Downlink Multiuser Beamforming and Power Control for Base Stations Empowered by Renewable Energy

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***Abstract*—This work examines offline and online power control policies for efficient usage of renewable energy in the downlink of a multi-antenna wireless system. Two multiuser beamforming schemes are considered, namely, channel inversion (CI) and maximal ratio transmit (MRT) beamforming schemes. Power control policies are derived for both schemes, respectively, with the goal of maximizing the sum throughput by a deadline subject to energy causality and battery storage constraints. With CI beamforming, the power control problem can be formulated as a convex optimization problem, whose solution can be obtained using the directional water-filling algorithm. With MRT beamforming, the power control problem becomes non-convex, due to interference between the signals intended for different users, and, thus, is difficult to solve exactly. However, an efficient solution can be obtained by performing successive approximation into a sequence of geometric programming problems, i.e., by employing the condensation method. Offline power control policies are first derived assuming non-causal knowledge of the energy arrival and channel coefficients over time. Online power control policies are then proposed based on observations gained from the offline policy. The performance of the proposed schemes are demonstrated through numerical simulations.**

I. Introduction

In recent years, small cell networks [1] have emerged as a promising solution to meet the increasing demand for high throughput and high date-rate wireless communications. However, the exponential increase in data traffic is accompanied by a rapid increase in energy and environmental costs [2]. Interestingly, the small cell architecture is not only energy efficiency but the low transmit power required also allows for the employment of renewable energy sources at the base-stations (BSs). However, the efficient use of renewable energy is often hindered by uncertainties of the energy arrival and limitations of the battery storage capacity.

The main objective of this work is to derive power control policies for efficient usage of renewable energy in the downlink of a multiantenna wireless system. We consider the use of a multiantenna BS that is powered by renewable energy through a rechargeable battery. The BS serves all users in the cell simultaneously and power control policies are derived to maximize the sum throughput by a deadline subject to the energy causality and the battery storage capacity constraints. The deadline can be viewed as either a QoS requirement or the time before users leave the BS coverage. The energy causality constraint refers to the fact that energy cannot be used before it arrives whereas the battery capacity constraint refers to the maximum energy that can be stored in the battery at any given time. Offline power control policies are first derived by assuming non-causal knowledge of the energy arrival and channel coefficients over time. Online policies are then derived based on observations made from the solution of the offline problem. Two multiuser beamforming schemes are considered, namely, the channel inversion (CI) [3] and the maximal ratio transmit (MRT) [4] beamforming schemes. In the CI beamforming case, the offline power control problem can be formulated as a convex optimization problem, whose solution is given by the directional water-filling algorithm [5]. In this algorithm, the energy in each slot is shared with later time slots to make the water-levels between consecutive time slots as even as possible. In the MRT beamforming case, the power control problem is non-convex due to interference between the signals intended for different users. This type of problem was shown to be NP hard in [6], but an efficient solution can be obtained using the condensation method [7], which approximates the problem successively into a sequence geometric programming (GP) problems. Based on the fact that energy should be shared with later time slots to even the water-levels between consecutive time slots, an online power control policy is proposed where power allocation is performed in each time slot with total power given by the average of the current available energy plus the energy arrival in the remaining time slots. The effectiveness of the proposed schemes are demonstrated through numerical simulations.

Power control policies for energy harvesting transmitters (i.e., transmitters empowered by renewable energy) was investigated recently in [5], [8], [9]. In [5], online and offline power control policies were derived for systems with only a single receiver. The directional water-filling algorithm was proposed to maximize the sum throughput by a deadline subject to energy arrival and battery storage constraints. This problem was shown in [8] to be closely related to the problem of minimizing the transmission completion time for a given amount of data. The latter problem was extended to the K-user AWGN broadcast channel in [9]. It was shown that the total power allocated to each time slot is the same as that in the single-user scenario and can be found using the directional water-filling algorithm. However, the transmitter is assumed to have only a single antenna in these works. Different from these works, we consider a multiantenna BS and derive the power control policy for different multiuser beamforming schemes.

II. System Model and Problem Formulation

Let us consider a downlink wireless cellular system that consists of a single base-station (BS) equipped with M antennas and K single-antenna users, as illustrated in Fig. 1. The BS is supported by a rechargeable battery that is continuously charged by a renewable energy source. Here, we consider a time-slotted system where all users are to be served simultaneously in each time slot.

