Harmonic Signal Processing Method Based on the Windowing Interpolated DFT Algorithm

XIANGUI WU AND ANNA WANG
Department of Information Science and Engineering
Northeastern University
Shenyang, 110819 P.R. China
E-mail: waxiangui1980128@163.com

The discrete Fourier transform (DFT) has become a main method of the harmonic analysis because it can be easily implemented in embedded system, but the conventional DFT is afflicted by the spectral leakage and the picket fence effect (PFE) in the asynchronous sampling. In this paper, the frequency-domain estimation method of the harmonic parameters, which is based on the windowing interpolated DFT (IpDFT) algorithm, is considered. In the modulus-based windowing IpDFT algorithm, a novel approach of frequency estimation error caused by the asynchronous sampling is proposed, and it is obtained by using the dichotomy approach algorithm. The proposed method can be easily carried out to solve the high order equations by microcontroller. In order to reduce the other harmonic measurement error caused by the fundamental component spectral interference, the rectification formula of frequency estimation error is derived for the Blackman window. The feasibility and validity of the proposed methods are confirmed by computer simulations and actual experiments of multi-frequency signals.

Keywords: windowing interpolated discrete Fourier transform, harmonic estimation, spectral interference, frequency estimation error, dichotomy approach algorithm

1. INTRODUCTION

Recently, harmonic estimation has become a more serious issue because of the nonlinear loads growth in many scientific and engineering applications. Thus, real-time detection of the harmonic can provide a scientific basis to analyze the harmonic components. Harmonic estimation means the reliable measurement of frequencies, phases and amplitudes of every frequency components existed in the multi-frequency input signal [1, 2]. The harmonic estimation can be classified as time-domain (parametric) and frequency-domain (non-parametric) methods. The parameter estimation of a single tone (and several tones) from discrete-time observations was considered as a maximum-likelihood problem and the Cramer-Rao lower bounds were derived to reduce estimation errors by Rife and Boorstyn [3, 4]. The non-parametric methods, such as DFT algorithm, have the advantages of robustness towards signal model inaccuracies and the computational load is low [5, 6].

Because DFT algorithm has the particularly attractive characteristic to perform a rapid spectral analysis, it is a practical estimation method widely used in harmonic analysis [7, 8]. But this algorithm has some innate performance restrictions, such as the spectral leakage and the PFE, due to the asynchronous sampling and the finite measurement length [9, 10]. In the actual measurement system, to implement the synchronous sampling is very difficult, thus the errors which cannot be ignored are involved in the results. The calculated parame-
ters cannot satisfy the accuracy requirement of the harmonic analysis [11]. The windowing IpDFT algorithms have been presented to overcome these shortcomings in many literatures. To improve the precision of DFT algorithm, Jain et al. [12] presented the earliest interpolation algorithm that could rectify the calculated result of DFT and effectively improve the computing precision. Many interpolation algorithms were based on the concepts in [12]. Zhang et al. [9] proposed an algorithm of interpolating signal windowed with poly-items cosine, which can greatly increase the accuracy of DFT to meet the precision demand of harmonic measurement. Grandke [13] applied the Hanning window to reduce the spectral leakage and further enhanced the computing precision. Interpolation DFT techniques was investigated for real-time multi-frequency waveform analysis [14, 15]. Agrež [15] suggested a weighted multi-point interpolation of the DFT algorithm with the Hanning window to improve the amplitude estimation of the signal tones. To reduce the PFE, Belega [16] presented the algorithm based on multi-spectrum-lines interpolation DFT.

The common principle of these is that the spectral leakage can be reduced by windowing the signal in time-domain and the PFE can be reduced by interpolating in frequency-domain. According to this principle, many windows with good property have been adopted, and the multi-point IpDFT algorithms have been proposed.

Recently, study on the IpDFT mainly focuses on the three issues: firstly, a better window is chosen [17-20]; secondly, the more spectral lines are used [14, 15, 21]; thirdly, the novel approaches for solving the high order equation of the frequency estimation error are studied [22-24]. But the rapid side-lobe decaying and main-lobe width of a window are contradictory to each other. The computational complexity is greater according as the interpolation spectral lines increases, and it is not suitable for the embedded system. Some methods for solving the equation of frequency estimation error, such as the least square curve fitting, polynomial approximation and Chebyshev best approximation, were presented [22-24].

