Liveness Enforcing Supervision in Video Streaming Systems Using Siphons*

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The Internet motivated video streaming systems are largely complicated by issues such as a high degree of network resource sharing amongst many flows, which potentially leads to deadlocks. Using concepts of siphons along with their corresponding dangerous markings, we derive an algebraically necessary and sufficient characterization for such a much undesirable situation. The target system is assumed to be described in a Petri net formalism, whose markings provide the information on the current interactions among related network operations and resources. The theoretic materials allow us to introduce the control laws iteratively. At each iteration step, we produce a generalized mutual exclusion constraint which contains only markings for which liveness can be enforced. Since the explicit enumeration of all siphons is avoided, the proposed method can greatly reduce the complexity of off-line computation for the on-line restriction policy. Furthermore, a generalized elementary siphon control investigation is involved such that the final supervisor can be structurally simplified. Examples are demonstrated in this paper to validate the effectiveness and efficiency of the proposed approach.

Keywords: liveness enforcing supervision, Petri nets, deadlock prevention, video stream systems, siphon

1. INTRODUCTION

On the Internet, one of the most important media for communications and entertainment is video [1-7]. The digitization of video together with the advent of large scale digital integrated circuits and computers facilitates the compression of video and thus its transmission among various geographically distributed network nodes. Moreover, recent growth and popularity of wide bandwidth communication and high-speed network technologies make the real-time video streaming systems over Internet feasible [4, 5]. However, only best-effort services are available over the current Internet, which can be much complicated by additional issues such as reasonable sharing of network resources during data delivery in a streaming session and excellent performance of peer-to-peer communication for special content [4, 5].

Basically, a video stream is spitted into a set of frames, which are formally fixed-size packets that must be transmitted from the sender to the receiver [1, 2, 4, 5]. In a connection oriented service, a virtual circuit subnet is constructed before the delivery of

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the whole video. All packets of the same virtual circuit follow the same preconfigured path and arrive in the order by which they are sent. In a congested network, additional queuing delays can be experienced at busy intermediate network nodes. In addition, all involved nodes are dedicated to the circuit so that their capacities are somewhat wasted. In order to maximize the network resource utilization ration while reduce the transmission delay, the resource allocation policy must be improved. Consequently, the path of each packet is determined independently and dynamically. The number of intermediate nodes along the path of each packet may be totally different so that the packet can be forwarded along an optimal route with least congestion and delay. Moreover, once a packet is transmitted from one node to another, the required resource must be released and can be reacquired by the following packets. Obviously, the new transmission control protocol promises a high degree of resource sharing since each resource can accept and manage a higher amount of concurrent streams thus minimizing resource idling. However, when the resources are inappropriately allocated, the whole network can stumble at a deadlock condition, which is a most unfavorable situation since some of the packets will be idle due to their indefinite waiting for the release of other resources.

Informally speaking, a video streaming system falls into the same framework of a resource allocation system (RAS) since it allows the construction of system models bi-partitioned into processes and resources [8-34]. Abundant literatures show that Petri nets are a powerful mathematical tool for modelling, analysis, and synthesis of RAS, with an accent in automated production systems [17, 20, 22]. In particular, siphons are a special structure object tightly related to deadlock occurrence in a Petri net. Thus, the fundamental idea motivating the development of deadlock control is to prevent the reachability of a marking under which at least one siphon is insufficiently marked [12-14, 17, 19-22, 26-28, 30, 31]. Obviously, this method can be easily implemented via a set of linear inequalities whose basic elements represent markings associated with operation places in siphons in the Petri net model of a video streaming system. Solutions and transformations of these inequalities lead to a set of nearly optimal control places (monitors) for the plant Petri net. In this paper, an iterative approach to automatically generate all the control places without the complete enumeration of all siphons is proposed. The method introduced in this paper is based on the solution of a set of mixed integer programming (MIP) formulations, and implemented by a set of invariant-imposing control places imposed upon the original or plant Petri net so as to prevent the target system from entering into either the deadlock zone or the livelock zone of a reachability graph (RG). Specifically, an additional control place behaves as a "virtual" resource imposing some generalized mutual exclusion constraints defined on a set of critical operation places.

The remainder of this paper is organized as follows. Section 2 reviews the basic definitions and notations of Petri nets used throughout this paper. Also, a well-known class of Petri nets, namely S^4PR, on which the experiment of our method is conducted, is introduced. Section 3 is devoted to the description of a mathematical programming methodology for the efficient derivation of insufficiently marked siphons. Also, an algebraic method to control these siphons is discussed. Section 4 investigates a generalized technique of elementary siphons, through which the structure of the expected supervisor can be greatly reduced. An explanatory example is illustrated in section 5. Section 6 concludes the paper and suggests directions for future research.

2. PRELIMINARIES

2.1 Petri Nets

A Petri net is a four-tuple $N = (P, T, F, W)$ where P and T are finite, nonempty, and disjoint sets. Specifically, P is the set of places and T is the set of transitions. $F \subseteq (P \times T) \cup (T \times P)$ is called the flow relation or the set of directed arcs. W: $(P \times T) \cup (T \times P) \rightarrow \mathbb{N}$ is the weight function, where $\mathbb{N}$ is the set of non-negative integers. Ordinary nets are those where W: $(P \times T) \cup (T \times P) \rightarrow \{0, 1\}$ while general nets are those where W: $(P \times T) \cup (T \times P) \rightarrow \mathbb{N}$. If the restriction of W to $(P \times T)$ maps on $\{0, 1\}$, the Petri net is said to be PT-ordinary. Unless otherwise stated, when talking about a net, we refer to a general one. The preset of a node $x \in P \cup T$ is defined as ${}^{\bullet}x = \{y \in P \cup T \,|\, (y, x) \in F\}$. The postset of a node $x \in P \cup T$ is defined as $x^{\bullet} = \{y \in P \cup T \,|\, (x, y) \in F\}$. The preset (postset) of a set is defined as the union of the presets (postsets) of their elements. An ordinary net $N = (P, T, F, W)$ is a state machine iff $|{}^{\bullet}t| = |t^{\bullet}| = 1$. The input and output incidence matrices $[N]^{-} = \{a_{ij}^{-}\}$ and $[N]^{+} = \{a_{ij}^{+}\}$ of the Petri net N are defined as $a_{ij}^{-} = W(p_i, t_j)$ and $a_{ij}^{+} = W(t_j, p_i)$. The incidence matrix $[N] = \{a_{ij}\}$ of the Petri net N is defined as $[N] = [N^{+}] - [N^{-}]$. $[N_{p_j}]$ ($[N_{p_j}^{-}]$, $[N_{p_j}^{+}]$) is the jth row of $[N]$ ($[N^{-}]$, $[N^{+}]$). $[N_{t_j}\}]$ ($[N_{t_j}^{-}], [N_{t_j}^{+}]$) is the jth column of $[N]$ ($[N^{-}]$, $[N^{+}]$).

A marking of N is a mapping M: $P \rightarrow \mathbb{N}$. (N, M_0) is called a net system or a marked net, where M_0 means the initial marking or state. A transition t_j is fireable iff $\forall p \in {}^{\bullet}t_j$, $M(p) \geq W(p, t)$. This is denoted as $M[t_j\rangle$. A marking M is reachable from another one M' iff there exists a firing sequence $\sigma = <t_1t_2\ldots t_n>$ so that $M[t_1\rangle M_1[t_2\rangle M_2\ldots M_{n\text{-}1}[t_n\rangle M'$. This fact is also denoted by $M[\sigma\rangle M'$. The length of σ, which is denoted by $|\sigma|$, is defined by the number of transitions in it. The firing sequence vector of σ is a $|T|$-dimensional vector, namely y, whose each component, *i.e.*, $y(t)$, $t \in T$, states the number of appearance of transition t in σ. The set of all markings reachable from a marking M_0, in symbol $R(N, M_0)$, is the smallest set in which $M \in R(N, M_0)$ and $M' \in R(N, M_0)$ if both $M \in R(N, M_0)$ and $M[t\rangle M'$ hold.

Let (N, M_0) be a net system and $N = (P, T, F, W)$. A transition $t \in T$ is live under M_0 iff $\forall M \in R(N, M_0)$, $\exists M' \in R(N, M)$, $M'[t\rangle$ holds. (N, M_0) is deadlock-free iff $\forall M \in R(N, M_0)$, $\exists t \in T$, $M[t\rangle$. (N, M_0) is livelock iff it is deadlock-free and $\exists M \in R(N, M_0)$, $\exists t \in T$, t is dead at M. (N, M_0) is live iff $\forall t \in T$, t is live under M_0. A marking $M \in R(N, M_0)$ is deadlock or total deadlock iff $\forall t \in T$, t is dead. A marked net is quasilive iff $\forall t \in T$, $\exists M \in R(N, M_0)$, $M[t\rangle$. (N, M_0) is bounded iff $\exists k \in \mathbb{N}^{+}$, $\forall M \in R(N, M_0)$, $\forall p \in P$, $M(p) \leq k$, where $\mathbb{N}^{+} = \mathbb{N}\backslash\{0\}$. (N, M_0) is pure iff $\forall(x, y) \in (P \times T) \cup (T \times P)$, $W(x, y) > 0 \Rightarrow W(y, x) = 0$.

