better.

2. CLASSICAL EDP FORMULATION

In power stations, every generator has its input/output curve. It has the fuel input as a function of the power output. But if the ordinates are multiplied by the cost of $/Btu, the result gives the fuel cost per hour as a function of power output [6].

In the practical cases, the fuel cost of generator i may be represented as a quadratic function of real power generation P_{Gi}.

$$F_i(P_{Gi}) = a_i P_{Gi}^2 + b_i P_{Gi} + c_i \tag{1}$$

where a_i, b_i and c_i are the cost coefficients of ith generator.

The total cost of active power generation may be expressed by:

$$G(P_{Gi}) = \sum_{i=1}^{ng} F_i(P_{Gi}) \tag{2}$$

where ng is the number of generation including the slack bus.

The standard economic dispatch problem can be mathematically represented as:

$$\text{Minimize } G(P_{Gi}). \tag{3}$$

Under the following constraints

$$P_{Gi\min} \le P_{Gi} \le P_{Gi\max} \tag{4}$$

$$\sum_{i=1}^{ng} P_{Gi} = P_{ch} + P_L \tag{5}$$

where $P_{Gi\min}$ and $P_{Gi\max}$ are the upper and the lower bounds of the active power generation at all generation buses, P_{ch} is the total load demand and P_L is the real active loss in the network. The equality constraints consist of the power-flow equations, whereas the inequality constraints reflect the limits of real generation.

3. NEW EDP FORMULATION

In order to achieve the NEDPF, we apply two eliminations separately: Firstly, to eliminate the linear inequality constraints, new variable θ has to be introduced. The inequality constraints given by Eq. (4) can be formulated as

$$0 \le \frac{P_{Gi} - P_{Gi\min}}{P_{Gi\max} - P_{Gi\min}} \le 1. \tag{6}$$

The function limited between 0 and 1 is the function $\sin^2\theta$:

$$0 \le \sin^2\theta \le 1. \tag{7}$$

Comparing Eqs. (6) and (7)

$$P_{Gi} = P_{Gi\min} + D_i \sin^2\theta_i \tag{8}$$

where $D_i = P_{Gi\max} - P_{Gi\min}$ and θ is an unconstrained variable (angle).

Secondly, to eliminate the linear equality constraints, we express P_{Gng} as a function of P_{Gj} (j =1, 2, …, $ng-1$)

$$P_{Gng} = P_{ch} + P_L - \sum_{j=1}^{ng-1} (P_{Gj\min} + D_j \sin^2\theta_j) \tag{9}$$

$$P_{Gng} = E - \sum_{j=1}^{ng-1} D_j \sin^2\theta_j \tag{10}$$

where $E = P_{ch} + P_L - \sum_{j=1}^{ng-1} P_{Gj\min}$.

Substitution of the expressions of Eqs. (8) and (10) in Eq. (2) gives the real power generation expressed as:

$$\begin{aligned} G(\theta_j) = {} & \sum_{j=1}^{ng-1} [a_j (P_{Gj\min} + D_j \sin^2\theta_j)^2 + b_j (P_{Gj\min} + D_j \sin^2\theta_j) + c_j] \\ & + a_{ng}(E - \sum_{j=1}^{ng-1} D_j \sin^2\theta_j)^2 + b_{ng}(E - \sum_{j=1}^{ng-1} D_j \sin^2\theta_j) + c_{ng}. \end{aligned} \tag{11}$$

After development, Eq. (11) can be represented in the following general form:

$$G(\theta_j) = (\sin^2\theta_j)^T M (\sin^2\theta_j) + (\sin^2\theta_j)^T V + K. \tag{12}$$

M and V are ($ng-1$)-by-($ng-1$) and ($ng-1$)-by-1 array of total cost coefficients and K is a constant total coefficient scalar.

The off-diagonal elements of matrix M are:

$$M_{ij} = D_i D_j a_{ng} \tag{13}$$

and the diagonal elements of matrix M are:

$$M_{jj} = D_j^2 (a_j + a_{ng}). \tag{14}$$

The elements of vector V are:

$$V_j = D_j(2a_j P_{Gj\min} + b_j - 2Ea_{ng} - b_{ng}). \tag{15}$$

The constant K is:

$$K = a_{ng}E^2 + b_{ng}E + c_{ng} + \sum_{j=1}^{ng-1} F_j(P_{Gj\min}). \tag{16}$$

The new economic dispatch problem is stated mathematically as follow:

$$\text{Minimize } G(\theta_i). \tag{17}$$

4. HOOKE-JEEVES' METHOD

Among the search methods in non-linear numerical optimization, Hooke and Jeeves pattern search method stands out to be simple yet very effective optimization technique. This technique consists of two major routines: the exploratory search routine and the pattern move routine. The exploratory routine search the local proximity in the directions parallel to the coordinate axes for an improved objective function value, and the pattern routine accelerate the search by moving to a new improved position in the direction of the previous optimal point obtained by the exploratory routine [4].