In this paper, the windowing IpDFT algorithm is discussed and a novel approach of the frequency estimation error is proposed, it can be accurately calculated by the dichotomy approach algorithm. Also, the measurement errors of the other harmonics caused by the spectral interference of the fundamental component are considered and the rectification formula of frequency estimation error is derived for the Blackman window.

2. THE WINDOWING IpDFT ALGORITHM

Let us consider a multi-frequency sine wave signal of the time-domain.

\[ x(t) = \sum_{n=1}^{M} A_n \sin(2\pi f_n t + \phi_n) \]  

(1)

where \(A_n, f_n\) and \(\phi_n\) are the amplitude, frequency and phase of \(m\)th harmonic, respectively; \(M\) is the number of harmonic components. The discrete sampled data can be obtained from the original continuous signal by using the sampling frequency \(f_s\) [23, 24]:

\[ x(nT_s) = \sum_{n=1}^{M} A_n \sin(2\pi f_n nT_s + \phi_n) \quad n = 0, 1, \cdots, N - 1 \]  

(2)

where \(N\) is the length of sampled data; \(T_s = 1/f_s\) is the sampling interval.
The harmonic estimation is usually based on the transform from the time-domain to the frequency-domain (DFT). The DFT of sampled data Eq. (2) is given by

$$X(k) = \frac{1}{2} \sum_{m=1}^{M} A_m \exp \left( j \pi W(k - \lambda_m) \right)$$

where $k = 0, 1, ..., N - 1$; $W(\theta) = (\sin(\pi \theta) / \sin(\pi \theta / N)) \exp(-j \frac{N-1}{N} \pi \theta)$; $\lambda_m$ is the harmonic component frequency divided by the frequency resolution $\Delta f = f_s / N$, and it is expressed by $\lambda_m = f_m / \Delta f = f_m NT_s$, namely, represents the $m$th harmonic frequency expressed in bins of the frequency axis. The second term in square bracket of Eq. (3) is due to the negative frequency component, and it is usually ignored. Thus, Eq. (3) can be rewritten as follows:

$$X(k) = \frac{1}{2} \sum_{m=1}^{M} A_m \exp \left( j \pi W(k - \lambda_m) \right).$$

Also, $\lambda_m$ can be divided in two parts by

$$\lambda_m = f_m / \Delta f = l_m + \gamma_m, \quad 0 \leq \gamma_m < 1$$

where $l_m$ and $\gamma_m$ are the integer and decimal parts of the normalized $m$th harmonic frequency respectively.

Because the fundamental frequency can be varied with time by several factors, it is very difficult to achieve the synchronous sampling. Thus, $\gamma_m$ is not zero and is also called the frequency estimation error. The harmonic spectrums are not placed at the integer bins of frequency axis in asynchronous sampling, and the DFT results of a signal are obtained only at integer values of frequency bins. So, the correct parameters of a signal cannot be obtained by DFT at this time. The accurate results can be obtained by determining $l_m$ and $\gamma_m$ separately (see Fig. 1). In the modulus-based windowing IpDFT, the main step for estimating the parameters is the position determination of the frequency estimation error on the frequency axis.

On the other hand, windows are applied to sampled data to reduce the spectral leakage due to the finite measurement length. The cosine windows are commonly characterized as a sum of weighted cosines. Its time-domain expression is as follows:

![Fig. 1. The DFT spectrum in the asynchronous sampling.](image-url)
\[ w_s(n) = \sum_{s=0}^{S_s} (-1)^s a_s \cos(2\pi s n / N) \quad n = 0, 1, \ldots, N-1 \]  
(6)

where \( S_s \) is the term number; \( a_s \) are the coefficients and has the following limitations.

\[ \sum_{s=0}^{S_s} a_s = 1, \quad \sum_{s=0}^{S_s} (-1)^s a_s = 0 \]  
(7)