A P-vector (place vector) is a column vector I: $P \rightarrow \mathbb{Z}$ indexed by P while a T-vector (transition vector) is a column vector J: $T \rightarrow \mathbb{Z}$ indexed by T, where $\mathbb{Z}$ is the set of integers. We denote a column vector where every entry equals 0(1) by $\mathbf{0(1)}$. I^T and $[N]^T$ are the transposed versions of vector I and matrix $[N]$, respectively. I is a P-invariant (place invariant) iff $I \neq \mathbf{0}$ and $I^T \cdot [N] = \mathbf{0}^T$ hold. J is a T-invariant (transition invariant) iff $J \neq 0$ and $[N] \cdot J = \mathbf{0}$ hold. $||I|| = \{p \in P \,|\, I(p) \neq 0\}$($||J|| = \{t \in T \,|\, J(t) \neq 0\}$) is called the support of $I(J)$, where $||I||^{+} = \{p \in P \,|\, I(p) > 0\}$($||J||^{+} = \{t \in T \,|\, J(t) > 0\}$) is called the positive support while $||I||^{-} = \{p \in P \,|\, I(p) < 0\}$($||J||^{-} = \{t \in T \,|\, J(t) < 0\}$) is called the negative support. A P-invariant (T-invariant) is minimal iff it contain no P-invariant (T-invariant) as a proper subset. P-invariant (T-invariant) is said to be a P-semiflow (T-semiflow) iff no component of I is negative. For economy of space, $\sum_{p\in P}M(p) \cdot p$ is used to denote vector M without confusion. Similarly, $\sum_{p\in P}I(p) \cdot p$ ($\sum_{t\in T}J(t) \cdot t$) is used to denote vector I (J). In a structurally bounded net system, $SB(p)$ means the maximal number of tokens which is defined as $SB(p) = \max\{M(p) \,|\, M = M_0 + [N] \cdot y, M \geq \mathbf{0}, y \geq \mathbf{0}\}$, where y is the firing sequence vector.

A string $x_1, x_2, \ldots, x_n$ is called a path of N iff $\forall i \in \{1, 2, \ldots, n-1\}$, $x_{i+1} \in x_i^\bullet$. A simple path is a path whose nodes are all different (except for, perhaps, x_1 and x_n). A path $x_1, \ldots, x_n$ is called a circuit iff it is a simple path and $x_1 = x_n$ [17].

Definition 1 [17] Let $\overline{N_i}$ be a strongly connected state machine, c be a circuit of $\overline{N_i}$. The set of nodes in c is denoted by $\{c\}$ and the length of circuit c is defined as *length*(c) = $|\{c\}|$. Let x, y be two nodes in c, and $A \subseteq (P \times T)$ be a set of nodes in c. x is said to be previous to y in c, which is formally denoted by $x <_c y$, iff there exists a path in c from x to y whose length is greater than 1 and does not pass over p_{0_i}. x is said to be previous to y in $\overline{N_i}$, which is formally denoted by $x <_{\overline{N_i}} y$, iff there exists a circuit c in $\overline{N_i}$ so that $x <_c y$. $x <_{\overline{N_i}} A$ iff there exists a node $y \in A$ so that $x <_{\overline{N_i}} y$. $A <_{\overline{N_i}} x$ iff there exists a node node $y \in A$ so that $y <_{\overline{N_i}} x$.

A nonempty set $S \subseteq P$ ($Q \subseteq P$) is a siphon (trap) iff $^\bullet S \subseteq S^\bullet$ ($Q^\bullet \subseteq {}^\bullet Q$) holds. A siphon (trap) is minimal iff there is no siphon (trap) contained in S (Q) as a proper subset. A minimal siphon (trap) not containing the support of any P-invariant is called a strict minimal siphon (trap). $M(p)$ indicates the number of tokens in p under M. p is marked by M iff $M(p) > 0$. A subset $S \subseteq P$ ($Q \subseteq P$) is marked by M iff at least one place is marked by M. The sum of tokens in all places in S (Q) is denoted by $M(S)$ ($M(Q)$), where $M(S) = \sum_{p\in S}M(p)$ ($M(Q) = \sum_{p\in Q}M(p)$). A siphon (trap) S (Q) is said to be empty at marking M iff $M(S) = 0$ ($M(Q) = 0$). In the sequel, the set of all the strict minimal siphons (trap) in a net is denoted by Π (Ω). S or Q is said to be max-marked (min-marked) at a marking M iff $\exists p \in S$ or $\exists p \in Q$ such that $M(p) \geq \max_{p^\bullet}$ $M(p) \geq \min_{p^\bullet}$), where $\max_{p^\bullet} = \max_{t\in p^\bullet}(W(p, t))$ ($\min_{p^\bullet} = \min_{t\in p^\bullet}(W(p, t))$).

Let S be a subset of places of $N = (P, T, F, W)$. λ_S is called the characteristic P-vector of S iff $\forall p \in S$, $\lambda_S(p) = 1$; otherwise, $\lambda_S(p) = 0$. Furthermore, η_S is called the characteristic T-vector of S iff $\eta_S^T = \lambda_S^T \cdot [N]$.

2.2 S^4PR Nets

To design an RAS with shared resources, many methodologies are proposed. Among them, a special class of Petri nets, namely systems of simple sequential process with shared resources or S^4PR for simplicity, are the most general ones. They are proposed in [28] and cited in this section so as to make this paper self-contained. Its related properties are also illustrated such that a clear presentation can be provided. For more details on these topics, the reader is referred to [25]. Specifically, we remark that the systems of simple processes with general resource requirement (S^3PGR^2) proposed in [25] and the systems of sequential systems with shared resources (S^4R) proposed in [11] model the same class of resource acquisition behavior as the S^4PR net structure originally proposed in [28].

Definition 2 [27] An S^4PR is a connected generalized self-loop free Petri net $N = (P_0 \cup P_S \cup P_R, F, T, W)$ where: (1) $P = P_0 \cup P_S \cup P_R$ is a partition such that: (a) $P_S = \bigcup_{I \in \mathbb{N}^+} P_{S_i}$, where for each $i \in \mathbb{N}^+$, $P_{S_i} \neq \emptyset$, and for each $i, j \in \mathbb{N}^+$, $i \neq j$, $P_{S_i} \cap P_{S_j} = \emptyset$. (b) $P_0 = \bigcup_{i \in \mathbb{N}^+} \{p_{0_i}\}$. (c) $P_R = \{r_1, r_2, \ldots, r_n\}$, $n > 0$. (2) $T = \bigcup_{i \in \mathbb{N}^+} T_i$, where for each $i \in \mathbb{N}^+$, $T_i \neq \emptyset$, and for each $i, j \in \mathbb{N}^+$, $i \neq j$, $T_i \cap T_j \neq \emptyset$. (3) For each $i \in \mathbb{N}^+$, the subnet $\overline{N_i} = N|(\{p_{0_i}\} \cup P_{S_i}, T_i)$ is a strongly connected state machine such that every cycle contains p_{0_i}. (4) For each $r \in P_R$, there exists a unique minimal P-invariant $Y_r = \mathbb{N}^{+|P|}$ such that $\{r\} = ||Y_r|| \cap P_R$, $P_0 \cap |Y_r| = \emptyset$, $P_S \cap |Y_r| \neq \emptyset$, and $Y_r(r) = 1$. (5) $P_S = \bigcup_{r \in P_R}(||Y_r||\backslash\{r\})$.

In Definition 2, P_R is the set of resource places (in short, resources). The initial marking of a place in P_R denotes the capacity of a considered resource. P_S is the set of operation places. The marking of a place in P_S denotes the activity of an operation. P_0 is the set of idle places, whose initial marking indicates the maximum number of products that are allowed to be concurrently manufactured in the corresponding working process.

Definition 3 [27] Let (N, M_0), $N = (P_0 \cup P_S \cup P_R, T, F, W)$, be an initially marked S^4PR. Marking M_0 is said to be an acceptable initial marking for N iff (1) $M_0(p) \geq 1$, $\forall p \in P_0$; (2) $M_0(p) = 0$, $\forall p \in P_S$; and (3) $M_0(r) \geq \max\{Y_r(p)\}$, $\forall r \in P_R$, $\forall p \in P_S$.

Given an arbitrary marking $M \in R(N, M_0)$, a transition t is M-process-enabled iff $\exists p \in {}^{\bullet}t \cap P_S$ such that $M(p) > 0$. Correspondingly, a transition t is M-resource-enabled iff $\exists r \in {}^{\bullet}t \cap P_R$ such that $M(r) \geq W(r, t)$ [27].

Definition 4 [27] Let (N, M_0), $N = (P_0 \cup P_S \cup P_R, T, F, W)$, be an acceptably marked S^4PR. Let $r \in P_R$, the set of holders of r is the support of a minimal P-invariant Y_r without place r, *i.e.*, $H(r) = ||Y(r)||\backslash\{r\}$.

Definition 5 [27] Let (N, M_0), $N = (P_0 \cup P_S \cup P_R, T, F, W)$, be an acceptably marked S^4PR. Let S be an emptiable siphon, $Th(S) = H_{S_R}\backslash S$ is called the thieves of S, where $S_R = P_R \cap S$, and $H_{S_R} = \sum_{r \in S_R} H(r)$.

Proposition 1 [27] Let (N, M_0) be an acceptably marked S^4PR. The system is not live iff there exists a marking $M \in R(N, M_0)$ such that the set of M-process-enabled transitions is non-empty and each of them is M-resource-disabled.

Proposition 2 [27] Let (N, M_0) be an acceptably marked S^4PR. The system is not live iff there exists a marking $M \in R(N, M_0)$ and a siphon $S \in \Pi$ such that (1) $\forall r \in S_R$, $M(r) < W(r, t)$. (2) $\forall p \in Th(S)$, $M(p) > 0$. (3) $\forall p \in S_P$, $M(p) = 0$.