Specifically, the signal transmitted in the t-th time slot is given by $\mathbf{x}[t]=\sum_{i=1}^{K}\sqrt{p_i[t]}\mathbf{w}_i[t]s_i[t]$ where $s_i[t]$ represents the signal intended for user i, $\mathbf{w}_i[t]$ is the associated $M\times 1$ beamforming vector, and $p_i[t]$ is the power used to transmit the signal to user i. Here, we set $\mathrm{E}[\|s_i[t]\|^2]=1$ and $\|\mathbf{w}_i[t]\|=1$. The received signal at user i is given by

$$y_i[t]=\mathbf{h}_i[t]^H\sqrt{p_i[t]}\mathbf{w}_i[t]s_i[t]+\mathbf{h}_i[t]^H\sum_{j\neq i}\sqrt{p_j[t]}s_j[t]\mathbf{w}_j[t]+n_i[t], \quad (1)$$

where $\mathbf{h}_i[t]$ is the $M\times 1$ channel vector between the BS and user i and $n_i[t]$ is the additive white Gaussian noise (AWGN) with zero mean and variance σ_n^2, i.e., $n_i[t]\sim\mathcal{CN}(0,\sigma_n^2)$. The first term in (1) contains the signal of interest for user i whereas the second term represents the interference from other signals. The received signal to interference plus noise ratio (SINR) can be expressed as

$$\gamma_i[t]\triangleq\frac{p_i[t]|\mathbf{h}_i[t]^H\mathbf{w}_i[t]|^2}{\sum_{j\neq i}p_j[t]|\mathbf{h}_i[t]^H\mathbf{w}_j[t]|^2+\sigma_n^2}. \quad (2)$$

The rate achieved by user i in the t-th time slot is given by $R_i[t]=\log(1+\gamma_i[t])$ and, thus, the total throughput achieved under the deadline constraint T is defined as

$$\sum_{t=1}^{T}\sum_{i=1}^{K}\log(1+\gamma_i[t]). \quad (3)$$

Our goal is to determine the optimal power control policy for a given multiuser beamforming scheme to maximize the sum throughput under a deadline constraint. The power utilized by the BS is subject to two renewable energy constraints, namely, *the energy causality constraint* and *the battery storage constraint*. The energy causality constraint refers to the fact that energy cannot be expended before it stored into the batter and the battery storage constraint refers to the fact that the amount of energy stored in the battery cannot exceed the maximum battery capacity, denoted by B_{max}. Let $\epsilon[t]$ be the energy arrival at the beginning of the t-th time slot and let $B_{in}[t]$ be the corresponding amount of energy actually stored into the battery[1]. Notice that $B_{in}[t]$ is used to model the fact that energy may not be stored if the battery capacity is exceeded and that $B_{in}[t]\leq\epsilon[t]$, for all t. The energy causality and the battery storage constraints are then given by

$$\sum_{t=1}^{\ell}\sum_{i=1}^{K}p_i[t]\leq\sum_{t=1}^{\ell}B_{in}[t],\quad \text{for } \ell=1,\cdots,T \quad (4)$$

and

$$\sum_{t=1}^{\ell+1}B_{in}[t]-\sum_{t=1}^{\ell}\sum_{i=1}^{K}p_i[t]\leq B_{max},\ \text{for } \ell=1,\cdots,T-1. \quad (5)$$

[1]In practice, energy does not arrival instantaneously. In this case, $\epsilon[t]$ can be viewed as the energy arriving in the previous time slot, i.e. slot $t-1$, but is not used until the t-th time slot. However, the instantaneous energy arrival at the beginning of each time slot is assumed here for tractability.

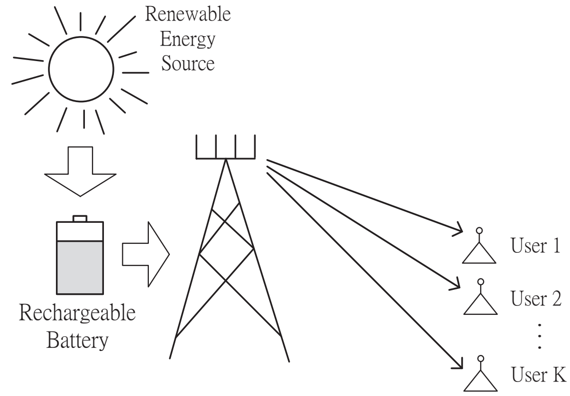

Fig. 1. System Model

Let us first consider the offline power control policy where the transmit powers $\{p_i[t],\forall i,t\}$ are determined by assuming non-causal knowledge of the energy arrivals $\{\epsilon_i[t],\forall i,t\}$ and channel conditions $\{\mathbf{h}_i[t],\forall i,t\}$. The problem is formulated as follows:

