Short Paper

Bit and Power Allocation for Multi-user MIMO OFDM Systems with Subcarrier Reuse

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An adaptive resource allocation for multiaccess MIMO/OFDM systems was developed to enhance power efficiency by exploiting multiuser diversity, and channel variations in the time, frequency, and space domains [1]. However, by increasing the number of antennas, the scheme has a limitation in the affordable maximum data rate due to constraints on the maximum modulation order even without transmit power constraint. More antennas should allow more users to share the same subcarriers such that total affordable maximum data rates should increase accordingly. The paper presents an enhanced version of the scheme to remedy this problem. After the subcarrier assignment, the proposed scheme relaxes the maximum modulation order constraint in the bit loading process and presents an iterative bit swapping procedure, which are the two points deviated from the scheme in [1] and solve the related above-mentioned problem. A model of the multi-user channel on uplink with subcarrier reuse is considered. The proposed scheme aims to minimize the total user transmit power while satisfying the required data rates, the maximum transmit power constraint, and the bit error rate of each user. The proposed algorithm offers better performance than the high complexity greedy multi-user algorithm as illustrated in the simulation results. 0-35% improvement in terms of percentage in meeting the required data rates is observed from the simulation results and 0~5 dB transmit power gain is achieved.

Keywords: resource allocation, subcarrier reuse, multiple-input multiple-output (MIMO), orthogonal frequency division multiple access (OFDMA), space division multiple access (SDMA)

1. INTRODUCTION

Multi-carrier modulation schemes like orthogonal frequency division multiplexing (OFDM) have drawn a lot of attention recently [2]. OFDM is a technique to combat inter-symbol interference (ISI) in wideband transmissions over multipath fading channels.
Multiple-input multiple-output (MIMO) systems where multiple antennas are used at both the transmitter and the receiver are viewed as a transmission technique to achieve high capacity. MIMO systems may achieve spatial multi-user access using the singular value decomposition (SVD) MIMO technique [1, 4]. The MIMO OFDM systems can multiplex users both in the frequency and the spatial domains. A resource allocation algorithm is proposed for orthogonal frequency division multiple access combined with space division multiple access (OFDMA/SDMA) systems for downlink where orthogonality in the spatial domain among users can be assumed [3]. However, due to imperfect orthogonality in the spatial domain on uplink, the co-channel interference caused by the subcarrier reuse may lower the system’s performance.

For multiuser MIMO OFDM systems with co-channel interference in the uplink, the combination of power control and adaptive modulation is desirable to reduce the effect of the co-channel interference. Zhang and Letaief proposed an allocation algorithm [1] by starting an OFDMA-based allocation procedure so that the amount of the co-channel interference is mitigated by the subcarrier allocation. The scheme used a neighborhood search scheme to increase power efficiency. However, the proposed scheme has a limitation on the affordable maximum data rates even without transmit power constraint when increasing the number of antennas. The systems with more antennas should allow more users to share the same subcarriers such that total affordable maximum data rates should increase accordingly. In this paper, we present an enhanced version of the scheme to remedy this problem with the co-channel interference in the uplink for the multiuser MIMO OFDM systems. The proposed scheme aims to minimize the total user transmit power while satisfying the required data rate, the maximum transmit power constraint, and the bit error rate (BER) of each user. The proposed algorithm offers better performance than that of the high complexity multi-user greedy algorithm [5] as illustrated in the simulation results. Referring to [5], the multi-user greedy algorithm loads one bit at a time by choosing the subcarrier associated with a user with the least incremental transmit power, which involves complex operations of calculating the incremental power and updating related parameters in the co-channel interference scenarios. Each loaded bit requires the complicated process. Greedy search-based approaches usually have simple descriptions, but have high computational complexities. The proposed scheme first uses OFDMA allocation idea where the subcarriers can not be shared; and it redistributes some bits from the users with subcarrier reuse by utilizing the search for spatial separable groups. The redistribution operation involves the similar complicated process as that of the multi-user greedy algorithm, but the number of the redistributed bits is much less. These manners reduce the computational complexity.

2. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a single cell multiple access MIMO/OFDM system consisting of $M$ users distributed randomly to communicate with a base station on an uplink as described by the model in [1]. There are $P$ receive antennas at the base station and $J$ transmit antennas at each user. The gain of each omni-directional antenna element is unity. The intra-cell bandwidth is reused by employing spatial division to separate mobiles. We divide the considered spectrum into $N$ orthogonal subcarriers. The subscript $m$ is the index of a
mobile user; the subscript $c$ is the index of a subcarrier; boldface means vector or matrix. We assume the channel is fixed in the coherence time, and its impulse response using a $L$-ray model between the $i$th received antenna and the $j$th transmit antenna is defined as:

$$
h_{m_1,j}(t) = G_{m_1,j} \sum_{l=1}^{L} \beta_{m_1,j}^l e^{j\phi_{m_1,j}^l} \delta(t - \tau_{m_1,j}^l) a^R_j (\Theta_{m_1,j}^R) a^T_j (\Theta_{m_1,j}^T)$$

where $\tau_{m_1,j}^l$ is the $l$th path delay; the terms $\beta_{m_1,j}^l$ and $\phi_{m_1,j}^l$ are independent and identically distributed (i.i.d.) Rayleigh amplitude and i.i.d. uniform phase on the interval $[0, 2\pi]$ of the $l$th path; $a^R_j$ and $a^T_j$ are array responses of path $l$ with the arrival angle $\Theta_{m_1,j}^R$ and the departure angle $\Theta_{m_1,j}^T$, respectively. $G_{m_1,j}$ is a real-valued random variable representing the path loss and the log-normal shadow fading.

The channel gain $H_m^c$ at subcarrier $c$ can be written as

$$
H_m^c = \begin{bmatrix}
H_{m,11}^c & H_{m,12}^c & \cdots & H_{m,1L}^c \\
H_{m,21}^c & \ddots & \cdots & \vdots \\
\vdots & \ddots & \ddots & \vdots \\
H_{m,PL}^c & \cdots & \cdots & H_{m,PL}^c
\end{bmatrix},
$$

where $h_{m_1,j}(k)$ is the sampled version of $h_{m_1,j}(t)$ and the gain at a subcarrier is obtained through the discrete Fourier transform (DFT) operation. The channel gain $H_m^c$ at subcarrier $c$ can be decomposed using SVD

$$
H_m^c = U_m^c S_m^c V_m^c H
$$

where $u_m^c$ and $v_m^c$ are the left and the right singular vectors, respectively; $s_m^c$ denotes the $k$th singular value that is arranged in a descending order. Note that, in an outdoor environment, the maximum singular value is generally much larger than the other singular values due to the lack of local scatters around a base station [4].

There are many methods which would be adopted to compute beamforming vectors in base stations to separate signals transmitted from multiple mobiles, such as the zero forcing (ZF) or the minimum mean square error (MMSE) approaches [6]. However, the beamforming weight vector is a function of users’ related parameters such as transmit
power. To perform the joint optimization of beamforming and resource allocation has a high computational complexity. To simplify the optimization task, the matched filter (MF) receivers at the base stations that match each user’s spatial signature are adopted [1]. Hence the beamforming vector is only the function of the desired user channel matrix. The output decision statistics is expressed as

$$\gamma^c_m = (u^c_{m1})^H r^c = s^c_m \sqrt{p^c_m} d^c_m + \sum_{k=1}^{M} \rho^c_{m,k} s^c_k \sqrt{p^c_k} d^c_k + (u^c_{m1})^H n^c,$$  

(5)

where the correlation between the spatial signatures of users $k_1$ and $k_2$ on subcarrier $c$ is represented as

$$\rho^c_{k_1,k_2} = (u^c_{k_1})^H u^c_{k_2}.$$  

(6)

Note that, the number of active users on a subcarrier should not exceed the number of antennas at the base station. Otherwise, they cannot be separated in the spatial domain.