To find the minimum of the objective function $G(\theta)$, we use this method which relies only on evaluating $G(\theta)$ on a sequence θ_1, θ_2, … and comparing values in order to calculate a minimum of G.

The direct search algorithm operates in the following manner [4, 5]:

1. Define:
   - Starting base point θ^{k-1} ($k = 1$) and $\theta = \{\theta_1, \theta_2, \ldots, \theta_j\}^T$.
   - Incremental change $\Delta\theta$ for all variables.
   - Step reduction factor q.
   - Termination parameter ε.
2. To initiate an exploratory search, $G(\theta^{k-1})$ is evaluated.
3. Perform type I exploratory search:
   (a) θ_1^{k-1} is changed by an amount $+\Delta\theta_1^{k-1}$, so that $\theta_1^k = \theta_1^{k-1} + \Delta\theta_1^{k-1}$. If the objective function is reduced, $\theta_1^{k-1} + \Delta\theta_1^{k-1}$ is adopted as the new element in θ. If the increment fails to improve the objective function, θ_1^{k-1} is changed by $-\Delta\theta_1^{k-1}$, and the value of $G(\theta)$ again checked as before. If the value of $G(\theta)$ is not improved by either $\theta_1^{k-1} \pm \Delta\theta_1^{k-1}$, θ_1^{k-1} is left unchanged.
   (b) Repeat the procedure in step 3-(a) for second variable θ_2 (note that $G(\theta)$ has (ng – 1) variables) by considering variations $+(-)\Delta\theta$ from the base point which results from step 3-(a). Apply the procedure to each variable in turn, finally arriving at a new base point θ^k.
4. If exploratory move successful (New base point $\theta^k \neq$ old base point θ^{k-1}), go to 6. If not ($\theta^k = \theta^{k-1}$), continue.

5. Check for termination: Is $||\Delta\theta|| < \varepsilon$?
   – Yes: Stop. The minimum (optimum) is assumed to have been reached.
   – No: Incremental change $\Delta\theta$ will be reduced $\Delta\theta = \dfrac{\Delta\theta}{q}$, and the search is repeated (go to step 3).
6. Perform pattern move: Initial base point θ^{k-1} and the base point obtained using the exploratory move θ^{k} define the "pattern" of the search direction. Pattern move takes a single step from present base point in the direction specified by the pattern (move from θ^{k} to $\theta_p^{k+1} = 2\theta^{k} - \theta^{k-1}$). This become the new starting point for type II exploratory move.
7. New sequence of exploratory (type II) moves about θ_p^{k+1} as the base point. Let result be θ^{k+1}.
8. If the lowest function value obtained during the pattern and exploratory moves $G(\theta^{k+1})$ is less than $G(\theta^{k})$, then a new base point has been reached. Set $\theta^{k-1} = \theta^{k}$ and $\theta^{k} = \theta^{k+1}$ go to step 6.
   – If $G(\theta^{k+1})$ is not decreased after type II exploratory search, the pattern is said to fail, and a new type I exploratory search is made in order to define a new successful direction (such a move is called a type I exploratory search in contrast to the type II exploratory search that follows a pattern search. After a type II exploratory move, a decision is made as to whether the previous pattern moves were a success or failure).

5. CASE STUDIES

The tests were applied in three example systems having different numbers of generators. The performance of the new formulation was evaluated through its application on 14-bus, 9-bus and 30-bus test systems. The single-line diagrams of those systems and their detailed data are given in [6, 8-11].

The cost coefficients and rating of each generator are shown in Table 1.

The initial conditions of the optimization are the results of the load flow solution.

The computational results are obtained using a Pentium IV, 1.7 GHz TURBO PASCAL compiler.

14-bus test system

The transmission line losses are calculated by Gauss-Seidel method [12] and maintained constant: $P_L = 17.87$ *MW*. The cost coefficients of the modified optimal power flow are:

$K = 1020.69$, $V = [-91.96]$, $M = [54]$.