Propositions 1 and 2 attribute the nonliveness of an S^4PR net to the presence of an insufficiently marked siphon and the occurrence of a corresponding marking. This is specially useful since we can detect and forbid these siphons instead of enumerating and controlling all the strict minimal siphons, whose number is, in theory, proved to be exponential with the size of a Petri net model.

The net system depicted in Fig. 2 is an example of S^4PR. It models a video streaming system in Fig. 1 consisting of three networks resource types $\Re_1$, $\Re_2$, and $\Re_3$ with

Fig. 1. A small video streaming system.

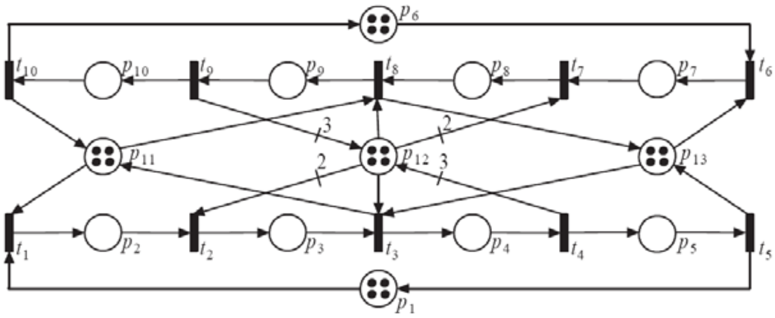


Fig. 2. An example S^4PR net.

capacities $C(\Re_1) = C(\Re_2) = C(\Re_3) = 4$. The resources support two job types, *i.e.*, Job 1 and Job 2, and are denoted by p_{11}, p_{12}, and p_{13}, respectively. The flow of process type for Job 1 is defined by a set of places $\{p_1, p_2, p_3, p_4, p_5\}$ which are strictly ordered through the flow relations. Specifically, a process of Job 1 can take the route defined by the sequences of places $<p_1, p_2, p_3, p_4, p_5>$. The resource requirements for the operation places are defined as follows: $a_{p_2} = [1, 0, 0]^T$, $a_{p_3} = [1, 2, 0]^T$, $a_{p_4} = [0, 3, 1]^T$, and $a_{p_5} = [0, 0, 1]^T$, where $a_{p_i}(j)$ indicates the required number of resource $\Re_j$ to support the execution of operation p_i. The flow of process type for Job 2 is defined by a set of places $\{p_6, p_7, p_8, p_9, p_{10}\}$ which are strictly ordered through the flow relations. Specifically, a process of Job 2 can take the route defined by the sequences of places $<p_6, p_7, p_8, p_9, p_{10}>$. The resource requirements for the operation places are defined as follows: $a_{p_7} = [0, 0, 1]^T$, $a_{p_8} = [0, 2, 1]^T$, $a_{p_9} = [1, 3, 0]^T$, $a_{p_{10}} = [1, 0, 0]^T$. It should be noticed that, throughout this paper, the bandwidth between any two nodes are assumed unlimited, which is practically reasonable due to the widely used optical fiber transmission system. Finally, assuming that places are ordered in the incidence matrix according to the sequence $<p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}, p_{11}, p_{12}, p_{13}>$, we have $M_0 = 4p_1 + 4p_6 + 4p_{11} + 4p_{12} + 4p_{13}$. In addition, $M_{D_1} = 2p_2 + 2p_3 + 4p_7$, $M_{D_2} = 3p_2 + p_3 + 3p_7 + p_8$, and $M_{D_3} = 4p_2 + 2p_7 + 2p_8$ are reachable states under which no transitions are enabled. Therefore, the net is not live.

3. SIPHONS AND THEIR RESPECTIVE BAD MARKINGS

The RG of the Petri net model of an RAS can be distinguished by the live zone, livelock zone, and deadlock zone [29]. The deadlock zone contains states that are deadlock or that inevitably lead to deadlock states, which constitute the most undesirable behavior in a system. The livelock zone contains states that are livelock or that inevitably lead to livelock states, which constitute a set of partially deadlock behavior in a system.

At a livelock or deadlock state, the initial state is unreachable. The live zone contains states that are live, which constitute the optimal behavior of a system. Normally, the live states are also called good ones, while the all the other states are called bad ones, among which the states that inevitably lead to livelock or deadlock states are called the dangerous ones.

Siphons and their respective bad markings are widely exploited in various deadlock prevention methods. In this section, we investigate them in an integrity fashion, which can facilitate the derivation of a potentially insufficiently marked siphon along with its corresponding bad marking using a set of mathematical programming formulations.

3.1 Computation of Siphons and their Respective Bad Markings

As mentioned above, a bad marking, *i.e.*, $M \in R(N, M_0)$, in S^4PR implies that there exists at least one transition, *i.e.*, $t \in T$, which is either M-resource-disabled or M-process-disabled [27]. Therefore, in [27], the algorithm to identify an insufficiently marked siphon together with its respective bad markings can be realized using an MIP approach with four binary variables introduced, which are v_p, e_{pt}, e_{rt}, and e_t. Several attributes should be properly assigned to them. $v_p = 0$ iff $p \in S$, $e_{pt} = 1$ iff t is M-process-enabled while $e_{rt} = 1$ iff t is M-resource-enabled, $e_t = 1$ iff t is both M-process-enabled and M-resource-enabled [27]. The properties of these four variables together with the fact that S is an insufficiently marked siphon lead to

$$v_p \geq \sum_{q \in {}^\bullet t} v_q - |{}^\bullet t| + 1, \ \forall p \in P, \ \forall t \in {}^\bullet p, \tag{1}$$

$$e_{pt} \geq \frac{M(p)}{B(p)}, \ \forall t \in T, \forall p \in {}^\bullet t \cap P_S, \tag{2}$$

$$e_{pt} \leq M(p), \ \forall t \in T, \ \forall p \in {}^\bullet t \cap P_S, \tag{3}$$

$$e_{pt} \geq v_p, \ \forall W(p, t) > 0, \tag{4}$$

$$e_{rt} \geq \frac{M(r) - W(r, t) + 1}{M_0(r) - W(r, t) + 1}, \ \forall r \in P_R, \forall t \in r^\bullet, \tag{5}$$

$$e_{rt} \geq \frac{M(r)}{W(r, t)} + v_r, \ \forall r \in P_R, \forall t \in r^\bullet, \tag{6}$$

$$e_t \geq \sum_{p \in {}^\bullet t \cap P_S} e_{pt} + \sum_{r \in {}^\bullet t \cap P_R} e_{rt} - |{}^\bullet t| + 1, \ \forall t \in T, \tag{7}$$

$$v_p \geq e_t, \ \forall W(t, p) > 0, \tag{8}$$

$$e_{pt} \in \{0, 1\}, \ \forall t \in T, \tag{9}$$

$$e_{rt} \in \{0, 1\}, \ \forall r \in P_R, \forall t \in r^\bullet. \tag{10}$$

For clarification, we re-address some comments in [27] on the variables involved in the above IP formulations since they are helpful for understanding of the whole formalism and this paper.

Eq. (1) indicates that a siphon is a set of places the values of whose respective variables v_p's in presented inequalities are 0's. The formulation of these inequalities is straightforward from the well established algebraic trap characterization in [31].

The value of e_{pt} is determined by Eqs. (2)-(4), which indicates whether or not a transition t is M-process-enabled. In case that t is M-process-enabled, $M(p) \geq 1$ and $0 < M(p)/SB(p) \leq 1$. Since $M(p)/SB(p) \leq e_{pt} \leq M(p)$, e_{pt} must be 1 due to its binary property. In case that t is M-process-disabled, $M(p) = 0$ and $M(p)/SB(p) = 0$. Since $M(p)/SB(p) \leq e_{pt} \leq M(p)$, e_{pt} must be 0. Furthermore, in case $\forall W(p, t) > 0$, $v_p = 1$, e_{pt} must be 1 due to the fact that $\nexists S \subseteq \Pi$ such that $p \in S$.

The value of e_{rt} is determined by Eqs. (5) and (6), which indicates whether or not a transition t is M-resource-enabled by a resource r. In case that t is M-resource-enabled, $M(r)/W(r, t) \geq 1$, $(M(r) - W(r, t) + 1)/(M_0(r) - W(r, t) + 1) \geq 0$. Since $(M(r) - W(r, t) + 1)/(M_0(r) - W(r, t) + 1) \leq e_{rt} \leq M(r)/W(r, t)$, e_{rt} must be 1 due to its binary property. In case that t is M-resource-disabled by a resource r, $0 \leq M(r)/W(r, t) \leq 1$, $(M(r) - W(r, t) + 1)/(M_0(r) - W(r, t) + 1) \leq 0$. e_{rt} must be 0.

The value of e_t is determined by Eq. (7), which indicates whether or not a transition t is firable. In case that $\forall t \in {}^\bullet t \cap P_S$, $e_{pt} = 1$, and $\forall t \in {}^\bullet t \cap P_R$, $e_{rt} = 1$, e_t must be 1, which means that t is potentially firable. In case that $\exists t \in {}^\bullet t \cap P_S$, $e_{pt} = 0$, or $\exists t \in {}^\bullet t \cap P_R$, $e_{rt} = 0$, e_t must be 0, which means that t is dead at marking $M \in R(N, M_0)$.

