This paper presents a novel variational and Partial Differential Equation (PDE)-based method to link edge points into closed contours. First, an ending point restrained energy function, which improves the one used in the Gradient Vector Flow (GVF) Snake model, is proposed to derive an ending point restrained Gradient Vector Flow (ERGVF) field. When there are broken edges in the edge map, the GVF field cannot recover the object’s original shape, especially the broken corners of the object’s boundary. The new ERGVF field can solve this problem by diffusing the gradient vectors in the continuous edge regions of the edge map into the broken edge regions. To detect the linked edge map based on the ERGVF field, this paper proposes a vector-based Mean Vector Difference (VMVD) method, which optimizes the original Mean Vector Difference (MVD) method by employing a new vector-based formula. The VMVD method has better performance than the MVD method when coping with images with broken edges. The proposed contour closure method can link broken edges belonging to multiple objects simultaneously and recover the objects’ original shapes, especially the corners. Synthetic images with different edge structures and real cell images have been tested to show its validity and effectiveness in closed contour extraction and shape recovery.

Keywords: partial differential equation, gradient vector flow, mean vector difference, closed contour extraction, shape recovery

1. INTRODUCTION

Closed contour extraction is a fundamental procedure in image segmentation and computer vision applications [1-3]. To extract the contour of an object in an image, edge detection is usually the first step. Most edge operators usually apply partial derivatives to the image. Due to the low contrast in some regions of the image, there may be a few gaps on the contour of the object in the detected edge map [3]. These small gaps make the object’s contour unclosed. To obtain high quality edge maps and objects’ closed contours, edge linking techniques need to be applied to link the broken edge segments. Although linking broken edges seems to be simple for human vision, it is a difficult task to be performed automatically by computers [4].

Various methods have been proposed to address the edge linking problem, such as adaptive morphological method [5], smart routing method [6], cost-based method [4, 7],...
and ant-based method [8]. In general, these methods first find the edge gaps and label the ending points after edge detection. Then the ending points are connected by different approaches. Zhu et al. [9] proposed a method to model the edge map as a potential force field and used a potential function to direct edge linking. Farag and Delpc [10] presented a sequential search method, which treats the edge linking as a graph search problem. In this method, each edge point is associated with its corresponding gradient information (direction and magnitude) to form a search graph containing arcs between ending points, and then a search algorithm is applied. Promising experimental results were presented in the method, but many image-dependent parameters must be tuned.

Ghita and Whelan [11] proposed linking ending points based on local information. They defined a cost function that has a minimum value in the linking path. The method proposed by Wang and Zhang [12] incorporates geodesic distance into the cost function. Sappa and Vintimilla [4] also used the cost-based method. This method introduces the graph theory into edge linking, and obtains good results for closed contour extraction. However, the cost function still has to be improved. When the broken edges appear in the corners of the object’s boundary, most of the above cost-based methods may link the ending points in an optimal path resulting in the loss of corner information.

Since 1990s, the variational and PDE-based methods have attracted much attention in image processing and computer vision [13, 14]. Active contour or Snake model, introduced by Kass et al. [15] in 1988, is one of the most powerful variational models in the literature. The Gradient Vector Flow (GVF) Snake, proposed by Xu and Prince [16] in 1998, introduces a new external force that extends the gradient vector field of the edge map in the Snake model. Since then, the active contour models have been improved in different ways [17-24]. The Snake model, which uses a moving closed contour to detect the object’s boundary, can be used in contour closure applications. However, the classical Snake model cannot deal with the case when the image contains multiple objects with broken edges. Moreover, this method cannot recover the broken corners of the objects.

Recently, Yang-Mao et al. [25] proposed a Mean Vector Difference (MVD) method based on the GVF field to enhance the weak edges in segmentation of cell images. Inspired by the GVF Snake model and the MVD method, we present a new contour closure method in this paper. One of the main contributions of this paper is the development of a variational and PDE-based method to cope with the edge linking problem. Instead of linking broken edges directly in the edge map, the proposed algorithm performs the main linking procedure in the gradient vector field of the edge map. In addition, we propose an improved MVD method to detect edges based on the gradient vector field. Although several improved GVF Snake models have been developed to address the weak edge leakage problem [20, 22, 23], our method is different from them in two aspects. First, our method “recovers” the original GVF vectors in the broken edge regions, and hence is able to recover the broken corners of the object. The methods in [20, 22, 23] may not have this ability. Second, our method does not use the Snake model but the GVF vectors to detect the object contours.