The DFT of Eq. (6) can be expressed as the algebraic sum of Dirichlet kernel:

\[ W_s(k) = \sum_{s=0}^{S_s} (-1)^s a_s [D(k-s) + D(k+s)]/2 \]  
(8)

where \( D(\theta) = W(\theta) / N = (\sin(\pi \theta) / N \sin(\pi \theta / N)) \exp(-j \frac{N-1}{N} \pi \theta) \).  
(9)

According to the product theorem of Fourier transform, the DFT of sampled data truncated by a window is equal to the convolution of Eqs. (4) and (8).

\[ X_w(k) = \sum_{s=0}^{S_s} (-1)^s a_s [X(k-s) + X(k+s)]/2. \]  
(10)

The DFT spectrum of \( f_m \) neighborhood is obtained by Eqs. (4) and (5).

\[ X(l_m + b) = -\frac{j}{2} A_m e^{j\phi_m} W(b - \gamma_m) \]  
(11)

where \( b \) is a integer. By substituting Eq. (11) into Eq. (10), we can obtain Eq. (12).

\[ X_w(l_m + b) = -\frac{j}{2} A_m e^{j\phi_m} \sum_{s=0}^{S_s} (-1)^s a_s [W(b-s - \gamma_m) + W(b+s - \gamma_m)]/2 \]  
(12)

Because the actual frequency \( f_m \) is located between two spectral lines \( l_m \) and \( l_m+1 \), these are clearly the largest spectral lines that are located around the peak point of actual frequency.

Thus, we can consider the following ratio:

\[ \alpha = \left| X_w(l_m + 1) / X_w(l_m) \right| \]  
(13)

where \( | \cdot | \) indicates the magnitude of a complex.

From Eqs. (12) and (13), the high order equations on \( \alpha \) and \( \gamma_m \) can be given as follows.

<table>
<thead>
<tr>
<th>Windows</th>
<th>( S_s )</th>
<th>( a_0 )</th>
<th>( a_1 )</th>
<th>( a_2 )</th>
<th>( a_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangle</td>
<td>0</td>
<td>1</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Hanning</td>
<td>1</td>
<td>0.5</td>
<td>0.5</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Blackman</td>
<td>2</td>
<td>0.42</td>
<td>0.5</td>
<td>0.08</td>
<td>–</td>
</tr>
<tr>
<td>Blackman-Harris</td>
<td>3</td>
<td>0.35875</td>
<td>0.48829</td>
<td>0.14128</td>
<td>0.01168</td>
</tr>
</tbody>
</table>

Table 1. Coefficients of the several windows [23].
For example, when \( S_0 = 1; \ a_0 = a_1 = 0.5 \), namely, Hanning window is adopted, by substituting Eq. (12) into Eq. (13), we can obtain the following expression:

\[
\alpha = a_0\frac{\left| W\left(1 - \gamma_m\right) + W\left(1 - \gamma_m\right)\right|/2 - a_1\left| W\left(-\gamma_m\right) + W\left(-\gamma_m\right)\right|/2}{a_0\left| W\left(-\gamma_m\right) + W\left(-\gamma_m\right)\right|/2 - a_1\left| W\left(-1 - \gamma_m\right) + W\left(-1 - \gamma_m\right)\right|/2} = \frac{1 + \gamma_m}{2 - \gamma_m}
\]

where if \( N \gg 1, \ W\left(\theta\right) \approx \frac{N\sin(\pi\theta)}{\pi\theta} \cdot \exp(-\pi\theta). \) So, from the above equation, \( \gamma_m = \frac{2\alpha - 1}{1 + \alpha} \).