Theorem 1 [27] Let (N, M_0) be an acceptably marked S^4PR, $N = (P_0 \cup P_S \cup P_R, T, F, W)$, $P = P_0 \cup P_S \cup P_R$. Then, given a marking $M \in R(N, M_0)$, an insufficiently marked siphon (not necessarily minimal) is determined by $S = \{p \in P \mid v_p = 0\}$, where v_p, $p \in P$, is determined by the following MIP formulations:

$$G(M) = \max \sum v_p$$

subject to (1)-(10).

Definition 6 [27] Let (N, M_0) be an acceptably marked S^4PR, $N = (P_0 \cup P_S \cup P_R, T, F, W)$, $P = P_0 \cup P_S \cup P_R$, S be an insufficiently marked siphon at certain marking $M \in R(N, M_0)$. $M_S^{\max} \in R(N, M_0)$ is called a critical marking with respect to S such that

$$M_S^{\max}(S_R) = \max \sum_{r \in S_R} M(r)$$

subject to constraints (1)-(10).

In practice, an insufficiently marked siphon S may correspond to many bad markings, among which the one, namely $M_S^{\max}$, with the maximum quantity of $M_S^{\max}(S_R)$ is not only of theoretical interest but also very expensive in practical application, since such a marking is evidently the first met marking at which S is insufficiently marked. Mathematically, an intuitive method to obtain $M_S^{\max}$ is to introduce another objective function that is $Q(M) = \max\sum(1 - v_p) \cdot M(p)$ according to optimization theory. Many experimental results show that S along with $M_S^{\max}$ can be derived from such a multiobjective programming model efficiently. If there are no multiobjective software package available, such a mathematical programming model can be converted to a singleobjective MIP model by introducing a Lagrangian coefficient α ($0 < \alpha < 1$). Consequently, the objective function for such a single objective programming model can be constituted by $\alpha \cdot G(M) - (1 - \alpha) \cdot Q(M)$. Specifically, α is much greater than $1 - \alpha$, which means the fitness function $G(M)$ is of a higher priority. Thereby, an immediate implication is that the bad marking M with maximum quantity of $M_S^{\max}(S_R)$ such that S is insufficiently marked can be determined by the following integer programming problem iff there exists a solution

$$S(M) = \max\{\alpha \cdot G(M) - (1 - \alpha) \cdot Q(M)\}$$

subject to (1)-(10).

For simplicity and without confusion, we also denote the $M_S^{\max}(S_R)$ by $M_S^{\max}$ in the sequel.

Corollary 1 Let (N, M_0) be an acceptably marked S^4PR, $N = (P_0 \cup P_S \cup P_R, T, F, W)$, $P = P_0 \cup P_S \cup P_R$. (N, M_0) is live if the following integer programming (IP) formulations have no solutions.

$$S(M) = \max\{\alpha \cdot G(M) - (1 - \alpha) \cdot Q(M)\}$$

subject to constraints (1)-(10), and

$$M = M_0 + [N] \cdot y,\ M \geq \mathbf{0},\ y \geq \mathbf{0}. \tag{11}$$

Proof: By contradiction, suppose that (N, M_0) is live whereas an integer solution is found such that $\exists p \in P$, $v_p = 0$. As indicated by Eq. (8), $\exists t \in {}^{\bullet}p$, $e_t = 0$, which means that t is dead at a marking $M \in R(N, M_0)$. That contradicts the liveness of (N, M_0). ❑

Since siphons obtained from an MIP algorithm may be neither strict nor minimal, an algorithm to generate their corresponding strict minimal ones is well investigated and can be accomplished in polynomial time [20]. According to this research, an iterative method is exploited to remove places, whose absence has no effect on the other places, so that the remaining places form a strict minimal siphon.

Different from many traditional deadlock prevention or avoidance policies, the proposed MIP-based algorithm evolves iteratively, which means that an insufficiently marked siphon is controlled immediately once it is detected. Furthermore, the process to derive the deadlock marking is accompanied by an algorithm to detect the liveness of the Petri net model, which means the absence of an integer solution can be used as the termination condition for such an iterative process. This phenomenon claims that the whole policy can be terminated without necessarily deriving all the siphons. Since the number of deadlock markings is bounded by the size of RG of a Petri net, the iteration counts in theory do not exceed the number of reachable nodes due to the boundedness of S^4PR. Obviously, such a conclusion ensures the convergence of the iterative procedure suggested in our approach.

Another important question is the way in which the detected insufficiently marked siphons and their respective bad markings are controlled. In this work, we conduct such an approach from an algebraic perspective, which tries to obtain a maximally permissive supervisor.

3.2 Control of Siphons Using Algebraic Method

This section is devoted to the synthesis of a liveness enforcing supervisor. A set of new places, namely control places (monitors), are iteratively introduced such that the behavior of the system are restricted within the limits of the legal behavior. In fact, any liveness enforcing policy can be represented by a supervisor implemented as control places connected to the transitions of a plant Petri net. A supervisor synthesis approach aims to control all the minimal siphons which are potentially insufficiently marked. An insufficiently marked siphon is controlled if an additional specification in the paradigm of a set of inequalities initially proposed in [27] is properly associated to the plant Petri net. Previous literatures [11, 27] show that such a set of specifications can be easily translated to the constraints in terms of the Petri net, which defines the supervisor for liveness enforcement. In the sequel, Definitions 7-9 are somewhat a summation from [11, 27] which facilitates the establish of our innovative technique in the section 4.

Definition 7 Let (N, M_0), $N = (P_0 \cup P_S \cup P_R, T, F, W)$, be an acceptably marked S^4PR, S be an insufficiently marked siphon at a marking $M \in R(N, M_0)$, (N^*, M_0^*), $N^* = (P_0 \cup P_S \cup P_R \cup \{p_c\}, T, F^*, W^*)$ is called the augmented net with respect to (N, M_0) through the addition of a control place p_c such that $F^* = F \cup (\{p_c\} \times p_c^\bullet) \cup ({}^\bullet p_c \times \{p_c\})$, $\forall p \in P_0 \cup P_S \cup P_R$, $M_0^*(p) = M_0(p)$.

Definition 8 Let (N, M_0), $N = (P_0 \cup P_S \cup P_R, T, F, W)$, be an acceptably marked S^4PR, S be an insufficiently marked siphon at a marking $M \in R(N, M_0)$, (N^*, M_0^*) is the augmented net. l is called a monitor P-vector of S such that

$$l(p) = \begin{cases} \sum_{r \in S_R} \sum_{p \in Th(s)} Y_r(p), & \forall p \in Th(S) \\ 0, & \text{otherwise} \end{cases}.$$

In addition, $\theta = [l^T\ 1]^T$ is called a monitor P-invariant in N^*.

Definition 9 Let (N, M_0), $N = (P_0 \cup P_S \cup P_R, T, F, W)$, be an acceptably marked S^4PR, S be an insufficiently marked siphon at a marking $M \in R(N, M_0)$, δ is called a siphon-resource P-invariant of S such that $\delta = \sum_{r \in S_R} Y_r$.

Since a siphon in N remains a siphon in N^*, its respective siphon-resource P-invariant remains a P-invariant although the dimension is increased by one. For simplicity, we do not distinguish them in denotation and suppose the readers can identify their difference from their context.

Definition 10 Let (N, M_0) be an acceptably marked S^4PR, $N = (P_0 \cup P_S \cup P_R, T, F, W)$, $P = P_0 \cup P_S \cup P_R$. A siphon S is said to be critically controlled by P-invariant I iff $\forall M \in R(N, M_0)$, $I^T \cdot M > M_S^{\max}$ and $I(p) \leq 0$ for $\forall p \in P \backslash S$ hold, or equivalently, $\forall M \in R(N, M_0)$, $I^T \cdot M > M_S^{\max}$, and $\{p \in P \mid I(p) > 0\}$. Such a siphon is called also called a critically controlled siphon.

Lemma 1 Let (N, M_0), $N = (P_0 \cup P_S \cup P_R, T, F, W)$, be an acceptably marked S^4PR, S be an insufficiently marked siphon at a marking $M \in R(N, M_0)$, and l be the monitor P-vector of S. A monitor P-invariant $\theta = [l^T\ 1]^T$ can be introduced in N by the associated place, namely p_c, defined in terms of the addition of the incidence matrix row $[N_{p_c}] = -\, l^T \cdot [N]$.

Proof: $[l^T\ 1]^T \cdot [N^T | N_{p_c}^T = l^T \cdot [N] - [N_{p_c}] = \mathbf{0}^T$. Thus, $[l^T\ 1]^T$ is a newly introduced P-invariant in N. ❑

Lemma 2 Let (N, M_0), $N = (P_0 \cup P_S \cup P_R, T, F, W)$, be an acceptably marked S^4PR, S an insufficiently marked siphon at a marking $M \in R(N, M_0)$, (N^*, M_0^*) the augmented net, θ and δ be the monitor and siphon-resource P-invariant of S in N^*, respectively, $I = \delta - \theta$. S is invariant controlled if $I^T \cdot M_0 > M_S^{\max}$.

Proof: Several remarks must be argued before a formal proof. Since $\delta = \sum_{r\in S_R} Y_r = \sum_{r\in S_R} \sum_{p\in P_S \cup P_R} Y_r(p)$ and $\theta = \sum_{r\in S_R} \sum_{p\in Th(S)} Y_r(p)$, it is obvious that

$$I(p) = \delta(p) - \theta(p) \begin{cases} = \sum_{r\in S_R} Y_r(p), & \forall p \in S \backslash S_R \\ = 1, & \forall p \in S_R \\ = -1, & \forall p = p_c \\ = 0, & \text{otherwise} \end{cases}.$$

Thus, we have $\|I\|^+ \subseteq S$ and $\nexists p \in S \cap \|I\|^-$.