$$\max_{p_i[t],B_{in}[t],\forall i,\forall t}\ \sum_{t=1}^{T}\sum_{i=1}^{K}\log(1+\gamma_i[t]) \quad (6a)$$

$$\text{subject to}\quad \sum_{t=1}^{\ell}\sum_{i=1}^{K}p_i[t]\leq\sum_{t=1}^{\ell}B_{in}[t], \quad (6b)$$

$$\sum_{t=1}^{\ell+1}B_{in}[t]-\sum_{t=1}^{\ell}\sum_{i=1}^{K}p_i[t]\leq B_{max}, \quad (6c)$$

$$B_{in}[t]\leq\epsilon[t],\ p_i[t]\geq 0,\ \forall i,\forall\ell,\forall t. \quad (6d)$$

In the following lemma, we show that, in the offline optimization problem, the optimal energy stored in each time slot can actually be solved explicitly.

Lemma 1: For the offline problem in (6)*, the optimal amount of energy stored into the battery in the t-th time slot is given by $B_{in}^*[t]=\min\{\epsilon[t],B_{max}\}$, for $t=1,\ldots,T$.*

Due to space limitations, we provide only a sketch of the proof in the following. In particular, this lemma follows from the fact that the sum rate in each time slot increases monotonically with the total transmit power. To show this, let $\alpha_i[t]$ be the portion of power allocated to user i in the t-th time slot. In this case, the power used to transmit user i's message can be written as $p_i[t]=\alpha_i[t]p_{tot}[t]$, where $p_{tot}[t]$ is the total transmit power in slot t, and the sum rate is given by

$$R_{sum}[t]=\sum_{i=1}^{K}\log\left(1+\frac{\alpha_i[t]p_{tot}[t]|\mathbf{h}_i[t]^H\mathbf{w}_i[t]|^2}{\sum_{j\neq i}\alpha_j[t]p_{tot}[t]|\mathbf{h}_i[t]^H\mathbf{w}_j[t]|^2+\sigma_n^2}\right).$$

It follows straightforwardly from this expression that the sum rate $R_{sum}[t]$ increases monotonically with $p_{tot}[t]$.

Moreover, let $\{p_i^*[t], B_{in}^*[t], \forall i, \forall t\}$ be a solution to the offline power control problem in (6). Suppose that there exists t' such that $B_{in}^*[t'] < \min\{\epsilon[t'], B_{max}\}$. If the maximum battery capacity B_{max} was not met in slot t', a larger sum rate can be achieved by increasing both $B_{in}^*[t']$ and $p_{tot}^*[t']$ by the same amount without altering the sum rate in other time slots. On the other hand, if B_{max} was met in slot t, the power consumed in the previous time slot $t'-1$ can be increased instead to allow for an increase in $B_{in}[t']$. This achieves a larger sum rate in slot $t'-1$ without altering the sum rate in other slots. This contradicts the fact that $\{p_i^*[t], B_{in}^*[t], \forall i, \forall t\}$ is a solution for (6). Hence, the optimal solution must yield $B_{in}^*[t] = \min\{\epsilon[t], B_{max}\}$, for all t.

This lemma shows that the optimal power control should allow all energy arrivals to be stored completely into the battery unless the instantaneous energy arrival exceeds the maximum battery capacity, i.e., B_{max}. The offline power control problem then reduces to

$$\max_{p_i[t], \forall i, \forall t} \sum_{t=1}^{T}\sum_{i=1}^{K} \log(1+\gamma_i[t]) \tag{7a}$$

$$\text{subject to} \quad \sum_{t=1}^{\ell}\sum_{i=1}^{K} p_i[t] \le \sum_{t=1}^{\ell} B_{in}^*[t], \tag{7b}$$

$$\sum_{t=1}^{\ell+1} B_{in}^*[t] - \sum_{t=1}^{\ell}\sum_{i=1}^{K} p_i[t] \le B_{max}, \tag{7c}$$

$$p_i[t] \ge 0, \ \forall i, \forall \ell, \forall t. \tag{7d}$$

where $B_{in}^*[t]$ is given as in Lemma 1.