Similarly, for Rectangle window,

\[
\gamma_m(1 + \alpha) - \alpha = 0.
\]

For Hanning window,

\[
\gamma_m(1 + \alpha) + (1 - 2\alpha) = 0.
\]

For Blackman window,

\[
3\gamma_m^2(1 + \alpha) - 9\gamma_m^2\alpha - \gamma_m(37 + 28\alpha) + (84\alpha - 50) = 0.
\]

For Blackman-Harris window,

\[
(35.15688 + 18.72114)\gamma_m - 0.81963\gamma_m^2 - 1.39719\gamma_m^3 - 0.03063\gamma_m^4 + 0.02931\gamma_m^5 + 0.0018\gamma_m^6 - 0.00006\gamma_m^7 - \alpha(51.66 - 12.915\gamma_m - 4.90044\gamma_m^2 + 1.22511\gamma_m^3 + 0.11652\gamma_m^4 - 0.02913\gamma_m^5 - 0.00024\gamma_m^6 + 0.00006\gamma_m^7) = 0
\]

Eqs. (14) and (15) are very simple, but it is extremely difficult to solve the high order equations of the complex windows such as Blackman and Blackman-Harris etc. In this paper, these high order equations are solved by using the dichotomy approach algorithm. After calculating the value of \( \gamma_m \), we can obtain the accurate frequency and amplitude of \( m \)th harmonic component from Eqs. (5) and (12).

The phase can be obtained by

\[
\phi_m = \text{phase}[X_w(l_m)] - \pi\gamma_m.
\]

### 3. THE PROPOSED METHODS

#### 3.1 The Dichotomy Approach Algorithm

The high order equations of the frequency estimation error are solved by the dichotomy approach algorithm, and it is very simple to understand. The main principle is that if \( f(x) \) is continuous and is a monotone increasing (or decreasing) function in the range \([a, b]\), and \( f(a) * f(b) < 0 \), then the equation \( f(x) = 0 \) has the only real solution \( \xi \) in this range. This algorithm consists of the following steps:
1. Take the middle point $\xi_1 = (a+b)/2$ in $[a, b]$ and calculate $f(\xi_1)$. Since $\gamma_m$ takes a value in the range $[0,1)$, we firstly can take $\xi_1 = 0.5$. If $f(\xi_1) = 0$, then the solution $\xi$ is just $\xi_1$.

2. If $f(\xi_1) * f(a) > 0$, then take $a_1 = \xi_1$ and $b_1 = b$. If $f(\xi_1) * f(b) > 0$, then take $a_1 = a$ and $b_1 = \xi_1$.

3. Since $f(a_1) * f(b_1) < 0$, repeat the 1, 2 steps in the new range $[a_1, b_1]$. The range of $\xi$ is reduced as $b_1 - a_1 = (b-a)/2$. If $\xi$ is not equal to $\xi_2 = (a_1+b_1)/2$, then $\xi$ will exist in the new range $[a_2, b_2]$.

4. If this process is repeated $n$ times, then we will obtain $a_n < \xi < b_n$. We can find that if $a_n$ (or $b_n$) is the approximation of $\xi$, its error will be smaller than $(b-a)/2^n$.

These Eqs. (14)-(17) are continuous in $[0,1)$ and the feature of monotone increasing (or decreasing) is confirmed, e.g. the high order equation of Blackman window has the feature of monotone decreasing. Also, because the proposed method have to satisfy the condition $f(a) * f(b) < 0$, the range of ratio $\alpha$ is considered. For example, when Blackman window is adopted, this range is $(25/42, 42/25)$. When $\alpha$ calculated by Eq. (13) does not satisfy this range, we can change the sampled data length $N$ or use the other window. In fact, the algorithms reported by [23], [24] too contains the above problem in obtaining the inverse function on the frequency estimation error. The proposed method can avoid the difficulty of initial value choice in iterative process and reduce the computation complexity.

3.2 The Influence of Spectral Interference

The accuracy of the harmonic estimation obtained by the windowing IpDFT algorithm is affected by the spectral interference from the other harmonic components. The feature of a harmonic signal is as follows: firstly, the amplitude of the fundamental component is much larger than the other harmonic one; secondly, the amplitudes of odd harmonics are larger than even ones. These phenomena can be usually seen in electric power system. In power system, wide applications of power electronics based non-linear loads has produced a large amount of harmonic components, which deteriorates the quality of electric energy and greatly affects the safe and economical operation of power system and electric equipment. The harmonic analysis of power system can provide a scientific basis for electric signal processing and harmonic estimation. Because the fundamental component of electric signal is the largest, the other harmonic estimation error caused by the spectral interference of the fundamental component is very large.