Unfolding the expression $I^T \cdot M_0 > M_S^{\max}(S_R)$ (remember the fact that $\vee_{r\in S_R} M(r) > 0 \vee M(p_c) > 0 \vee_{p\in S_R \cup \{p_c\}} I(p) = 1$), we have $M_0(S_R) - M_0(p_c) > M_S^{\max}$, which is equivalent to $M(S_R) + [M_0(S_R) - M(S_R)] - M_0(p_c) > M_S^{\max}$. Owing to $M_0(S_R) = \delta^T \cdot M = M(S_R) + \sum_{p\in S\cap P_S} \delta(p) \cdot M(p) + \sum_{p\in Th(S)} \delta(p) \cdot M(p)$, we have $M_0(S_R) - M(S_R) = \sum_{p\in S\cap P_S} \delta(p) \cdot M(p) + \sum_{p\in Th(S)} \delta(p) \cdot M(p)$, which consequently leads to $M(S_R) + \sum_{p\in S\cap P_S} \delta(p) \cdot M(p) + \sum_{p\in Th(S)} \delta(p) \cdot M(p) - M_0(p_c)$. By $\forall p \in Th(S)$, $l(p) = \delta(p)$ is true. Furthermore, $\theta = [l^T\ 1]$ is a P-invariant whose support is composed of all p's in $Th(S)$ and p_c. Thereby, $\sum_{p\in Th(S)} \delta(p) \cdot M(p) - M_0(p_c) = -\, M(p_c)$. As a result, we have $M(S_R) + \sum_{p\in S\cap P_S} \delta(p) \cdot M(p) - M(p_c) > M_S^{\max}$, which evidently leads to $I \cdot M > M_S^{\max}$. ❑

4. SIMPLIFICATION OF SUPERVISOR STRUCTURE

It should be noticed that on the base of the aforementioned approach, each siphon introduces one additional places. This can lead to a much complex controlled Petri net. In this section, we explore ways to minimize the addition of new places so that a much simpler supervisor is expected, while achieving the same control purpose. In the sequel, strict minimal siphons are distinguished by elementary and dependent ones in a Petri net. Π_E and Π_D are the sets of elementary and dependent ones, respectively. The concept of elementary siphons is initially proposed in [22]. However, we redefine them in a more general form, which can facilitate the supervisor synthesis process.

Definition 11 Let (N, M_0), $N = (P_0 \cup P_S \cup P_R, T, F, W)$, be an acceptably marked S^4PR, S an insufficiently marked siphon at a marking $M \in R(N, M_0)$, l and δ be the monitor P-vector and siphon-resource P-invariant of S in N, respectively. ϑ is called the control P-vector iff $\vartheta = \delta - l$.

Lemma 3 Let (N, M_0), $N = (P_0 \cup P_S \cup P_R, T, F, W)$, be an acceptably marked S^4PR, S an insufficiently marked siphon at a marking $M \in R(N, M_0)$, l, δ, and ϑ be the control P-vector, siphon-resource P-invariant and monitor P-vector of S in N, respectively. We have $\vartheta^T \cdot [N] = -l^T \cdot [N]$.

Proof: The definition of control P-vector leads to $\vartheta^T + l^T = \delta^T$. Multiplying both sides by the incidence matrix $[N]$, we have $\vartheta^T \cdot [N] + l^T \cdot [N] = \delta^T \cdot [N]$. Since δ is a P-invariant in N, we have $\vartheta^T \cdot [N] + l^T \cdot [N] = \mathbf{0}^T$. Therefore, $\vartheta^T \cdot [N] = -l^T \cdot [N]$ holds trivially. ❑

Definition 12 Let (N, M_0), $N = (P_0 \cup P_S \cup P_R, T, F, W)$, be an acceptably marked S^4PR, S an insufficiently marked siphon at a marking $M \in R(N, M_0)$, and ϑ a control P-vector of S. γ is called the control T-vector of S iff $\gamma^T = \vartheta^T \cdot [N]$.

Definition 13 Let S_1, S_2, …, and S_n be the siphons in an S^4PR net N. Their control T-vectors γ_{S_1}, γ_{S_2}, …, and γ_{S_n} form a vector space. The base of the vector space is denoted by $\gamma_B = \{\gamma_{S_{B_1}}, \gamma_{S_{B_2}}, \ldots, \gamma_{S_{B_k}}\}$, where k is the rank of the vector space. Then S_{B_1}, S_{B_2}, … , and S_{B_k} are called elementary siphons in net N.

Definition 14 Let $S \in \Pi \backslash \Pi_E$ be a siphon in an S^4PR net N and $S_1, S_2, \ldots, S_n \in \Pi_E$. S is strongly dependent on S_1, S_2, …, and S_n, if $\gamma_S = \sum_{i=1}^{n} a_i \cdot \gamma_{S_i}$ holds, where a_1, a_2, …, and $a_n \geq 0$.

Definition 15 Let $S \in \Pi \backslash \Pi_E$ be a siphon of an S^4PR net N and $S_1, S_2, \ldots, S_n, S_{n+1}, S_{n+2}, \ldots$, and $S_{n+m} \in \Pi_E$ $(n \geq 1, m \geq 1)$. S is weakly dependent on elementary siphons S_1, S_2, …, S_n, S_{n+1}, S_{n+2}, …, and S_{n+m} if $\gamma_S = \sum_{i=1}^{n} a_i \cdot \gamma_{S_i} - \sum_{j=n+1}^{n+m} a_j \cdot \gamma_{S_j}$ holds, where $\forall i, j \in \{1, 2, \ldots, n + m\}$, $a_i \geq 0$, $a_j \geq 0$.

If S is dependent on S_{B_1}, S_{B_2}, …, and S_{B_k}, we say that S_{B_1}, S_{B_2}, …, and S_{B_k} are the elementary siphons of S. Next results show the conditions under which dependent siphons are controlled when their corresponding elementary ones are critically controlled. In the sequel, the initial net system is denoted by (N_0, M_0) and the net system with additional places is denoted by (N_1, M_1).

Theorem 2 Let (N_0, M_0) be a net system and siphon S be strongly dependent on elementary siphons S_1, S_2, …, and S_n. If S_1, S_2, …, and S_n are critically controlled by the addition of control places p_{c_1}, p_{c_2}, …, and p_{c_n}, and $M_0(S) > \sum_{i=1}^{n} a_i \cdot M(p_{c_i}) + M_S^{\max}$ holds, S is critically controlled.

Proof: Suppose that $I = [\vartheta^T, -a_1, -a_2, \ldots, -a_n]^T$, $\vartheta_{S_i}(\gamma_{S_i})$ is the control $P(T)$-vector of S_i, where $i = \{1, 2, \ldots, n\}$, and (N_1, M_1) is the new net with additional places p_{c_1}, p_{c_2}, …, and

p_{c_n}. It is trivial that $[N_1] = [N_0^T | \gamma_{S_1} | \gamma_{S_2} | \ldots | \gamma_{S_n}]^T$, $\gamma_{S_i}^T = \vartheta_{S_i}^T \cdot [N]$, where $i \in \{1, 2, \ldots, n\}$. Therefore, we have $I^T \cdot [N_1] = [\vartheta^T, -a_1, -a_2, \ldots, -a_n] \cdot [N_1] = [\vartheta^T, -a_1, -a_2, \ldots, -a_n] \cdot [N_0^T | \gamma_{S_1} | \gamma_{S_2} | \ldots | \gamma_{S_n}]^T = \vartheta_S^T \cdot [N_0] - a_1 \cdot \gamma_{S_1}^T - a_2 \cdot \gamma_{S_2}^T - \ldots - a_n \cdot \gamma_{S_n}^T = \gamma_S^T - \sum_{i=1}^{n} a_i \cdot \gamma_{S_i}^T = \mathbf{0}^T$. Clearly, $I = [\vartheta^T, -a_1, -a_2, \ldots, -a_n]^T$ is a P-invariant.

Next, we prove that S is critically controlled under I. According to the definition of control P-vector, we know that $||I||^+ \subseteq S$. The fact that $\nexists p \in ||I||^- \cap S$ and $\forall p \in (||I||^- \cap S)$ makes $\max_{p\bullet} = 1$ implicitly hold. $I^T \cdot M_1 = [\vartheta_S^T, -a_1, -a_2, \ldots, -a_n] \cdot [M_0 | M(p_{c_1}) | M(p_{c_2}) | \ldots | M(p_{c_n})] = \vartheta_S^T \cdot M_0 - a_1 \cdot M(p_{c_1}) - a_2 \cdot M(p_{c_2}) - \ldots - a_n \cdot M(p_{c_n}) = M_0(S) - \sum_{i=1}^{n} a_i \cdot M(p_{c_i}) > M_S^{\max}$. Therefore, S is critically controlled. ❑

Theorem 3 Let (N_0, M_0) be a net system and siphon S be weakly dependent on elementary siphons $S_1, S_2, \ldots, S_n, S_{n+1}, S_{n+2}, \ldots$, and S_{n+m}. If $S_1, S_2, \ldots, S_n, S_{n+1}, S_{n+2}, \ldots$, and S_{n+m} are controlled by the addition of control places $p_{c_1}, p_{c_2}, \ldots, p_{c_n}, p_{c_{n+1}}, p_{c_{n+2}}, \ldots$ and $p_{c_{n+m}}$, and $M_0(S) > \sum_{i=1}^{n} a_i \cdot M(p_{c_i}) + M_S^{\max}$ holds, S is controlled.