Our method is based on an ending point restrained energy function, newly proposed in this paper, which improves that used in the GVF Snake model [16]. By minimizing the energy function, the gradient edge map becomes a new dense vector field called ending point restrained Gradient Vector Flow (ERGVF) field. This field diffuses the gradient vectors of the continuous edge region into the broken edge region in the edge map so
that the edge gaps are filled. Compared with the GVF field, the ERGVF field can recover the object’s original shape, especially the corner information, in the image with broken edges. To detect the edges based on the ERGVF field, we improved the MVD method [25] and developed a vector-based MVD (VMVD) method. The proposed contour closure method can not only link edge segments belonging to multiple objects simultaneously, but also recover the object’s original shape. It is suitable for closed contour extraction and shape recovery applications.

The rest of this paper is organized as follows: Section 2 describes the proposed contour closure method in detail. Section 3 analyzes the performance of the proposed method and shows some applications in segmentation of real cell images. Conclusions and future work of this paper are presented in Section 4.

2. THE PROPOSED METHOD

This paper aims at linking broken edges in the one-pixel thickness edge map of an image. We assume that the input of the proposed method is a binary edge map, which can be viewed as the edge detection result of a real image. Many effective edge detectors such as the Sobel edge detector and the Otsu thresholding technique [26] can be used to obtain the binary edge map. In the binary edge map, edges are assigned with value 1 (white), and the background with value 0 (black). The framework of the proposed method is illustrated in Fig. 1. First, a morphological thinning algorithm [27] is applied to the input binary edge map to obtain the one-pixel thickness edge map, and the corresponding gradient edge map is calculated as well. Second, the ending points are labeled to construct an ending point restrained map. Then, an ending point restrained energy function is proposed to derive the ERGVF field. Finally, an improved MVD method (VMVD) is performed to detect the linked edges based on the gradient vectors in the ERGVF field.

2.1 Ending Point Labeling

In order to obtain the one-pixel thickness edge map $f$, this paper first adopts morphological thinning [27] to the original binary edge image (or called edge map). After that, a labeling algorithm is performed to label the ending points of $f$. We define an ending point as the edge pixel with only one 8-connected neighbor. Each ending point is marked with the broken edge direction according to its 8-connected neighbor. The broken edge direction (namely ending point direction) represents the direction along which the edge is broken. Eight different ending point directions are defined as N, E, S, W, NE, SE, NW, and SW, which are similar to those defined in [9]. The ending point labeling
process is as follows: The algorithm scans each edge pixel in the edge map, if an edge pixel has only one 8-connected neighbor pixel, then this pixel is labeled as an ending point, and its ending point direction is marked according to its 8-connected neighbor pixel’s position.

2.2 Ending Point Restrained GVF Field

The classical active contour model [15] has the drawbacks of limited capture range and poor convergence of boundary concavities. In order to solve these problems, Xu and Prince [16] proposed a new external force field, called GVF field, to extend the original gradient edge map. The GVF is a vector field \( V(x, y) = [u(x, y), v(x, y)] \) that minimizes the energy function below:

\[
E = \int \mu \| \nabla V \|^2 + \| \nabla f \|^2 \| V - \nabla f \|^2 \, dxdy
\]

(1)

where \( \nabla f \) is the gradient of the original edge map, \( \mu \) is a regularization parameter, \( V(x, y) \) is the GVF field to be solved. The Euler equation to solve the GVF field is

\[
\mu \Delta u - (u - f_x)(f_x^2 + f_y^2) = 0
\]

\[
\mu \Delta v - (v - f_y)(f_x^2 + f_y^2) = 0
\]

(2)

where \( \Delta \) is the Laplacian operator.