Taking into account all properties of a window, such as the main-lobe width, the peak side-lobe level and the side-lobe decaying rate, the rectification formula of frequency estimation error of Blackman window is derived to reduce this error. From Eq. (4), the spectral interference of the fundamental component on the second harmonic can be expressed as the following:

$$X_i(k_2) = -j\frac{A_i e^{j\phi}}{2} W(k_2 - \lambda_i).$$  \hfill (19)

According to Eq. (10),

$$X_w(k_2) = \sum_{s=0}^{N_s} (-1)^s a_s [X_i(k_2 - s) + X_i(k_2 + s)]/2 = -j\frac{A_i e^{j\phi}}{2}$$  \hfill (20)
\[ \sum_{s=\pm 1} (-1)^s a_s [W(k_2 - s - k_1 - \gamma_1) + W(k_2 + s - k_1 - \gamma_1)] / 2. \]

When the Blackman window is adopted, Eq. (20) is derived as follows:

\[ X_{w_1}(k_2) = -\frac{j}{2} A_{1} e^{j\pi \gamma_1} \frac{N \sin(\pi \gamma_1)}{\pi} e^{j\pi \gamma_1} \frac{E^2(4a_0 - 3a_1) - 4a_0}{E(E^2 - 1)(E^2 - 4)} \]

(21)

where \( E = k_2 - k_1 - \gamma_1 \).

By Eq. (12),

\[ X_{w_1}(k_2) = -\frac{j}{2} A_{1} e^{j\pi \gamma_1} \frac{N \sin(\pi \gamma_1)}{\pi} e^{j\pi \gamma_1} \frac{4a_0 + \gamma_1^2 (-a_0 + 3a_2)}{\gamma_1^2 (1 - \gamma_1^2)(4 - \gamma_1^2)}. \]

(22)

From Eqs. (21) and (22),

\[ X_{w_1}(k_2) = \frac{\gamma_1^2 (1 - \gamma_1^2)(4 - \gamma_1^2)}{4a_0 + \gamma_1^2 (-a_0 + 3a_2)} \frac{E^2(4a_0 - 3a_1) - 4a_0}{E(E^2 - 1)(E^2 - 4)} X_{w_1}(k_2). \]

(23)

Similarly,

\[ X_{w_1}(k_2 + 1) = \frac{\gamma_1^2 (1 - \gamma_1^2)(4 - \gamma_1^2)}{4a_0 + \gamma_1^2 (-a_0 + 3a_2)} \frac{(E + 1)^2(4a_0 - 3a_1) - 4a_0}{E(E^2 - 1)(E^2 + 2)(E + 3)} X_{w_1}(k_2). \]

(24)

The second harmonic spectrum calculated by the windowing IpDFT algorithm is equal to a sum of the fundamental component spectral interference and the actual second harmonic spectrum. Thus, Eq. (13) can be rewritten to reduce the spectral interference.

\[ \alpha = |X_{w_1}(k_2 + 1) - X_{w_1}(k_2 + 1)| / |X_{w_1}(k_2) - X_{w_1}(k_2)| \]

(25)

4. SIMULATION AND EXPERIMENTAL RESULT ANALYSIS

4.1 Comparisons with the Previous Algorithms

To verify the validity and feasibility of the proposed method, several simulations are executed, and the results are compared with the previous algorithms presented by [12, 13, 23, 24].

Firstly, the signal reported in [23] is simulated and its parameters are as follows: \( A_1 = 1 \) (normalized value), \( f_1 = 49.85 \text{ Hz}, \varphi_1 = 0.9 \text{ rad}; A_2 = 0.07, f_2 = 99.7 \text{ Hz}, \varphi_2 = 1.2 \text{ rad}; A_3 = 0.2, f_3 = 149.55 \text{ Hz}, \varphi_3 = 0.75 \text{ rad}; f_s = 1500 \text{ Hz}, N = 512. \) From the parameters, we can find that this sampling is clearly the asynchronous one. The simulation results of the proposed method and the results for Blackman-Harris window in [23] are given in Table 2.