The results of the active optimal generated power, minimum fuel cost, optimal angles, stage number and computing time are grouped in the Table 2.

The convergences of active power generation and the fuel cost are illustrated in Figs. 1 and 2.

Table 1. Cost coefficients and rating of generators.

14-bus system					
bus	a	b	c	$P_{G\max}$ (MW)	$P_{G\min}$ (MW)
1	0.006	1.5	100	300	135
2	0.009	2.1	130	145	30
9-bus system					
bus	a	b	c	$P_{G\max}$ (MW)	$P_{G\min}$ (MW)
1	0.1100	5.0	150	250	10
2	0.0850	1.2	600	300	10
3	0.1225	1.0	335	270	10
30-bus system					
bus	a	b	c	$P_{G\max}$ (MW)	$P_{G\min}$ (MW)
1	0.00375	2.0	0	200	50
2	0.00175	1.5	0	80	20
5	0.06250	1.8	0	50	15
8	0.00834	2.0	0	40	10
11	0.02500	1.5	0	30	10
13	0.02500	1.8	0	40	12

Table 2. Summary of optimization process.

Bus	P_{G0} (MW)	θ_0 (rd)	θ_{opt} (rd)	P_{Gopt} (MW)
1	236.87	0.904	0.590	186.10
P_{G2opt} (MW)				90.77
Optimal cost $G(\theta)$ (\$/h)				981.72
Stage number				41
Computing time (s)				0.001

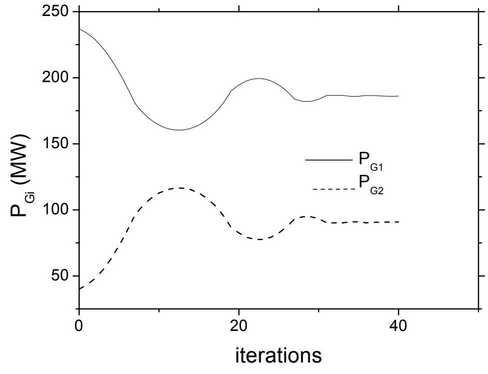

Fig. 1. Active power generation.

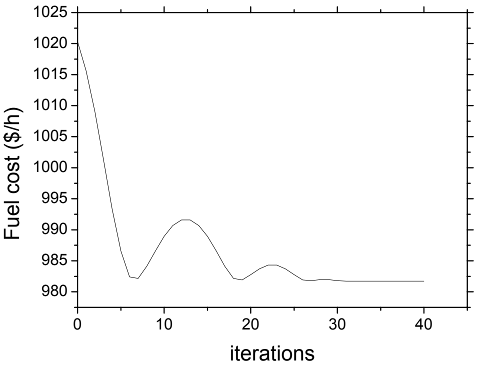

Fig. 2. Fuelcost.

9-bus test system

The transmission line losses are calculated by Gauss-Seidel method and maintained fixed: $P_L = 4.64$ *MW*. The cost coefficients of the modified optimal power flow are:

$$M = \begin{bmatrix} 13392 & 8526 \\ 8526 & 17450.75 \end{bmatrix}, \; V = \begin{bmatrix} -16149.35 \\ -20760.80 \end{bmatrix}, K = 12488.15.$$

The optimization process of this case is summarized in Table 3.

Figs. 3 and 4 plots respectively active power generation and fuel cost with respect of the number of the iterations.

Table 3. Summary of optimization process.

Bus	P_{G0} *(MW)*	θ_0 *(rd)*	θ_{opt} *(rd)*	P_{Gopt} *(MW)*
1	71.64	0.531	0.606	87.96
2	163.00	0.813	0.721	136.23
P_{G3opt} *(MW)*				95.44
Optimal cost $G(\theta)$ (\$/h)				5328.33
Stage number				52
Computing time (s)				0.001

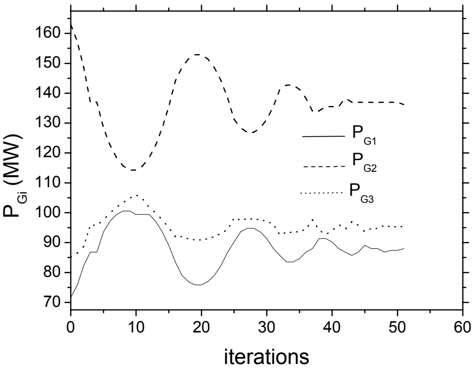

Fig. 3. Active power generation.