It should be noted that \( \| \nabla f \| \) in Eq. (1) is large in the edge regions and hence the energy function (1) can be minimized by the second term when setting \( V = \nabla f \). In the non-edge regions, \( \| \nabla f \| \) is small and hence the energy function (1) is dominated by the first term, which results in a slowly-varying field [16]. Examples of the GVF field are shown in Fig. 2. Each point in Fig. 2 (c) corresponds to a pixel in the edge map, and the vector at that point corresponds to the GVF vector of that pixel. As shown in Fig. 2 (c), the gradient vectors near the edge pixels flow into the edge line. While in the non-edge regions, the gradient vectors are almost emanative or parallel without pointing to any single pixel. Consequently, the edge in the GVF field is similar to a river into which the neighbor vectors flow [25].

However, the drawbacks of the GVF field appear when the edge is broken into several segments. The GVF vectors in the broken edge regions all flow into the ending points instead of the original unbroken edge points, as shown in Fig. 2 (c). Can we recover the gradient vectors of the original unbroken edge regions or extend the gradient vectors of the continuous edge regions into the broken edge regions? Inspired by this idea and based on the GVF energy function (1), this paper proposes an ending point restrained energy function to extend the edge map’s gradient vectors. In this function, we introduce a new term \( G \), called an ending point restrained map, into the GVF energy function (1). This new term \( G \) makes the gradient vectors diffuse in a new way so that the gradient vectors in the broken edge regions can be recovered according to those in the continuous edge regions. The ending point restrained energy function is

\[
E' = \int \mu \| \nabla V_{ex} \|^2 + |G(x,y) \cdot \nabla f|^2 \| V_{ex} - G(x,y) \cdot \nabla f \|^2 \, dxdy
\]

(3)
where $G$ is the ending point restrained map, $V_{es}(x, y) = [u(x, y), v(x, y)]$ is the new vector field to be solved, $\nabla f$ is the gradient of the one-pixel thickness edge map $f$ obtained above, and $\mu$ is a regularization parameter. We call the new vector field $V_{es}(x, y)$ the ending point restrained GVF field (ERGVF). The new term $G$ is introduced to restrain the gradient near the ending points in $\nabla f$. We will describe how to obtain $G$ in detail later.

According to the deduced procedure of the GVF [16], using calculus of variations [28], the Euler equations to solve the ERGVF field are

\[
\mu \Delta u(x, y) - [u(x, y) - G(x, y) \cdot f_x(x, y)] \cdot |G(x, y) \cdot \nabla f|^2 = 0
\]

\[
\mu \Delta v(x, y) - [v(x, y) - G(x, y) \cdot f_y(x, y)] \cdot |G(x, y) \cdot \nabla f|^2 = 0,
\]

\[
|G(x, y) \cdot \nabla f|^2 = [G(x, y) \cdot f_x(x, y)]^2 + [G(x, y) \cdot f_y(x, y)]^2.
\]
By treating $u$ and $v$ as time varying functions

$$u = u(x, y, t),$$
$$v = v(x, y, t).$$  \hspace{1cm} (6)$$

Eq. (6) can be solved using the steepest decent method [29]. Due to the scalar product in $G(x, y) \cdot f_x(x, y)$ and $G(x, y) \cdot f_y(x, y)$, the numerical implementation of the equations is similar to that used in the GVF model, except that $f_x(x, y)$ and $f_y(x, y)$ are multiplied with $G(x, y)$, respectively. We refer the reader to [16] for details.

First, we describe how to obtain the ending point restrained map $G$ as follows. For the numerical implementation of $G$, we replace the Cartesian coordinates $x$ and $y$ respectively by the pixel indices $i$ and $j$ for convenience, i.e., $G(x, y) = G(i, j)$. We define eight structural elements. Each element is a window, called restraining window $W_r$ consisting of $3 \times 3$ pixels. These pixels are assigned with different restraining coefficients: $\alpha_1, \alpha_2, \alpha_3,$ and 1, respectively, as shown in Fig. 3. Each restraining window is assigned with a direction corresponding to the ending point direction defined in Section 2.1. $W_r$ is used to restrain the gradient vectors’ magnitudes along the ending point direction. Therefore, we use eight different restraining windows to meet different ending point directions, respectively.