Secondly, the signal which contains an inter-harmonic component is also simulated, and the amplitude differences of the harmonic components of this signal are very large. The proposed method is compared with some algorithms reported by [12, 13, 23], the parameters of the simulation signal and the results are shown in Table 3. The sampling parameters are \( f_s = 640 \text{ Hz} \) and \( N = 128. \)
Table 2. Comparison of the simulation results.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Conventional DFT</th>
<th>Blackman-Harris in [23]</th>
<th>Hanning in this paper</th>
<th>Blackman</th>
<th>Blackman-Harris</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_1$ [Hz]</td>
<td>$A_1$ [rad]</td>
<td>$f_2$ [Hz]</td>
<td>$A_2$ [rad]</td>
<td>$f_3$ [Hz]</td>
</tr>
<tr>
<td>Conventional DFT</td>
<td>49.8047 1.0004</td>
<td>0.9493 99.6094 0.0699</td>
<td>1.3039 149.4141 0.1989</td>
<td>0.8395</td>
<td></td>
</tr>
<tr>
<td>Blackman-Harris</td>
<td>49.8493 1.0000</td>
<td>0.9008 99.6982 0.0700</td>
<td>1.2023 149.5479 0.2000</td>
<td>0.7523</td>
<td></td>
</tr>
<tr>
<td>Hanning in this</td>
<td>49.8500 0.9999</td>
<td>0.8999 99.7600 0.0700</td>
<td>1.2000 149.5499 0.2000</td>
<td>0.7500</td>
<td></td>
</tr>
<tr>
<td>Blackman</td>
<td>49.8500 1.0000</td>
<td>0.8999 99.7600 0.0700</td>
<td>1.2000 149.5499 0.2000</td>
<td>0.7500</td>
<td></td>
</tr>
<tr>
<td>Blackman-Harris</td>
<td>49.8499 1.0000</td>
<td>0.9000 99.6999 0.0700</td>
<td>1.2000 149.5499 0.2000</td>
<td>0.7500</td>
<td></td>
</tr>
</tbody>
</table>

Table 3. Simulation comparison of several algorithms.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Rectangle (%)</th>
<th>Hanning (%)</th>
<th>Blackman in this paper (%)</th>
<th>Blackman-Harris in this paper (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000V</td>
<td>0.026</td>
<td>0.012</td>
<td>0.014</td>
<td>0.0025</td>
</tr>
<tr>
<td>50Hz</td>
<td>0.013</td>
<td>0.15</td>
<td>0.176</td>
<td>0.00033</td>
</tr>
<tr>
<td>10V</td>
<td>-0.18</td>
<td>0.83</td>
<td>-1.934</td>
<td>0.00016</td>
</tr>
<tr>
<td>68.125Hz</td>
<td>-0.068</td>
<td>0.12</td>
<td>0.295</td>
<td>0.00014</td>
</tr>
<tr>
<td>1V</td>
<td>9.8</td>
<td>9.8</td>
<td>8.694</td>
<td>0.027</td>
</tr>
<tr>
<td>150Hz</td>
<td>0.32</td>
<td>-0.31</td>
<td>0.375</td>
<td>0.002</td>
</tr>
<tr>
<td>0.1V</td>
<td>28.0</td>
<td>25.0</td>
<td>30.436</td>
<td>0.022</td>
</tr>
<tr>
<td>250Hz</td>
<td>0.33</td>
<td>0.23</td>
<td>0.525</td>
<td>0.00096</td>
</tr>
</tbody>
</table>

From Table 3, we can find that when the simulation signal contains the closely spaced harmonic components with large amplitude differences, although the sample number is not very large, estimation results are close to the actual values. But the result errors of the inter-harmonic component are larger, because the proposed method is based on the DFT and short length of sampled data reduces the frequency resolution, the accuracy is not too high. Especially, when the Rectangle window is adopted, the estimation errors are the largest. On the other hand, because the amplitudes of other harmonic components are too small, these are embedded by the spectral leakage of the fundamental component that has large amplitude. Therefore, the windows with better side-lobe features are needed.