Proof: Omitted due to its similarity with the proof of Theorem 2. ❑

Theorem 4 [22] Let $N = (P, T, F, W)$ be a Petri net and w be the number of elementary siphons of N. We have $w \le \min(|P|, |T|)$.

The reader familiar with [22] can notice that the definition of elementary siphons herein possesses many difference. We must point out that the previous one, although correct mathematically, cannot be directly deployed to synthesize the liveness enforcing supervisor algebraically, which makes it less powerful. Also, we should notice that for a subclass of S^4PR nets, known as S^3PR nets, the control P-vector (control T-vector) coincides with the characteristic P-vector (characteristic T-vector), thus the two kinds of definition is equivalent. We leave the relevant proof to the reader.

As a conclusion, the synthesis method for obtaining the control elements of a specified video streaming system can be characterized by the following steps:

(1) Let $i = 0$, $j = 0$, and $ES = \emptyset$. Design the plant or uncontrolled Petri net model which is denoted by (N_0, M_0) with n places and m transitions.
(2) Design and solve the corresponding integer programming model. Check whether an insufficiently siphon exists. If yes, go to step 3. If no, go to step 7.
(3) The obtained insufficiently marked siphon (if available) is denoted by $\hat{S}_i$ whose respective strict minimal one is S_i.
(4) Establish the monitor P-vector l_{S_i} and the siphon-resource P-vector δ_{S_i} so that the monitor P-invariant $\theta_{S_i} = [l_{S_i}^T\, 1]^T$, $I_{S_i} = \delta_{S_i} - \theta_{S_i}$, and the control P-vector $\vartheta_{S_i} = \delta_{S_i} - l_{S_i}$.
(5) Check whether S_i is a dependent siphon with respect to $ES_1, ES_2, \ldots, ES_j$. Do as follows.
    (5.1) In the case "yes", make reassignment of the markings $M(p_{c_1}), M(p_{c_2}), \ldots, M(p_{c_j})$ such that $M_j(S_i) - \sum_{k=1}^{j} a_k \cdot M(p_{c_k}) > M_{S_i}^{\max}$ and $M_j = [M_0^T\, M(p_{c_1})\, M(p_{c_2}) \ldots M(p_{c_j})]^T$.
    (5.2) In the case "no", $j = j + 1$, $ES = ES \cup \{S_i\}$. Control S_i with p_{c_j} (or p_{n+j}, for clear

presentation) such that $[N_{p_{c_j}}] = \vartheta_{S_i}^T \cdot [N_{j\text{-}1}]$, $M(p_{c_j}) = M_{j\text{-}1}(S_i) - (M_{S_i}^{\max} + 1)$. Furthermore, establish a new net model (N_j, M_j) such that $[N_j] = [[N_{j\text{-}1}]^T \,|\, [N_{p_{c_j}}]^T]^T$ and $M_j = [M_{j\text{-}1}^T\ M(p_{c_j})]^T$.

(6) Let $i = i + 1$. Go to step 2.
(7) The resultant net system is live.
(8) Exit.

Theorem 5 Let (N_0, M_0), $N = (P_0 \cup P_S \cup P_R, T, F, W)$, be an acceptably marked S^4PR, (N_1, M_1), (N_2, M_2), …, and (N_k, M_k) be k augmented nets with elementary siphons ES_1, ES_2, …, and ES_k iteratively controlled by p_{c_1}, p_{c_2}, …, and p_{c_k}, where only (N_k, M_k), $N_k = (P_0 \cup P_S \cup P_R \bigcup_{i=1}^{k}\{p_{c_i}\}, T_k, F_k, W_k)$ is live. We have $k \leq |T|$.

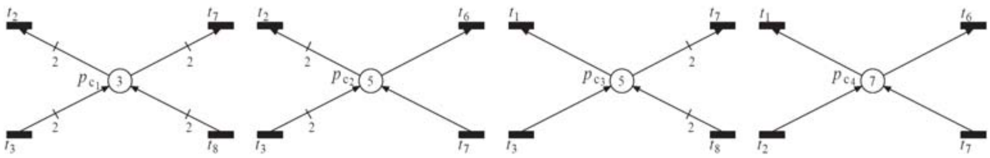
Proof: Considering (N_k, M_k), we have $|T_k| = |T|$ since no additional transitions are introduced during the iteration. Furthermore, the fact that ES_1, ES_2, …, and ES_k are elementary siphons in N_k leads to $k \leq \min\{|P_k|, |T_k|\} \leq |T_k|$. By $|T_k| = |T|$, we have $k \leq |T|$. ❑

Now, an example is illustrated to demonstrate the feedback control of a Petri net model using inequality constraints. Consider the Petri net model shown in Fig. 2, whose incidence matrix $[N_0]$ is as follows:

$$[N_0] = \begin{pmatrix} -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & -2 & -1 & 3 & 0 & 0 & -2 & -1 & 3 & 0 \\ 0 & 0 & -1 & 0 & 1 & -1 & 0 & 1 & 0 & 0 \end{pmatrix}.$$

Since this system is small in size, the supervisor simplification operation is ignored during our demonstration. At the first step, it has be verified that there exists an insufficiently siphon $\hat{S}_0 = S_0 = \{p_4, p_9, p_{12}\}$ with $M_{S_0}^{\max} = 0$. Then, the augmented net N_1 is built by adding a control place p_{c_1} associated with S_0. $\vartheta_{S_0} = 3p_4 + 3p_9 + p_{12}$ is the control P-vector with respect to S_0. The product $\vartheta_{S_0}^T \cdot [N_0] = [0\ -2\ 2\ 0\ 0\ 0\ -2\ 2\ 0\ 0]$ determines that $p_{c_1}^{\bullet} = \{t_2, t_7\}$, ${}^{\bullet}p_{c_1} = \{t_3, t_8\}$, and $W(p_{c_1}, t_2) = W(t_3, p_{c_1}) = W(p_{c_1}, t_7) = W(t_8, p_{c_1}) = 2$. In addition, we obtain $M(p_{c_1}) = M_0(S_0) - (M_{S_0}^{\max} + 1) = 3$. The control subnet enforcing such a constraint is shown in Fig. 3 (a).

At the second step, it has be verified that there exists an insufficiently siphon $\hat{S}_1 = \{p_4, p_5, p_6, p_8, p_9, p_{10}, p_{13}, p_{14}\}$, whose corresponding strict minimal one is $\{p_4, p_5, p_8, p_{13},$



(a) (b) (c) (d)

Fig. 3. The control places for Petri net shown in Fig. 2.

$p_{14}\}$ with $M_{S_1}^{\max} = 1$. Then, the augmented net N_2 is built by adding a control place p_{c_2} associated with S_1. $\vartheta_{S_1} = p_4 + p_5 + 3p_8 + p_{13} + p_{14}$ is the control P-vector with respect to S_1. The product $\vartheta_{S_1}^T \cdot [N_1] = -2t_2 + 2t_3 - t_6 + t_7$ determines that $p_{c_2}^{\bullet} = \{t_2, t_6\}$, ${}^{\bullet}p_{c_2} = \{t_3, t_7\}$, and $W(p_{c_2}, t_2) = W(t_3, p_{c_2}) = 2$, $W(p_{c_2}, t_6) = W(t_7, p_{c_2}) = 1$. In addition, we obtain $M(p_{c_2}) = M_1(S_1) - (M_{S_1}^{\max} + 1) = 5$. The control subnet enforcing such a constraint is shown in Fig. 3 (b).

At the third step, it has be verified that there exists an insufficiently siphon $\hat{S}_2 = \{p_1, p_3, p_4, p_5, p_9, p_{10}, p_{11}, p_{14}\}$, whose corresponding strict minimal one is $S_2 = \{p_3, p_9, p_{10}, p_{11}, p_{14}\}$ with $M_{S_2}^{\max} = 1$. Then, the augmented net N_3 is built by adding a control place p_{c_3} associated with S_2. $\vartheta_{S_2} = 2p_3 + p_9 + p_{10} + p_{11} + p_{14}$ is the control P-vector with respect to S_3. The product $\vartheta_{S_2}^T \cdot [N_2] = -t_1 + t_3 - 2t_7 + 2t_8$ determines that $p_{c_3}^{\bullet} = \{t_1, t_7\}$, ${}^{\bullet}p_{c_3} = \{t_3, t_8\}$, and $W(p_{c_3}, t_1) = W(t_3, p_{c_3}) = 1$, $W(p_{c_3}, t_7) = W(t_8, p_{c_2}) = 2$. In addition, we obtain $M(p_{c_3}) = M_2(S_2) - (M_{S_2}^{\max} + 1) = 5$. The control subnet enforcing such a constraint is shown in Fig. 3 (c).

At the fourth step, it has be verified that there exists an insufficiently siphon $\hat{S}_3 = \{p_1, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}, p_{15}, p_{16}\}$, whose corresponding strict minimal one is $S_3 = \{p_3, p_8, p_{15}, p_{16}\}$ with $M_{S_3}^{\max} = 2$. Then, the augmented net N_4 is built by adding a control place p_{c_4} associated with S_3. $\vartheta_{S_3} = 3p_3 + 2p_8 + p_{15} + p_{16}$ is the control P-vector with respect to S_4. The product $\vartheta_{S_3}^T \cdot [N_3] = -t_1 + t_2 - t_6 + t_7$ determines that $p_{c_4}^{\bullet} = \{t_1, t_6\}$, ${}^{\bullet}p_{c_4} = \{t_2, t_7\}$, and $W(p_{c_4}, t_1) = W(t_2, p_{c_4}) = W(p_{c_4}, t_6) = W(t_7, p_{c_4}) = 1$. In addition, we obtain $M(p_{c_4}) = M_3(S_3) - (M_{S_3}^{\max} + 1) = 7$. The control subnet enforcing such a constraint is shown in Fig. 3 (d).