Then $G$ is a matrix with the same size as the original edge map, and it is defined as

$$G(i, j) = \begin{cases} 
\text{corresponding value in } W_r(k), & (i, j) \in W_r(k) \\
1, & \text{otherwise}
\end{cases}$$  \hspace{1cm} (7)$$

where $k$ is the marked label of the ending point, $W_r(k)$ is the structural element defined in Fig. 3 whose direction corresponds to the $k$th ending point’s direction. The position of $W_r(k)$ in $G$ is determined by its center pixel whose position is equal to the $k$th ending point’s position in $f$. Actually, $G$ is such a weighting matrix: the elements of the matrix $G$ are 1 except those around the ending points. The elements around the ending points are redefined by $W_r(k)$. Therefore, $G$ restrains the gradients near the ending points and pre-
serves the gradients in other regions of $\nabla f$.

Next, we analyze the behavior of the new energy function (3). In this function, $\nabla f$ is redefined by $G$ around the ending points. $G(x,y) \cdot \nabla f$ is large only in continuous edge regions. Thus, in continuous edge regions, the function is dominated by the second term, which has the minimum value when $V_{gb}(x,y) = G(x,y) \cdot \nabla f$. However, in other regions including the ending point regions and the non-edge regions, $G(x,y) \cdot \nabla f$ is small, and the function is dominated by the first term, which makes the vector field vary slowly. As a result, the vectors in the continuous edge regions are diffused into the broken edge regions, and the broken edges can be linked by the diffused vector field. Fig. 2 shows the comparisons between the GVF and the ERGVF fields.

In the first row of Fig. 2, the white edge is broken into two segments (the left part and the right part), and its GVF vectors in the broken edge region all flow into the ending points, as shown in Fig. 2 (c). On the contrary, the ERGVF vectors in the broken edge region preserve the vector flow in the continuous edge line instead of flowing into the ending points. In other words, the gradient vectors around the original unbroken edge line are “recovered” by the ERGVF, as shown in Fig. 2 (d). Furthermore, the ERGVF can recover the corner’s information of the broken edge, as shown in the second row of Fig. 2. The corner of the rectangle edge is broken, and the ERGVF field preserves the corner’s gradient vectors in the way that it diffuses the vectors around the perpendicular line segments into the broken corner regions. The last three rows of Fig. 2 show some results of the cell edge images, which further illustrate the properties of the ERGVF field. Due to the low intensity contrast of the original grayscale cell images, the contours of the cell cytoplasm are broken after edge detection, as shown in Fig. 2 (a). We can see that the broken contour information is “recovered” in the ERGVF field.

2.3 Vector-Based MVD Method

Recently, Yang-Mao et al. [25] proposed a MVD method for edge enhancement in cell images. Unlike classical methods, the MVD method extracts edges based on the GVF field. As the authors depicted, the edge in the GVF field is a set of pixels into which the GVF vectors flow as a stream. The MVD [25] method is briefly summarized as follows:

First, the MVD method estimates the direction of an edge pixel in its corresponding window consisting of $M \times M$ pixels. Four edge directions including $0^\circ$, $45^\circ$, $90^\circ$, and $135^\circ$ are defined, as shown in the first row of Fig. 4 where $m_{w} = 5$ [25] (similar definitions are also used by Zhu et al. [30]). For each pixel, the corresponding window centered at it is divided into two regions according to the edge direction, namely black and white regions, respectively. The first row of Fig. 4 shows the four cases of different edge directions as well as their corresponding black and white regions. Each square represents an image pixel. Then the edge direction $\theta_e$ is estimated as the one that has the maximum difference between the average pixel intensities of the black and white regions.

Second, the MVD value $f_{MVD}$ of $(i,j)$ is calculated by the following equation

$$f_{MVD}(i,j) = \frac{\sin(\theta_e - \theta_s) + \sin(\theta_s - \theta_e) + 2}{4} \times 255$$

(8)
where \( \theta_b \) and \( \theta_w \) are the average directions of the GVF vectors in the black and white regions, respectively. The MVD result is, to some extent, the possibility of a pixel belonging to the edge pixels.