In the following sections, we first find solutions to the offline power control problem for two different beamforming schemes, namely, the channel inversion (CI) and the maximal ratio transmit (MRT) beamforming schemes.

III. Power Control with Directional Water-filling for Channel Inversion Beamforming

In this section, the optimal power control policy is derived for the case of channel inversion (CI) beamforming [3].

Specifically, in the case of CI beamforming, the beamforming vector $\mathbf{w}_i[t]$ is obtained by normalizing the i-th column of $\mathbf{H}[t]^H(\mathbf{H}[t]\mathbf{H}[t]^H)^{-1}$, where $\mathbf{H}[t] = [\mathbf{h}_1[t], \ldots, \mathbf{h}_K[t]]^H$. It follows that, since $\mathbf{h}_i[t]^H \mathbf{w}_j[t] = 0$, for all $i \neq j$, no interference will be experienced at each user. The objective in (7) is thus given by

$$R_{\text{CI}}(\{p_i[t]\}) = \sum_{t=1}^{T}\sum_{i=1}^{K} \log\left(1+\beta_i[t]p_i[t]\right) \tag{8}$$

where $\beta_i[t] \triangleq |\mathbf{h}_i[t]^H \mathbf{w}_i[t]|^2/\sigma_n^2$ is the effective channel gain of user i in slot t. Since the objective function is concave in the power and the constraints in (7) are linear, the optimization problem is a convex optimization problem. Let us define the Lagrangian function [10]

$$\mathcal{L}(\{p_i[t]\}) = R_{\text{CI}}(\{p_i[t]\}) - \sum_{\ell=1}^{T} \lambda_\ell \left(\sum_{t=1}^{\ell}\sum_{i=1}^{K} p_i[t] - \sum_{t=1}^{\ell} B_{in}^*[t]\right) - \sum_{\ell=1}^{T-1} \mu_\ell \left(\sum_{t=1}^{\ell+1} B_{in}^*[t] - \sum_{t=1}^{\ell}\sum_{i=1}^{K} p_i[t] - B_{max}\right) - \sum_{t=1}^{T}\sum_{i=1}^{K} \nu_i[t] p_i[t], \tag{9}$$

where $\lambda_\ell \ge$, $\mu_\ell \ge 0$, and $\nu_i[t] \ge 0$, for all ℓ, i, and t, are the Lagrange multipliers associated with the first, second, and third constraints in (7). The complimentary slackness conditions are given by

$$\lambda_\ell \left(\sum_{t=1}^{\ell}\sum_{i=1}^{K} p_i[t] - \sum_{t=1}^{\ell} B_{in}^*[t]\right) = 0, \ \ell = 1, \cdots, T-1 \tag{10}$$

$$\mu_\ell \left(\sum_{t=1}^{\ell+1} B_{in}^*[t] - \sum_{t=1}^{\ell}\sum_{i=1}^{K} p_i[t] - B_{max}\right) = 0, \ \ell = 1, \cdots, T-1. \tag{11}$$

$$\nu_i[t] p_i[t] = 0, \ t = 1, \ldots, T, \ i = 1, \ldots, K. \tag{12}$$

Notice that the slackness condition is not included in (10) for $\ell = T$ since the corresponding constraint must be satisfied with equality, i.e., all energy should be expended by the deadline.

By applying the KKT optimality conditions to the Lagrangian function in (9), the optimal power allocation $p_i^*[t]$ can be given in terms of the Lagrange multipliers as