Thirdly, the signal given by [24] is analyzed ($f_1 = 50$ Hz, $f_s = 3000$ Hz, $N = 1024$) and all the parameters are shown in Table 4. This signal has been used to verify the validity of algorithms in many references. The amplitudes $A_m$ were measured in the actual measurement system, and the phases $\phi_m$ were arbitrarily chosen.

Table 4. Parameters of the simulation signal.

<table>
<thead>
<tr>
<th>Harmonic order</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
<th>6th</th>
<th>7th</th>
<th>9th</th>
<th>11th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplitudes $A_m$ [V]</td>
<td>240</td>
<td>0.1</td>
<td>12</td>
<td>0.1</td>
<td>2.7</td>
<td>0.05</td>
<td>2.1</td>
<td>0.3</td>
<td>0.6</td>
</tr>
<tr>
<td>Phases $\phi_m$ [°]</td>
<td>0</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
<td>80</td>
<td>100</td>
</tr>
</tbody>
</table>

The results of the proposed method, DFT and the 6-order cosine window in [24] are compared respectively and they are shown in Table 5.

From the results of Tables 2 and 5, we can find that the accuracy of the conventional DFT is very low and especially, its phase error is very large in the asynchronous sampling. Also, the second harmonic errors are large. In the simulation signals, the amplitudes of the even order harmonics are very small, and those are easily affected by the spectral interference of the odd order harmonics with large amplitudes.
But the proposed algorithm can provide the accurate estimation of a multi-frequency signal including a very weak second harmonic component. The comparisons of the simulation result show that the accuracy of the parameters calculated by this algorithm is almost equal to the other algorithms.

### 4.2 Parameter Estimation of a Signal with Noise

The harmonic estimation error is also caused by the noise pollution. Thus, how to effectively suppress the influence of the noise signal on the measurement results is important. To estimate the performance of the proposed algorithm under the different noise conditions, simulation of the multi-frequency signal corrupted by white Gaussian noise is executed. The 11 order harmonics signal mentioned in section 4.1 is used (see Table 4). The simulation results of the amplitude and phase of the weak second harmonic component are shown in Figs. 2 and 3.

<table>
<thead>
<tr>
<th>Harmonic order</th>
<th>Frequency estimation $f_m$ [Hz]</th>
<th>Amplitude estimation $A_m$ [V]</th>
<th>Phase estimation $\phi_m$ [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>49.8046 50.0000 50.0000</td>
<td>237.88 240.000 240.000 240.000</td>
<td>12.0925 0.0000 0.00022 -0.0001</td>
</tr>
<tr>
<td>2nd</td>
<td>99.6093 99.9807 99.9976</td>
<td>1.0988 0.1000 0.1008 0.1000</td>
<td>170.6874 10.010 10.3002 10.1557</td>
</tr>
<tr>
<td>3rd</td>
<td>149.4141 149.999 150.000</td>
<td>10.935 11.999 12.000 12.000</td>
<td>59.3463 20.000 20.0034 19.9991</td>
</tr>
<tr>
<td>4th</td>
<td>199.2181 199.992 199.999</td>
<td>0.5756 0.1000 0.1001 0.1000</td>
<td>168.2137 30.001 30.1224 30.0802</td>
</tr>
<tr>
<td>5th</td>
<td>249.023 249.999 249.999</td>
<td>2.3979 2.7000 2.7000 2.7000</td>
<td>110.0265 40.000 40.0004 40.0058</td>
</tr>
<tr>
<td>6th</td>
<td>298.828 299.999 299.993</td>
<td>0.3857 0.0500 0.0500 0.0500</td>
<td>168.8865 50.000 49.9788 50.4393</td>
</tr>
<tr>
<td>7th</td>
<td>348.632 350.000 350.000</td>
<td>1.6832 2.0999 2.1000 2.1000</td>
<td>148.1192 60.000 59.9994 60.0109</td>
</tr>
<tr>
<td>9th</td>
<td>448.242 450.000 450.000</td>
<td>0.3422 0.3000 0.3000 0.3000</td>
<td>173.4164 80.000 79.9977 80.0484</td>
</tr>
<tr>
<td>11th</td>
<td>547.851 550.000 550.000</td>
<td>0.2777 0.6000 0.6000 0.6000</td>
<td>195.5806 99.999 99.9992 100.001</td>
</tr>
</tbody>
</table>