Through the Eq. (8), the MVD method calculates the convergence of the gradient vectors beside the edge line at \((i, j)\). However, when applying it to the edge linking problem, there are some drawbacks of the MVD method: (1) In broken edge regions, the low intensity contrast makes the estimation of the edge direction unreliable; (2) The MVD value calculated in Eq. (8) is actually an angle-based calculation method, which calculates the convergence of the gradient vectors through their angles. In this way, the angles of the gradient vectors must be calculated beforehand, which needs the calculation of inverse trigonometric function and results in additional calculation steps.

To cope with the drawbacks of the MVD method, we employ a new formula to measure the convergence of the gradient vectors by their inner product in the Cartesian coordinates. We call the improved MVD method a vector-based MVD (VMVD) method.

First, we define two normal vectors \( \mathbf{N}_b \) and \( \mathbf{N}_w \) in the black and white regions, respectively. They are perpendicular to the edge line in each window, as shown in the second row of Fig. 4. Then, the VMVD value \( f_{\text{VMVD}} \) of pixel \((i, j)\) is defined as

\[
f_{\text{VMVD}}(i, j) = \frac{V_b \cdot \mathbf{N}_b + V_w \cdot \mathbf{N}_w + 2}{4} \times 255
\]

where \( V_b \) and \( V_w \) are the unit sum of the ERGVF vectors in the black and white regions, respectively. They are defined as

\[
V_b = \frac{\sum_{i=1}^{M} V_{bi}}{\sum_{i=1}^{M} V_{bi}}, \quad V_w = \frac{\sum_{i=1}^{M} V_{wi}}{\sum_{i=1}^{M} V_{wi}}
\]

**Fig. 4.** Four edge directions (first row) and their corresponding normal vectors (second row): (a) 0°, (b) 45°, (c) 90°, and (d) 135°.
where $V_{bi}$ and $V_{wi}$ are the ERGVF vectors of the $i$th pixel in the black and white regions, respectively. $M$ is the total number of the pixels in each region. $|\cdot|$ is the Euclidean norm of a vector. In Eq. (9), we only use the ERGVF vectors without calculating the angle of each vector. Let $(x_b, y_b)$, $(x_w, y_w)$, $(x_{Nb}, y_{Nb})$ and $(x_{Nw}, y_{Nw})$ be the coordinates of $V_b$, $V_w$, $N_b$ and $N_w$, respectively. The $(x_{sb}, y_{sb})$ and $(x_{sw}, y_{sw})$ defined in each edge direction are summarized as: $(0,1)$ and $(0,-1)$ in $0^\circ$ direction; $(\sqrt{2}/2,\sqrt{2}/2)$ and $(-\sqrt{2}/2,\sqrt{2}/2)$ in $45^\circ$ direction; $(1,0)$ and $(-1,0)$ in $90^\circ$ direction; $(\sqrt{2}/2,\sqrt{2}/2)$ and $(-\sqrt{2}/2,\sqrt{2}/2)$ in $135^\circ$ direction. Then the inner product can be implemented by direct multiplication of two vector’s coordinates as

$$ V_b \cdot N_b = x_b \times x_{Nb} + y_b \times y_{Nb}, $$

$$ V_w \cdot N_w = x_w \times x_{Nw} + y_w \times y_{Nw}. \tag{11} $$

Since there is no obvious edge direction in the broken edge regions, we estimate the edge direction as the one maximizing the VMVD value. That is, we calculate the $f_{VMVD}$ in each edge direction and choose the maximum to be the final $f_{VMVD}$ instead of estimating the priori edge direction:

$$ f_{VMVD}(i, j) = \max_{\theta = 0, 45, 90, 135} \{ (f_{VMVD}(i, j))_{\theta} \}. \tag{12} $$

Fig. 5 shows some comparisons between the MVD method and the VMVD method. Both the MVD and the VMVD methods worked on the same ERGVF field. The input images can be found in Fig. 2 (a) including synthetic edge images and cell edge images. We can see that, due to the false estimation of the edge direction in the broken edge regions, the MVD method does not work well in these regions. On the contrary, the VMVD method detects continuous and smooth edges that successfully brighten the broken edge regions.