At the last iteration, when (N_4, M_4) is analyzed with its MIP model, no solutions can be found. This means that the whole net system is live. Furthermore, RG analysis shows the resultant net is composed of 537 states. It can be seen that only 87 bad states are eliminated and 100% (537/537) good states are preserved in during the iteration.

5. AN ILLUSTRATIVE EXAMPLE

In this section, we investigate a video streaming system where four video streams, *i.e.*, streams 1, 2, 3, and 4, can be simultaneously transmitted inside a pool of video servers, to demonstrate the methodologies proposed throughout this paper. Then, some experimental studies are conducted to show the computational efficiency and control performance (behavior permissiveness and supervisor structure) of the proposed methods.

Consider the system shown in Fig. 4. It is a video system consisting of three servers $\mathfrak{S}_1$, $\mathfrak{S}_2$, and $\mathfrak{S}_3$ with capacities $C(\mathfrak{S}_1) = C(\mathfrak{S}_3) = 1$, and $C(\mathfrak{S}_2) = 2$. These servers are connected by two router nodes, *i.e.*, $\Re_1$ and $\Re_2$ with capacities $C(\Re_1) = 2$ and $C(\Re_2) = 1$. These resources support the video transmission between $\mathfrak{S}_1$ and $\mathfrak{S}_2$, $\mathfrak{S}_2$ and $\mathfrak{S}_3$. Fig. 5 shows its Petri net model resulting from the four transmission processes for the videos in Fig. 4. The net model is an S^4PR in which resource types $\Re_1$, $\mathfrak{S}_1$, $\mathfrak{S}_2$, $\Re_2$, and $\mathfrak{S}_3$ are denoted by p_{19}, p_{20}, p_{21}, p_{22}, and p_{23}, respectively.

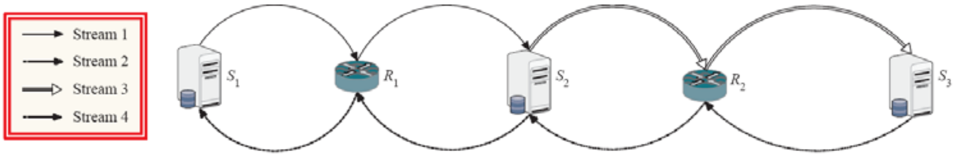


Fig. 4. The layout of a video streaming system.

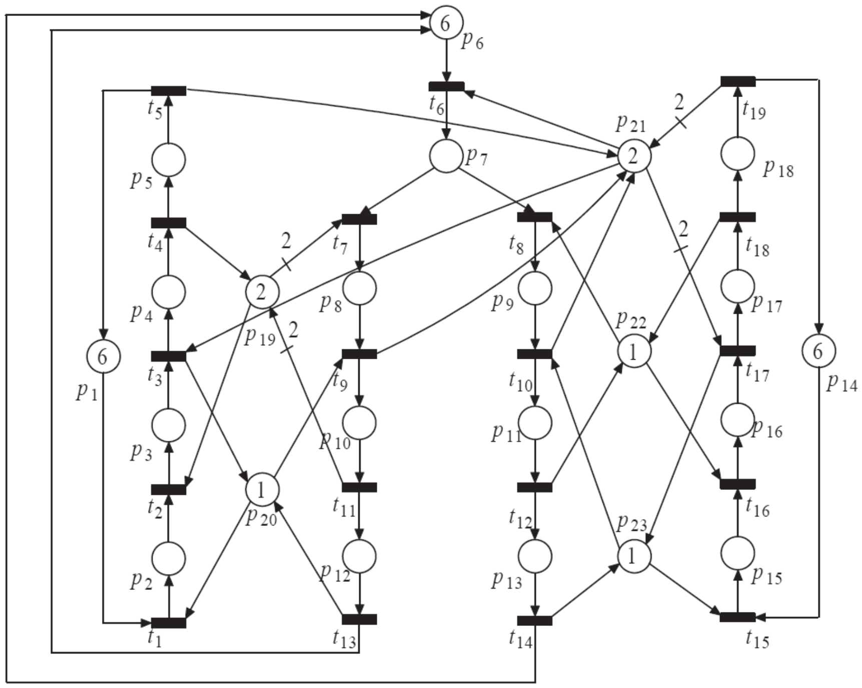


Fig. 5. The Petri net modeling the system in Fig. 4.

The transmission process for video 1 from $\mathfrak{S}_1$ to $\mathfrak{S}_2$ is defined by a set of places $\{p_1, p_2, p_3, p_4, p_5\}$ which is strictly ordered through the transmission relations. Specifically, a transmission process of video 1 can take the route defined by the sequence of places $<p_1, p_2, p_3, p_4, p_5>$. The transmission process for video 2 from $\mathfrak{S}_2$ to $\mathfrak{S}_1$ is defined by a set of places $\{p_6, p_7, p_8, p_{10}, p_{12}\}$ which is strictly ordered through the transmission relations. Specifically, a transmission process of Job 2 can take the route defined by the sequence of places $<p_6, p_7, p_8, p_{10}, p_{12}>$. The transmission process for video 3 from $\mathfrak{S}_2$ to $\mathfrak{S}_3$ is defined by a set of places $\{p_6, p_7, p_9, p_{11}, p_{13}\}$ which is strictly ordered through the transmission relations. Specifically, a transmission process of video 3 can take the route defined by the sequence of places <p_6, p_7, p_9, p_{11}, p_{13}>. The transmission process for video 4 from $\mathfrak{S}_3$ to $\mathfrak{S}_2$ is defined by a set of places {p_{14}, p_{15}, p_{16}, p_{17}, p_{18}} which is strictly ordered through the transmission relations. A transmission process of video 4 can take the route defined by the sequence of places <p_{14}, p_{15}, p_{16}, p_{17}, p_{18}>. The resource requirements for the operation places are defined as follows: $a_{p_2} = \mathfrak{S}_1$, $a_{p_3} = \mathfrak{S}_1 + \mathfrak{R}_1$, $a_{p_4} = \mathfrak{R}_1 + \mathfrak{S}_2$, $a_{p_5} = \mathfrak{S}_2$, $a_{p_7} = \mathfrak{S}_2$, $a_{p_8} = \mathfrak{S}_2 + 2\mathfrak{R}_1$, $a_{p_9} = \mathfrak{S}_2 + \mathfrak{R}_2$, $a_{p_{10}} = 2\mathfrak{R}_1 + \mathfrak{S}_1$, $a_{p_{11}} = \mathfrak{R}_2 + \mathfrak{S}_3$, $a_{p_{12}} = \mathfrak{S}_1$, $a_{p_{13}} = \mathfrak{S}_3$, $a_{p_{15}} = \mathfrak{S}_3$, $a_{p_{16}} = \mathfrak{S}_3 + \mathfrak{R}_2$, $a_{p_{17}} = \mathfrak{R}_2 + \mathfrak{S}_2$, and $a_{p_{18}} = \mathfrak{S}_2$. In deed, the establishment of a virtual circuit and the data delivery between two network nodes are denoted as an operation place by which two resources are shared. Finally, assuming that all the places are ordered in the incidence matrix according to the sequence <p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}, p_{11}, p_{12}, p_{13}, p_{14}, p_{15}, p_{16}, p_{17}, p_{18}, p_{19}, p_{20}, p_{21}, p_{22}, p_{23}>, we have the initial marking $M_0 = 6p_1 + 6p_6 + 6p_{14} + 2p_{19} + p_{20} + 2p_{21} + p_{22} + p_{23}$. Trivially, this net system is deadlock susceptible due to the circular-wait that occurs among relative operations and resources.

To control deadlocks, our policy, which is based on the insufficiently marked siphons and their integer programming derivation approach, is used to generate the siphons and control the elementary ones.

First, we obtain a siphon $\hat{S}_0$ = {p_3, p_4, p_5, p_7, p_{10}, p_{11}, p_{12}, p_{13}, p_{16}, p_{17}, p_{18}, p_{19}, p_{20}, p_{21}, p_{22}, p_{23}} which leads to a strict minimal siphon S_0 = {p_3, p_4, p_{10}, p_{12}, p_{19}, p_{20}} so that $\vartheta_{S_0} = 3p_3 + p_4 + 3p_{10} + p_{12} + p_{19} + p_{20}$, and $M_{S_0}^{\max} = 0$. Consequently, we have $[N_{p_{c_1}}] = \vartheta_{S_0}^T \cdot [N_0] = -t_1 + t_2 - 2t_7 + 2t_9$. $M(p_{c_1}) = M_0(S_0) - M_{S_0}^{\max} - 1 = 2$. As a result, a new net (N_1, M_1) with $[N_1] = [[N_0]^T | [N_{p_{c_1}}]^T]^T$ and $M_1 = [M_0^T M(p_{c_1})]^T$ is resulted.