The signal power to interference plus noise power ratio (SINR) of the $m$th user’s decision statistics at subcarrier $c$ can be written as

$$\text{SINR} \Gamma^c_m = \frac{(s^c_m)^2 p^c_m}{\sum_{k=1}^{M} [p^c_{m,k}]^2 (s^c_k)^2 p^c_k + \sigma^2}.$$  

(7)

The allocation problem under consideration is to minimize the total transmit power of all users with the required data rate $B_m$ and the BER $P_e$ constraint of user $m$. We also impose the maximum transmit power $P_{\text{max}}$ constraint which one user can support. The allocation problem can be formulated as

$$\begin{align*}
\text{minimize} & \sum_{m=1}^{M} \sum_{c=0}^{N-1} p^c_m, \quad \text{subject to} \sum_{c=0}^{N-1} b^c_m = B_m, \\
0 \leq & \sum_{c=0}^{N-1} p^c_m \leq P_{\text{max}}, \quad \forall m, b^c_m \in [0, 2, 4, 6], \forall m, c,
\end{align*}$$  

(8)

where $b^c_m$ is the number of bits for a moddulation mode per transmission.

Our goal is to find the assignment of bits and power. In other words, we have to find $p^c_m$ associated with $b^c_m$ for every user and every subcarrier to minimize the total transmit power in the interference channel. It has been shown that the required SINR $\Gamma^c_m$ at subcarrier $c$ for user $m$ can be approximated as a function of a desired BER $P_e$ and a bit loading $b^c_m$ [7]:

$$\text{SINR} \Gamma^c_m \approx \frac{(\ln 5 P_e (2^{b^c_m} - 1))}{-1.5}.$$  

(9)
In order to ensure the desired BER for every subcarrier, every user should have SINR no less than the required SINR $\Gamma_m$, i.e. $\Gamma_m \geq \Gamma_m$, $\forall m, c$. Then we can have

$$\sum_{k=1}^{M} \left( s_m^{c} \sigma_m \right)^2 p_m \geq \Gamma, \forall c.$$  \hspace{1cm} (10)

Write in a matrix form

$$(I - DF^c)P^c \geq \Lambda^c, \forall c$$  \hspace{1cm} (11)

where $I$ is an $M \times M$ identity matrix; $D^c$ is $\text{diag}\{\Gamma_1^c, \ldots, \Gamma_M^c\}$; $P^c$ is $[p_1^c, \ldots, p_M^c]^T$ $\Lambda^c$ is

$$[\lambda_1^c, \ldots, \lambda_M^c]^T$$ and $\lambda_m^c = \frac{\Gamma_m \sigma_m^2}{(s_m^{c})^2}$; and $[F_m^c] = \left\{ \frac{\left( s_m^{c} \right)^2 p_m^c}{\left( s_m^{c} \right)^2}, i \neq j \right\}$. The solution for the equality is

$$P^c = (I - DF^c)^{-1} \Lambda^c.$$  \hspace{1cm} (12)

This equation provides the required bit and power relationship to be satisfied for a particular subcarrier $c$ under the interference channel. The equation will be utilized in the proposed scheme.