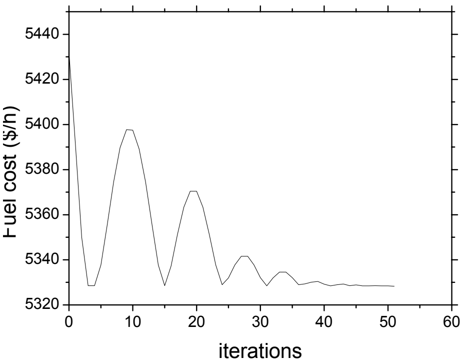

Fig. 4. Fuelcost.

30-bus test system

The transmission line losses are calculated by Gauss-Seidel method and maintained constant: $P_L = 9.04$ *MW*. The cost coefficients of the modified optimal power flow are:

$$M = \begin{bmatrix} 646.87 & 225 & 131.25 & 112.5 & 75 \\ 225 & 96.3 & 52.5 & 45 & 30 \\ 131.25 & 52.5 & 107.19 & 26.25 & 17.5 \\ 112.5 & 45 & 26.25 & 30 & 15 \\ 75 & 30 & 17.5 & 15 & 20 \end{bmatrix}, \; V = \begin{bmatrix} -1383.075 \\ -601.53 \\ -277.22 \\ -282.86 \\ -191.91 \end{bmatrix}, K = 1531.63.$$

The results of the active optimal generated power, minimum fuel cost, optimal angles, stage number and computing time are grouped in the Table 4.

Table 4. Summary of optimization process of 30-bus system.

Bus	P_{G0} (*MW*)	θ_0 (*rd*)	θ_{opt} (*rd*)	P_{Gopt} (*MW*)
1	151.48	0.966	0.687	110.34
2	60	0.955	1.570	80
5	30	0.714	0.000	15
8	20	0.615	1.570	40
11	15	0.524	1.999	26.54
P_{G13opt} (*MW*)				20.65
Optimal cost $G(\theta)$ (\$/h)				636.90
Stage number				151
Computing time (s)				0.30

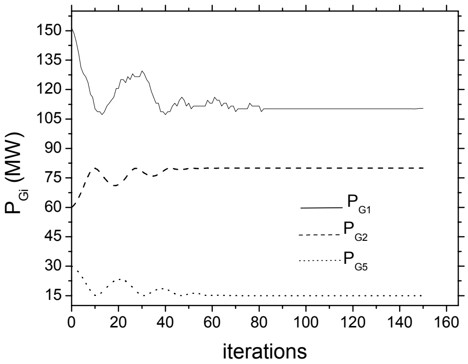

Fig. 5. Active power generation.

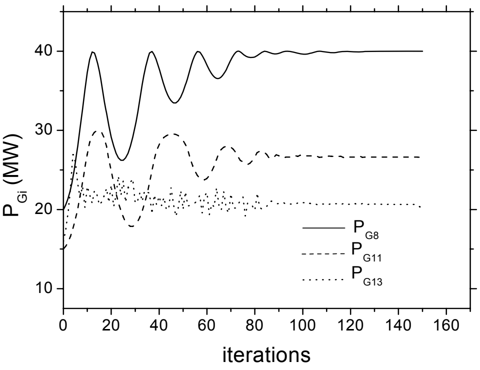

Fig. 6. Active power generation.

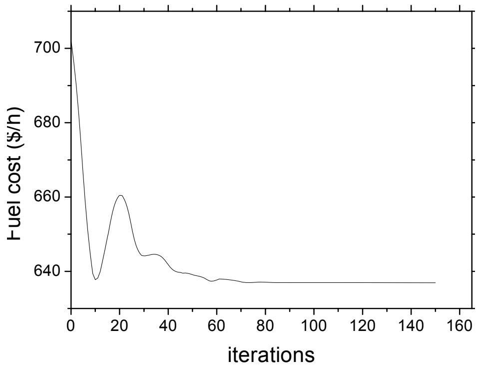

Fig. 7. Fuelcost.

The convergences of active power generation and the fuel cost are illustrated in Figs. 5, 6 and 7.

In order to demonstrate the performance of the proposed formulation, a comparison emerges between Hooke-Jeeves' method applied to the NEDPF and other optimization methods applied to the classical EDP formulation, as it is shown in Table 5.

Table 5. Comparison of NEDPF with other methods.