Second, we obtain a siphon $\hat{S}_1$ = {p_2, p_4, p_5, p_8, p_{10}, p_{11}, p_{12}, p_{13}, p_{16}, p_{17}, p_{18}, p_{19}, p_{20}, p_{21}, p_{22}, p_{23}} which leads to a strict minimal siphon S_1 = {p_4, p_5, p_8, p_{10}, p_{11}, p_{13}, p_{17}, p_{18}, p_{19}, p_{21}, p_{23}} so that $\vartheta_{S_1} = 2p_4 + p_5 + 3p_8 + 2p_{10} + p_{11} + p_{13} + 2p_{17} + 2p_{18} + p_{19} + p_{21} + p_{23}$, and $M_{S_1}^{\max} = 1$. Consequently, we have $[N_{p_{c_2}}] = \vartheta_{S_1}^T \cdot [N_1] = -t_2 + t_3 - t_6 + t_7 + t_{10} - t_{15} + t_{17}$. $M(p_{c_2}) = M_1(S_1) - M_{S_1}^{\max} - 1 = 2$. As a result, a new net (N_2, M_2) with $[N_2] = [[N_1]^T | [N_{p_{c_2}}]^T]^T$ and $M_2 = [M_1^T M(p_{c_2})]^T$ is resulted.

Third, we obtain a siphon $\hat{S}_2$ = {p_3, p_4, p_7, p_8, p_{10}, p_{11}, p_{12}, p_{13}, p_{16}, p_{17}, p_{18}, p_{22}, p_{23}, p_{25}} which leads to a strict minimal siphon S_2 = {p_{11}, p_{13}, p_{16}, p_{17}, p_{22}, p_{23}} so that $\vartheta_{S_2} = 2p_{11} + p_{13} + 2p_{16} + p_{17} + p_{22} + p_{23}$, $M_{S_2}^{\max} = 0$. Consequently, we have $[N_{p_{c_3}}] = \vartheta_{S_2}^T \cdot [N_2] = -t_8 + t_{10} - t_{15} + t_{16}$ and $M(p_{c_3}) = M_2(S_2) - M_{S_2}^{\max} - 1 = 1$. As a result, a new net (N_3, M_3) with $[N_3] = [[N_2]^T | [N_{p_{c_3}}]^T]^T$ and $M_3 = [M_2^T M(p_{c_3})]^T$ is resulted.

Fourth, we obtain a siphon $\hat{S}_3$ = {p_3, p_4, p_5, p_8, p_9, p_{10}, p_{11}, p_{12}, p_{13}, p_{15}, p_{17}, p_{18}, p_{21}, p_{22}, p_{24}, p_{25}} which leads to a strict minimal siphon S_3 = {p_4, p_5, p_8, p_9, p_{11}, p_{17}, p_{18}, p_{21}, p_{22}, p_{24}, p_{25}} so that $\vartheta_{S_3} = p_4 + p_5 + 3p_8 + 3p_9 + p_{11} + 3p_{17} + 2p_{18} + p_{21} + p_{22} + p_{24} + p_{25}$, $M_{S_3}^{\max} = 2$. $[N_{p_{c_4}}] = \vartheta_{S_3}^T \cdot [N_3] = -t_1 + t_3 - 2t_6 + 2t_7 + 2t_8 - t_{15} - t_{16} + t_{17}$, $M(p_{c_4}) = M_3(S_3) - M_{S_3}^{\max} - 1 = 4$. As a result, a new net (N_4, M_4) with $[N_4] = [[N_3]^T | [N_{p_{c_4}}]^T]^T$ and $M_4 = [M_3^T M(p_{c_4})]^T$ is resulted.

Fifth, we obtain a siphon $\hat{S}_4$ = {p_3, p_4, p_8, p_9, p_{10}, p_{11}, p_{12}, p_{13}, p_{16}, p_{17}, p_{18}, p_{24}, p_{25}, p_{26}, p_{27}} which leads to a strict minimal siphon S_4 = {p_3, p_8, p_9, p_{16}, p_{24}, p_{25}, p_{26}, p_{27}} so that $\vartheta_{S_4} = 2p_3 + 2p_8 + p_9 + 3p_{16} + p_{24} + p_{25} + p_{26} + p_{27}$, $M_{S_4}^{\max} = 1$. Consequently, we have $[N_{p_{c_5}}] = \vartheta_{S_4}^T \cdot [N_4] = -2t_1 + 2t_2 - 3t_6 + 3t_7 + 2t_8 + t_{10} - 3t_{15} - 3t_{16}$. $M(p_{c_5}) = M_4(S_4) - M_{S_4}^{\max}$

– 1 = 7. As a result, a new net (N_5, M_5) with $[N_5] = [[N_4]^T | [N_{p_{c_5}}]^T]^T$ and $M_5 = [M_4^T M(p_{c_5})]^T$ is resulted.

After p_{c_5} is added, no insufficiently marked siphons can be found. Thus, the resulting net is live with 91% (260/286) good states reserved. At each iteration step, it can be verified that the currently obtained siphon is independent of the previous ones in terms of their control T-vectors. Thus, the rearrangement of their initial markings is not necessary. Further analysis can show that such net system (N_5, M_5) contains twenty-two strict minimal siphons, among which eight ones are siphons contained in (N_0, M_0) while the others are induced due to the introduction of control places during the iteration. All the other eighteen siphons distinguished from $S_0, S_1, \ldots, S_4$ are implicitly controlled by p_{c_1}, $p_{c_2}, \ldots, p_{c_5}$.

Take a siphon $S = \{p_4, p_5, p_8, p_9, p_{10}, p_{11}, p_{17}, p_{18}, p_{19}, p_{21}, p_{22}\}$ for example. It is verified that $M_S^{\max} = 0$, $l_S = p_3 + p_7 + p_{16}$, and $\delta_S = p_3 + 2p_4 + p_5 + p_7 + 3p_8 + 2p_9 + 2p_{10} + p_{11} + p_{16} + 2p_{17} + 2p_{18} + p_{19} + p_{21} + p_{22}$ so that $\vartheta_S = \delta_S - l_S = \delta_S = 2p_4 + p_5 + 3p_8 + 2p_9 + 2p_{10} + p_{11} + 2p_{17} + 2p_{18} + p_{19} + p_{21} + p_{22}$ and $\gamma_S = \vartheta_S^T \cdot [N_5] = - t_2 + t_3 - t_6 + t_7 + t_8 - t_{16} + t_{17}$, which leads to $\gamma_S = \gamma_{S_1} - \gamma_{S_2}$. Therefore, S is a dependent siphon with respect to S_0, $S_1, \ldots, S_4$. Since $M_5(S) - M_5(p_{c_2}) = 5 - 2 = 3 > 0$. Consequently, S is critically controlled with no additional places introduced. In the same way, we can further verify that all the other dependent siphons are implicitly controlled. As a consequence, we can conclude that our approach can synthesize a much simpler supervisor which promises a as more permissive as possible resultant system.

Considering the statement in Theorems 4 and 5, we evidently anticipate that the supervisor structure can be greatly simplified when the considered Petri net is more complex. Fig. 6 shows the respective control places p_{c_i}'s introduced at each iteration step. As shown by the two small examples in this paper, our method and the one in [27] may lead to the similar result if no dependent siphons are found during the iteration. Nevertheless, our method is advantageous for more complex models, where the dependent siphons outnumber the elementary ones greatly. We leave the confirmation on this point to the reader thanks to its triviality.

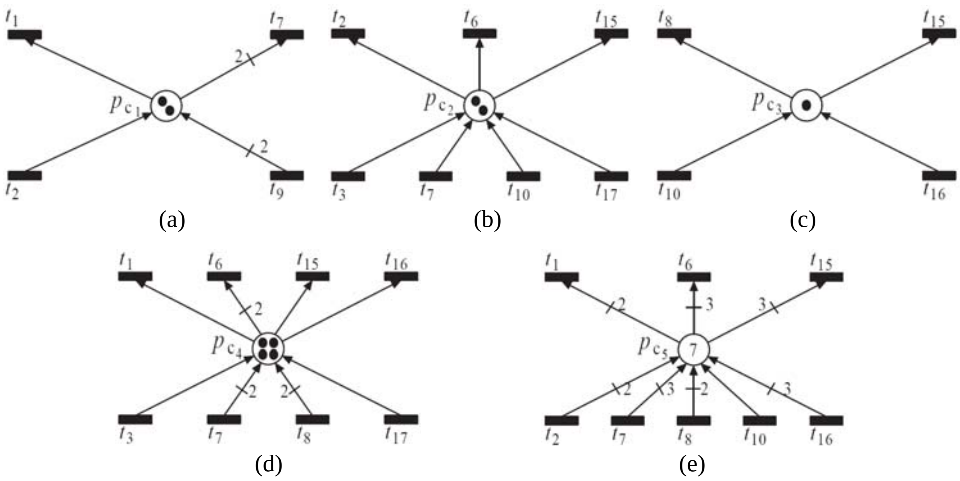


Fig. 6. The control places for Petri net shown in Fig. 5.

6. CONCLUSION

The liveness enforcing supervisor synthesis problem is studied for a class of video system, that allows multiple resource acquisitions and flexible routings. The effective characterization of liveness for the S^4PR nets, appropriately modeling the considered video system, through the notion of insufficiently marked siphons, allows the algebraic synthesis of liveness enforcing supervisor, which mathematically can be represented as a set of inequalities. Moreover, the investigation of generalized elementary siphon technique can simplify the supervisor structure on any specified video system. Future works aim at the extension of the derived results system with more general behavioral features, such as packet assembly/disassembly operations and uncontrollable/unobservable resource acquisition.

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