### 3. PROPOSED ALLOCATION ALGORITHM

The algorithm proposed in [1] deals with the allocation problem Eq. (8) without a maximum power constraint. The allocation process in [1] can be divided into three parts. First, an initial solution is generated to allocate subcarriers and each subcarrier only can be utilized by one user. After the subcarrier allocation, this power and bit problem can be easily solved by any single user allocation algorithm without co-channel interference [8, 9]. Then, the bit and power is assigned to satisfy the BER and the data rate constraints. Finally, the scheme repeats the bit swapping action to improve the power efficiency until no feasible swapping can be made to reduce transmit power. Each bit swapping operation reallocates two bits of a user from one subcarrier to another subcarrier while the other bits remain intact to reduce total user transmit power. The affordable total data rate in this spectrum over all users should be less than or equal to $P \cdot N \cdot \text{bit}_{\text{max}}$, where $\text{bit}_{\text{max}}$ is the maximum order of modulation. However, the scheme presents an OFDMA allocation approach to obtain the initial solution so the maximum total data rate where the system can support is limited to $N \cdot \text{bit}_{\text{max}}$ over all users even without power constraint as the subcarriers can not be shared. This total data rate limitation deters us from increasing the number of the received antennas for pursuing more capacity gain. The systems with more antennas should allow more users to share the same subcarriers. Besides, allocated transmit power for each user may exceed a maximum power constraint in
practice. Due to these problems, we propose an enhanced allocation algorithm to deal with these problems. The proposed method does not suffer from the data rate limitation problem and is imposed upon a maximum power constraint. We also allow users to reduce their transmit data rates in the case that the requested data rate exceeds the system’s affordable capacity under particular severe environments.

The proposed scheme is composed of the following three steps:

1. Subcarrier allocation, which adopts the same approach to achieve this allocation as the scheme in [1].
2. Bit loading process without imposing the maximum modulation order constraint for each user.
3. Iterative bit swapping.

The detailed explanation for the proposed processing steps is described as follows. The first step is the subcarrier allocation process, which is the same as the scheme in [1]. Quite a few heuristic algorithms can be utilized to achieve the task [8, 11, 13, 14]. We obtain the initial subcarrier allocation as the scheme in [1] before the procedure in considering the allocation for the spatially separable users.

Steps 2-3 are different from those in [1]. After the subcarrier assignment, the proposed scheme relaxes the maximum modulation order constraint in the bit loading process at steps 2 and 3 has to be proposed jointly, which are two points deviated from the scheme in [1]. Basically, the scheme in [1] performs subcarrier, bit, and power allocation in the first step as an OFDMA allocation scheme with one antenna case and then the power reduction is achieved by the neighbor search to allow the spatially separable users to share the same subcarriers. The bit and power allocation has the maximum modulation order constraint, whereas we relax the maximum modulation mode constraint in the second step and perform the greedy bit loading algorithm without imposing the constraint as:

Assume that user \( m \) uses subcarrier group \( J_m \). Denote the required received power for a particular BER as \( f(bc_m) \). The required transmit power is \( f(bc_m)/(s_{c}^{1})^2 \). Define the incremental power after adding two bits is \( \Delta p_{m}^{c} \). Set \( b_{m}^{*} = 0, c \in J_m \forall m \).

For each user \( m \):

**Step 1:** \( \Delta p_{m}^{c} = f(b_{m}^{*} + 2)/(s_{c}^{1})^2 - f(b_{m}^{*})/(s_{c}^{1})^2, c \in J_m \).

**Step 2:** Find \( c^{*} = \min \Delta p_{m}^{c} \). If \( \sum_{c \in J_m} b_{m}^{c} < B_{m} \) then update

\[
\Delta p_{m}^{c^{*}} = f(b_{m}^{*} + 2)/(s_{c^{*}}^{1})^2 - f(b_{m}^{*})/(s_{c}^{1})^2 \quad \text{and} \quad b_{m}^{c^{*}} = b_{m}^{c^{*}} + 2; \quad \text{and repeat step 2, else go to step 3.}
\]