![Fig. 5. (a) The MVD results of synthetic images; (b) the VMVD results of synthetic images; (c) the MVD results of cell images; (d) the VMVD results of cell images.](image-url)
After the VMVD result is obtained, we calculate the following

\[ f_r(i, j) = \lambda \times f_s(i, j) + (1 - \lambda) \times f_{MD}(i, j) \]  

(13)

where \( \lambda \) is a regularization parameter, \( f_s(i, j) \) is the intensity value at pixel \((i, j)\) of the original gradient edge map, and \( f_r \) is the new edge image in which the broken edge information has been recovered. To obtain the final binary edge map, we adopt the Otsu thresholding technique [26], which has been widely used in segmentation of intensity image with a two-model distributed histogram. Assume that \( T_s \) is the Otsu threshold, and \( r(i, j) \) is the intensity value at \((i, j)\) of the final edge map, then

\[ r(i, j) = \begin{cases} 1, & f_r(i, j) \geq T_s \\ 0, & \text{otherwise} \end{cases} \]  

(14)

3. EXPERIMENTAL RESULTS AND DISCUSSIONS

3.1 Contour Closure Results

The proposed method has been tested on various synthetic edge images with different edge structures including straight line, single object, and multiple objects with broken edges. These images can be viewed as the edge detection results of real images. Furthermore, we provide some results in segmentation of cervical cell images, which show potential applications of the proposed method. The algorithm was implemented in the Matlab R2011 software using a PC with CPU: Pentium(R) Dual-Core 2.60GHz, and RAM: 2GB. The size of the synthetic edge images is 128×128 pixels. The parameters set in this experiment are: \( \alpha_1 = \alpha_2 = \alpha_3 = 0.1, \mu = 0.2, m_p = 5, \) and \( \lambda = 0.5. \) In addition, these parameters are not very sensitive to different images, and if the input image changes, these parameters may still work well.

In order to investigate the performance of the proposed method, we compared it with the GVF Snake model [16], which can also be used in contour closure problem. Throughout the experiment, the initial circle (initial contour) of the GVF Snake model is placed in the center of the image, and the diameter is set to half of the image width.

Fig. 6 shows the results of the proposed method. A rectangle and a triangle with broken edges are shown in the top two rows of Fig. 6 (a), respectively. The broken edges appear at their borders and corners. For the proposed method, the broken edges are well linked to form a closed contour. Furthermore, the corners of the object’s original shape are also recovered. For the GVF Snake model, the edge gaps are also well filled, but the corner’s information is lost. Some results of multiple objects (one embedded in another) with broken edges are also shown in Fig. 6. Unfortunately, the GVF Snake model only links the edges belonging to a single object.

The last two rows of Fig. 6 (d) show the results of our method when the GVF Snake model cannot work well. Three objects are placed dispersely in the first image, and all the broken edges are well linked by the proposed method. The original shapes of the line, the “S” curve and the rectangle are all recovered. In addition, a complex helix curve is shown for edge linking. A complete and continuous helix curve is formed by using the
proposed method, as shown in Fig. 6 (f).

As shown in Fig. 6, the edge information in the broken edge regions is recovered, and the proposed method works well in the case when the object has a regular shape, such as the triangle and the rectangle. If the edges are irregular curves and the interval between two corresponding ending points is too large, the proposed method did not work so well.

![Fig. 6. (a) and (d) Original edge maps; (b) and (e) contour closure results of the GVF Snake model; (c) and (f) contour closure results of the proposed method.](image)

### 3.2 Applications in Cell Image Segmentation

The proposed method is useful for image segmentation, closed contour extraction, and shape recovery. In this section, we provide some contour closure results in segmentation of cervical cell images. These cell images are provided by the Traditional Chinese Medical Hospital of Hunan Province, China. The image size is approximately 256×256 pixels. Most of these cell images have obscure cell boundaries.

In cell image segmentation, the main task is to extract the closed contours of the nucleus and cytoplasm. The Edge Enhancement Nucleus and Cytoplasm Contour Detector (EENCC) [25] method is a recently proposed cell contour detector for single-cell image segmentation. However, when we used the EENCC method to cope with our tested cell images, it could not guarantee closed contours because a few obscure edges on the cell boundaries were not detected properly, as shown in Figs. 7 (a) and (b). In our experiments, we followed the EENCC method to detect the cell edges. In order to further brighten the weak edges, we slightly modified the EENCC method by replacing its MVD method with the VMVD method. However, there were still some small gaps on the cell contour, as shown in Fig. 7 (b).