The signal-to-noise ratio (SNR) is varied with an increment of 10 dB, from 10 to 90 dB. The relative errors by using the Blackman-Harris window are the lowest. For SNR < 30 dB, the effects of the white noise are significant. As shown in these figures, the amplitude errors are lower than the phase ones under the same noise condition, and the proposed method has a certain noise resisting ability.
4.3 Experimental Result Analysis

The proposed algorithm is also tested by a real measurement system. For this test, the multi-frequency signal whose range is 0–3.3 V, is supplied by the arbitrary waveform generator AWG 5002C and this signal is measured by the digital signal processor (DSP) development board LT 8503. The 32-bit microprocessor TMS320F2812PGFA is used, and this itself have a 12-bit analog-to-digital converter (ADC) whose maximum ADC clock is 25 MHz. The analog input signal is sampled by this ADC and the results are stored in 16-bit result registers. The XDS100USB DSP EMULATOR is also used. Take the sampling frequency \( f_s = 1500 \text{ Hz} \) and sampled data length \( N = 512 \). The experimental results measured by the proposed method and signal parameters are shown in Table 6. The experimental results are close to the expected values and this algorithm has a high accuracy practically.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( f_1[\text{Hz}] )</th>
<th>( A_1[\text{V}] )</th>
<th>( f_2[\text{Hz}] )</th>
<th>( A_2[\text{V}] )</th>
<th>( f_3[\text{Hz}] )</th>
<th>( A_3[\text{V}] )</th>
<th>( f_5[\text{Hz}] )</th>
<th>( A_5[\text{V}] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual values</td>
<td>50</td>
<td>1</td>
<td>100</td>
<td>0.1</td>
<td>150</td>
<td>0.2</td>
<td>250</td>
<td>0.15</td>
</tr>
<tr>
<td>Traditional DFT</td>
<td>49.8042</td>
<td>0.9927</td>
<td>99.6091</td>
<td>0.0954</td>
<td>149.413</td>
<td>0.1858</td>
<td>249.021</td>
<td>0.1235</td>
</tr>
<tr>
<td>Hanning in this paper</td>
<td>50.0124</td>
<td>0.9988</td>
<td>99.9897</td>
<td>0.1031</td>
<td>149.995</td>
<td>0.2013</td>
<td>249.995</td>
<td>0.1503</td>
</tr>
<tr>
<td>Blackman</td>
<td>50.0062</td>
<td>0.9991</td>
<td>99.9946</td>
<td>0.1014</td>
<td>149.997</td>
<td>0.2007</td>
<td>249.999</td>
<td>0.1500</td>
</tr>
<tr>
<td>Blackman-Harris</td>
<td>50.0037</td>
<td>0.9998</td>
<td>99.9985</td>
<td>0.1011</td>
<td>149.997</td>
<td>0.2001</td>
<td>249.999</td>
<td>0.1499</td>
</tr>
</tbody>
</table>

5. CONCLUSIONS

In this paper, the windowing IpDFT algorithm is discussed and a novel approach of the frequency estimation error formula is presented. This approach is easy to apply in the embedded system and its accuracy is also high. The interpolation high order equations are easily solved by the dichotomy algorithm. Also, in order to reduce the spectral interference on a weak harmonic component, the rectification formula of the frequency estimation error is derived for Blackman window.

The results of simulations and actual measurement ensure the validity of the proposed method, and show that this method can meet the accuracy requirement of harmonic analysis and has the high stability under the noise condition.

REFERENCES


Xiangui Wu (吴贤规) is a Ph.D. student in Department of Information Science and Engineering at Northeastern University, China. He received his Master degree at Kim Chaek University of Technology, D.P.R.Korea in 2006. His research focuses on electrical signal processing, harmonic detection and embedded system.

Anna Wang (王安娜) received her Ph.D. degree from Northeastern University, China. She is a Professor in Department of Information Science and Engineering of Northeastern University. Her research interests include electrical signal processing, fault diagnosis and embedded system.