**Step 3:** Power allocation \( p_{m}^{c} = f(b_{m}^{c})/(s_{c}^{1})^2 \) for each subcarrier \( c \).

Finally, in the third step, we use a proposed iterative bit swapping algorithm for the relaxed bit assignment to reach the goal of the minimization of total transmit power as described below. We hope to reach the required data rate for each user as close as possible. The proposed scheme does not reduce the data rate for any user unless it is necessary for a certain condition. The processing order is set in a sense of assigned priorities to determine which action should be chosen at a time. Then, we update the related parameters in the processing for the next iteration. Define the incremental power after add-
ing two bits to subcarrier $i$ of user $m$ ($b^{i'}_{m} = b^i_{m} + 2$) is $\Delta p^{i'}_m = \sum_{m=1}^{M} ((I - D^{i'}F^{i'})^{-1}\Lambda^{i'}) - \sum_{m=1}^{M} p^i_m$ and the reduced power after removing two bits from subcarrier $j$ of user $m$ ($b^{j'}_{m} = b^j_{m} - 2$) is $\Delta p^{j'}_m = \sum_{m=1}^{M} p^j_m - \sum_{m=1}^{M} ((I - D^{j'}F^{j'})^{-1}\Lambda^{j'})$, where $D^{i'}$ and $\Lambda^{i'}$ is calculated by bit vector $[b^i_1, b^i_m, \ldots, b^i_M]^T$.

This scheme along with the iterative bit swapping is summarized as the following diagram in Fig. 1. Referring to Fig. 1, we set the processing order as indicated in the diagram. The bit swapping operation to reduce total power has the highest priority. The bit swapping operation to reduce the number of subcarriers which exceed the maximum modulation mode constraint has the second priority. The operation of reducing the data rate for a user who exceeds the maximum power constraint has the third priority. The operation of decreasing the number of the bits at the subcarrier violating the maximum modulation order constraint has the fourth priority. Finally, we check if any user can be loaded with additional bits. Note that, if one meets many choices in one operation, the case which can reduce the maximum transmit power or increase the minimum transmit power over all users is selected. We repeat the operations until exit. The iteration stops if there is no condition specified in the diagram being satisfied. Note that, if it does not converge, it means one of the conditions would exist. However, during the iteration process, once the satisfactory conditions are met, the assignments are continuously adjusted and none of the conditions would exist after a few iterations.

As the adopted method of the subcarrier allocation is the same and the proposed iterative bit swapping procedures also include the neighbor search operation, these procedures in our proposed scheme will guarantee the performance of the proposed scheme as good as the method in [1] in the case of a low data rate request. Therefore, in the simulation, instead of being compared with the scheme in [1], we compare the performance of our proposed algorithm with the multi-user greedy algorithm [5] which is proposed for DMT-based DSL systems. The multi-user greedy algorithm assigns one bit at a time by choosing the case of the minimum incremental power associated with a user and a subcarrier. The method does not perform the assignment by jointly considering the total required numbers of bits for loading and the channel conditions for all users at a time. It may lead that the remaining bits for latter loading may be forced to be loaded into the channels with bad conditions.

Regarding the overhead issue, the signaling rate can be reduced by applying the grouping/sub-channelisation concept, where the grouping/sub-channelisation consists of a couple of subcarriers. The proposed scheme can be extended to solve the subchannel/channel-based allocation problem in terms of subchannels/channels for the problem formulation. The allocation unit, instead of one subcarrier, can be one subchannel/channel, which might be composed of a few subcarriers within a coherence bandwidth. The BER expressions for the required power are adjusted accordingly if the allocation unit becomes a subchannel/channel. The setting on the switching levels of the modulation types is beyond the scope. There are some research works dealing with the switching level of the modulation order problems. The system should be in a static or a slow time varying condition such that the signaling may not be sent out frequently. A blind detection technique can be observed in [15] without knowing modulation modes. The scheme of adaptive modulation with predicted wireless channels can be found in [16].
Precoding MIMO OFDM systems are considered in quite a few published papers. Further investigation may be followed to solve various issues, such as codebook-based or compression-based methods to reduce the overhead of reporting channel information. A design for an efficient feedback information scheme such as using compression falls in another research area. The update does not occur quite often due to either quite static or very slow time-varying channels for the applications. Besides, some papers deal with the prediction of mobile radio channels [17] and the long range prediction [18]. These techniques can be utilized in determining the future channel state information (e.g. utilizing the modified iterative autoregressive equation with various numbers of the prediction steps if using [18]). Besides, the system may send the coefficients of an AR model instead of channel gains to reduce the signaling bandwidth.
4. SIMULATION RESULTS