The proposed contour closure method is useful for improving the performance of the EENCC method. As shown in Fig. 7 (c), the proposed method linked the broken edge segments of the cell, and then the cell contour became continuous. In addition, we compared the proposed method with the original EENCC method by providing the one-pixel thickness edge maps obtained by different methods, as shown in Figs. 7 (d) and (e). To
get the one-pixel thickness edge map, we applied morphological thinning [27] on the linked edge map (in Fig. 7 (c)) of the proposed method. We found that the EENCC method failed to detect some parts of the cell contour. On the contrary, our method filled up the gaps and made the cell contours closed, as shown in Fig. 7 (e). Then, we used the morphological functions in the Matlab R2011 software to remove the spurs in the one-pixel thickness edge map of the proposed method. Finally, the biggest closed loop was selected as the contour of the cytoplasm, and the small closed loop within the cytoplasm was selected as the contour of the nucleus, as shown in Fig. 7 (f).

![Fig. 7](image-url)  
(a) Cervical cell image; (b) edge maps obtained by the modified EENCC method; (c) Contour closure results of the proposed method; (d) one-pixel thickness edge maps obtained by the original EENCC method [25]; (e) one-pixel thickness edge maps obtained by the proposed method; (f) final cell contour extraction results.
3.3 Discussions on the Impact of Image Noises

The presence of image noises may strongly affect the performance of the proposed contour closure method. It is critical to apply an effective noise-removal filter on the original grayscale image before our method is used. Removing the image noises can significantly reduce the extra edges in edge detection. The trim-meaning filter of the EENCC method [25] has been used in Section 3.2. In addition, we investigated the performance of the non-local means filter [31], which has been proved to be effective for removing microscopic cell image noises in [24]. Since we did not have the ground truth of cervical cell image noises, the conventional “add noise and then remove it” [24] strategy did not work. In this section, we directly investigate the impact of image noises through the contour closure results.

![Cell images denoised by the trim-meaning filter [25]; cell images denoised by the non-local means filter [31]; corresponding contour closure results; contour closure results without any noise-removal filter.](image)

Fig. 8 shows the contour closure results with and without the noise-removal filters. We can find that the trim-meaning filter and the non-local means filter both reduce the extra error edges in the results, as shown in Figs. 8 (b) and (d). When the noise-removal filter was not used, there were a large number of extra error edges that result in poor performance in the contour closure results, as shown in Fig. 8 (e). In addition, there were still extra edges even after the noise-removal filter was used. This may be caused by the uneven illumination of the microscopic image. Because the extra error edges are mostly small and unclosed, we can solve this problem by eliminating the small and unclosed edges before the proposed method is performed. Furthermore, a few extra edges may not dramatically influence the performance of the proposed method. Since the aim of the cell segmentation is to extract the closed cell contours, the small and unclosed extra edges can be excluded in the final contour extraction step, as show in Fig. 7 (f).

4. CONCLUSIONS AND FUTURE WORK

This paper presents a novel contour closure method. Unlike classical methods, we employ the variational and PDE-based method to address the edge linking problem. An improved energy function is proposed to obtain the ERGVF field, which solves the influence of the ending points on the GVF field. In this new ERGVF field, the gradient vectors in broken edge regions of the original edge map are recovered. We also proposed
an improved MVD method (VMVD) to detect the linked edge map using the ERGVF vectors. The proposed method is suitable for linking broken edges belonging to multiple objects and recovering the object’s original shape, especially when there are broken corners in the image. Various experimental results validated the proposed method.

Our future work will focus on introducing other new terms, which may depend on the information of the original color or grayscale images, into the ERGVF energy function to deal with more complex scenes. We believe that the variational and PDE-based methods would attract more attention in edge linking and contour closure applications.

REFERENCES

15. M. Kass, A. Witkin, and D. Terzopoulos, “Snakes: Active contour models,” Interna-
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28. R. Courant and D. Hilbert, Methods of Mathematical Physics, InterScience Inc., NY, 1953.

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