$$p_i^*[t] = \left(\frac{1}{\tau_t} - \frac{1}{\beta_i[t]}\right)^+, \tag{13}$$

for $i = 1, \ldots, K$ and $t = 1, \ldots, T$, where $\tau_t = \sum_{\ell=t}^{T} \lambda_i - \sum_{\ell=t}^{T-1} \mu_i$. Notice that, in time slot t, the power allocation among users is equivalent to a traditional water-filling solution with water-level equal to $1/\tau_t$. Hence, the power allocation can be performed in two steps, namely, by first determining the water-level in each time slot and then by computing the water-filling solution over different users. It can be shown, similar to that observed for the single user case in [5], that the water-level should be monotonically non-decreasing over time when there is no battery capacity constraint. This is due to the fact that, when $B_{max} = \infty$, the second constraint in (7) will never hold with equality. In this case, we should have $\mu_\ell = 0$, for all ℓ, and, thus, $1/\tau_t = 1/\sum_{\ell=t}^{T} \lambda_i$, for $t = 1, \ldots, T$, will form a non-increasing sequence. This implies that battery energy should be left for use in later time slots if future energy arrivals do not yield as good a performance as in earlier time slots. However, this is not completely the case when B_{max} is finite since the value of B_{max} will limit the energy that can be leaked to later time slots. The initial water-level at each time slot $1/\tau_\ell^{(0)}$ is found by traditional water-filling algorithm with $\sum_{i=1}^{K} p_i[\ell] = B_{in}^*[\ell], \forall \ell$. Following [5], the water-level associated with each time slot is found using the directional water-filling solution. In each iteration of the algorithm, say iteration k, we check for consecutive time slots ℓ and $\ell+1$ for which $1/\tau_\ell^{(k-1)} > 1/\tau_{\ell+1}^{(k-1)}$ and reset the water-levels so that $1/\tau_\ell^{(k)} = 1/\tau_{\ell+1}^{(k)}$ or to the point where the battery constraint B_{max} is met in slot $\ell+1$.

IV. Power Control with Geometric Programming for Maximal Ratio Transmit Beamforming

In this section, the optimal power control policy is derived for the case of maximal ratio transmit (MRT) beamforming.

Specifically, for MRT beamforming, the beamforming vector for user i is given by $\mathbf{w}_i[t] = \mathbf{h}_i[t]/\|\mathbf{h}_i[t]\|$ [4]. Here, the beamformer for user i's message is matched to the channel towards user i. This maximizes the signal energy at the intended receiver but does not avoid interference to other users. In this case, the objective in (7) can be written as

$$R_{\mathrm{MRT}}(\{p_i[t]\}) = \sum_{t=1}^{T}\sum_{i=1}^{K}\log\left(1+\frac{p_i[t]|\mathbf{h}_i[t]^H\mathbf{w}_i[t]|^2}{\sum_{j\neq i}p_j[t]|\mathbf{h}_i[t]^H\mathbf{w}_j[t]|^2+\sigma_n^2}\right). \quad (14)$$

Notice that, with (14) as the objective function, the optimization problem in (7) is no longer a convex optimization problem and has been shown in [6] to be NP-hard. However, the problem can be approximated by a sequence of geometric programming problems using the condensation method [7].

Specifically, let $f_i(\mathbf{p}[t],t) \triangleq \sum_{j\neq i}p_j[t]|\mathbf{h}_i[t]^H\mathbf{w}_j[t]|^2+\sigma_n^2$, $g_i(\mathbf{p}[t],t) \triangleq \sum_{j=1}^{K}p_j[t]|\mathbf{h}_i[t]^H\mathbf{w}_j[t]|^2+\sigma_n^2$, and$f(\mathbf{p}[t]) = \sum_{t=1}^{\ell}\sum_{i=1}^{K}p_i[t]$. Then, the optimization problem in (7) can be equivalently reformulated as

$$\min_{\mathbf{p}[t],\forall t} \quad \prod_{t=1}^{T}\prod_{i=1}^{K}\frac{f_i(\mathbf{p}[t],t)}{g_i(\mathbf{p}[t],t)} \quad (15a)$$

$$\text{subject to} \quad \sum_{t=1}^{\ell}\sum_{i=1}^{K}p_i[t] \leq \sum_{t=1}^{\ell}B_{in}^*[t], \quad (15b)$$

$$\frac{\sum_{t=1}^{\ell+1}B_{in}^*[t]-B_{max}}{f(\mathbf{p}[t])} \leq 1, \quad (15c)$$

$$p_i[t] \geq 0,\ \forall i,\forall \ell,\forall t \quad (15d)$$

where $\mathbf{p}[t] = [p_1[t],\ldots,p_K[t]]$. Notice that the above problem is not yet in a standard GP form due to the fact that $g_i(\mathbf{p}[t],t)$ and $f(\mathbf{p}[t])$ are posynomial functions. Let $\mathbf{p}^*[t]$ be a feasible solution of the problem in (15). By the relation between arithmetic and geometric means, we can obtain the following inequalities