In this section, we present the simulation results to demonstrate the efficiency of the proposed allocation algorithm. Instead of being compared with the scheme in [1], we compare the performance of our proposed algorithm with that of the multi-user greedy algorithm [5], which is adapted for the wireless communication application herein. The users are randomly distributed with a uniform distribution within the cell and the radius of the cell is 1000 m. The path loss gain is $-58.5$ dB at the reference distance of 10 m and no user is generated to be located within the reference distance. The path loss model: $PL(d) = PL(d_0) + 10n\log(d/d_0) + X_c$ is adopted, where $d_0$ is the reference distance, $n$ is the path loss exponent, and $X_c$ is a zero-mean Gaussian distributed random variable in dB. The path loss exponent is set to be 3.5 and the standard deviation of the lognormal shadowing fading is 7 dB. The maximum transmit power for one user is 30 dBm. The multi-path model follows the COST207 typical urban 6-ray channel model with the propagation delays of [0.0, 0.2, 0.5, 1.6, 2.3, 5.0] measured in $\mu$s and the normalized path power of [0.1894, 0.3788, 0.2386, 0.0947, 0.0606, 0.0379] [10]. There are 64 subcarriers in the OFDM system with subcarrier spacing of 31.25 kHz and the noise power at each subcarrier is $-131.9$ dBm per antenna. The OFDM symbol period which includes a cyclic prefix duration of 8 $\mu$s is 40 $\mu$s. A linear antenna array consists of five-wavelength spacing elements at the base station and of one-wavelength spacing elements at each user. The transmission and the reception angle spreads $\theta^T_m$ and $\theta^R_m$ of each multi-path for each user are 25 degrees. The desired BER is $10^{-3}$ for every subcarrier. There are two or four antennas in each user and six or eight antennas in the base station.

We compare the performance by varying different numbers of antennas where there are two, four, and eight users. To ensure fair comparison, the total number of transmitted bits during an OFDM symbol is set. We assume six modes: [128, 256, 384, 512, 640, 768] which is defined in bits per OFDM symbol for two and four users. We assume nine modes: [128, 256, 384, 512, 640, 768, 896, 1024, 1152] which is defined in bits per OFDM symbol for eight users. Each user has an equal transmit data rate. In Figs. 2-4, we compare the

![Fig. 2. Percentage of reached data rates versus requested total data rates with the number of users = 2.](image)

In Figs. 5-7, we compare the performance in terms of the average power per bit between them, where both of them use the same number of antennas for the numbers of users = 2, 4, and 8. Note that, the term “requested rate” in Figs. 2-4 means the user’s required data rate where the allocation scheme of the system tends to achieve. Due to bad
channel conditions, the required data rate may not be achievable. The “reachable total data rate” in the x-axis of Figs. 5-7 means the data rate which the system can offer in the operational environment. It shows that our proposed algorithm can use less power than the greedy algorithm except in the region of the higher requested data rate modes for two users. However, in the region, the proposed algorithm requires more power due to providing higher success rates to meeting the required data rate, which leads to total higher date rates. The proposed scheme is able to load more bit in bad channel conditions but
the required transmit power is not linear and higher, whereas the multi-user greedy method fails in these channel scenarios and the total power calculation does not include these cases of bad channel conditions.

5. CONCLUSIONS

In this paper, we propose a bit and power allocation scheme for multiuser MIMO OFDM systems with subcarrier reuse. The proposed scheme overcomes the limited affordable data rate problem of the existing algorithm [1] and an additional maximum power constraint is imposed on the proposed scheme. The performance of the proposed allocation scheme is improved as the number of antennas increases, whereas the existing algorithm [1] can not offer higher total data rates even thought the number of antennas and the available power increase. Our proposed algorithm uses less transmitting power and achieves higher total data rates compared with the multi-user greedy algorithm of a high computational complexity.

REFERENCES


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