IEEE 14-bus system

Method	Fuel cost ($/h)	Stage number	Computing time (s)
Hooke-Jeeves (NEDPF)	981.72	41	0.001
SUMT method [13]	985.38	20	0.000
Genetic algorithm [14]	982.76	6	0.001

IEEE 9-bus system

Method	Fuel cost ($/h)	Stage number	Computing time (s)
Hooke-Jeeves (NEDPF)	5328.33	52	0.001
Matpower [8]	5296.69	43	5.220

IEEE 30-bus system

Method	Fuel cost ($/h)	Stage number	Computing time (s)
Hooke-Jeeves (NEDPF)	636.90	151	0.30
Matpower [8]	771.74	30	3.50
Greenstadt method [15]	825.00	12	0.05
Steepest Descent [15]	870.51	6	0.01

We can say that the proposed method is performing well in the solution of economic dispatch problem regarding the great difference between the results of the Hooke-Jeeves's method and the other methods specially in the 30 bus test system where we have 6 generators (21.17% between Hooke-Jeeves and Matpower; 29.53% between Hooke-Jeeves and Greenstadt; 36.68% between Hooke-Jeeves and Steepest descent). For the other test systems, the difference is only 1.4% for the 14 bus system and 0.6% for the 9 bus system (the computing time for the Matpower method is very important than the Hooke-Jeeves method).

6. CONCLUSION

The New Economic Dispatch Problem Formulation has been developed and its performances are examined on various test systems. This NEDPF is based on the transformation of the classical EDP which is a constrained non linear programming problem of ng variables to an unconstrained one of $ng - 1$ variables.

The advantage of using Hooke-Jeeves' method to minimize the new unconstrained objective function is its simplicity and no necessity of gradients.

The NEDPF results of three IEEE test systems were compared with those obtained from classical EDP formulation.

Satisfactory results are obtained by adapting the new formulation to those test systems and found that it is very similar and even better to that the optimum dispatch methods applied to standard EDP formulation.

REFERENCES

1. M. Rahli, L. Abdelmalek, and P. Pirotte, "Economic dispatch using Zoutendijk' method," in *Proceedings of the 35th Universities Power Engineering Conference*, Queen's University of Belfast, Ireland, 2000.
2. M. Rahli, "Applied linear and nonlinear programming to economic dispatching," Ph.D. Thesis, The University of Sciences and Technology of Oran, Algeria, 1996.
3. M. Minoux, *Mathematical Programming – Theory and Algorithms*, Translated by S. Vajda, John Wiley Sons, 1986.
4. C. G. Ong, "Shaking and balance of a convertible one and two-cylinder reciprocating compressor," Master Thesis, Faculty of the Virginia Polytechnic and State University, Blacksburg, Virginia, 2000.
5. D. M. Himmelblau, *Applied Non Linear Programming*, McGraw-Hill, New York, 1972.
6. Y. Wallach and R. Even, *Calculations and Programs for Power Systems Network*, Prentice-Hall, Englewood Cliffs, 1986.
7. http://www.srl.gatech.edu/education/ME6172/Patten-Search.ppt.
8. R. D. Zimmerman, C. E. Murillo-Sánchez, and D. Gan, "MATPOWER, Matlab power system simulation package," Version 2.0, http://www.pserc.cornell.edu/matpower/.
9. L. L. Freris and A. M. Sasson, "Investigation of load-flow problem," in *Proceedings of IEE*, Vol. 115, 1968, pp. 1459-1470.
10. http://www.ee.washington.edu/research/pstca.
11. P. M. Anderson and A. A. Fouad, *Power System Control and Stability*, IEEE Press, 1994.
12. G. W. Stagg A. K. Ray, A. H. El-Abiad, K. M. Bhurchandi, *Computer Methods in Power System Analysis*, McGraw Hill Book Company, New York, 1968.
13. M. Rahli and P. Pirotte, "Optimal load flow using sequential unconstrained minimization technique (SUMT) method under minimization power losses," *Electric Power System Research Journal*, Vol. 52, 1999, pp. 61-64.
14. M. Abdelmalek and M. Rahli, "Optimal load flow using genetic algorithm," *CIMNA*, 2003.
15. L. Benasla and M. Rahli, "Economic dispatch: comparison between greenstadt' method and steepest descent' method," in *Proceedings of the 4th Conference of Applied Mathematics and Engineering Sciences*, *CIMASI*, 2002.

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