$$g_i(\mathbf{p}[t]) = \sum_{j=1}^{K}p_j[t]|\mathbf{h}_i[t]^H\mathbf{w}_j[t]|^2+\sigma_n^2 \geq \prod_{j=1}^{K}\left(\frac{|\mathbf{h}_i[t]^H\mathbf{w}_j[t]|^2p_j[t]}{\xi_j}\right)^{\xi_j}\left(\frac{\sigma_n^2}{\xi_{K+1}}\right)^{\xi_{K+1}} \triangleq \tilde{g}_i(\mathbf{p}[t]),$$

where $\xi_j = |\mathbf{h}_i[t]^H\mathbf{w}_j[t]|^2p_j[t]/g_i(\mathbf{p}^*[t])$ and $\xi_{K+1} = \sigma_n^2/g_i(\mathbf{p}^*[t])$, and, similarly,

$$f(\mathbf{p}[t]) = \sum_{t=1}^{\ell}\sum_{i=1}^{K}p_i[t] \geq \tilde{f}(\mathbf{p}[t]) = \prod_{t=1}^{\ell}\prod_{i=1}^{K}\left(\frac{p_i[t]}{\zeta_i[t]}\right)^{\zeta_i[t]}, \forall \ell,$$

where $\zeta_i[t] = p_i^*[t]/f(\mathbf{p}^*[t])$. Then, by approximating $g_i(\mathbf{p}[t])$ and $f(\mathbf{p}[t])$ with the monomial functions $\tilde{g}_i(\mathbf{p}[t])$ and $\tilde{f}(\mathbf{p}[t])$, we obtain the following problem

$$\min_{\mathbf{p}[t],\forall t} \quad \prod_{t=1}^{T}\prod_{i=1}^{K}\frac{f_i(\mathbf{p}[t],t)}{\tilde{g}_i(\mathbf{p}[t],t)} \quad (16a)$$

$$\text{subject to} \quad \sum_{t=1}^{\ell}\sum_{i=1}^{K}p_i[t] \leq \sum_{t=1}^{\ell}B_{in}^*[t], \quad (16b)$$

$$\frac{\sum_{t=1}^{\ell+1}B_{in}^*[t]-B_{max}}{\tilde{f}(\mathbf{p}[t])} \leq 1, \quad (16c)$$

$$p_i[t] \geq 0,\ \forall i,\forall \ell,\forall t. \quad (16d)$$

This problem is in the form of a standard GP problem and can be solved efficiently using general purpose interior point solvers such as CVX [11]. The condensation method finds an approximate solution of the original problem in (15) by iteratively solving a sequence of approximated problems, as in (16), with the feasible solution $\mathbf{p}^*[t]$ given by the approximate solution obtained in previous iteration [7].

V. Extensions to On-Line Power Control Policies for Multiuser Beamforming

In this section, an on-line power control policy is proposed to mimic the energy flow of the offline policies.

Specifically, based on the offline policy derived in section III, we can see that the maximum sum throughput is achieved by a directional water-filling solution where the energy should be left for use in later time slots to make the water-level in consecutive time slots as equal as possible. Based on this observation, we propose an online power control policy where power is determined instantaneously in each time slot. However, the power constraint is chosen such that the average energy that can be used from this point on is equally divided among remaining time slots. Specifically, recall that the energy available for user in the ℓ-th time slot is given by $B[\ell] \triangleq \sum_{t=1}^{\ell}B_{in}^*[t]-\sum_{t=1}^{\ell-1}\sum_{i=1}^{K}p_i[t]$ and the average energy arrival in each of the remaining time slots is given by $\mathrm{E}[\sum_{t=\ell+1}^{T}B_{in}^*[t]]/(T-\ell)$. If the energy available in slot ℓ is greater than the average energy arrival in each of the remaining time slots, then energy should be stored for use in later time slots. Otherwise, all energy should be expended in the current time slot. That is, the total energy expended in the ℓ-th slot should be given by

$$p_{tot}[\ell] = \min\left\{B[\ell], \frac{B[\ell]+\mathrm{E}\left[\sum_{t=\ell+1}^{T}B_{in}^*[t]\right]}{T-\ell+1}\right\} \quad (17)$$

The online power control in the ℓ-th time slot is then formulated as

$$\max_{p_i[\ell],\forall i,} \quad \sum_{i=1}^{K}\log(1+\gamma_i[\ell]) \quad (18a)$$

$$\text{subject to} \quad \sum_{i=1}^{K}p_i[\ell] \leq p_{tot}[\ell], \quad p_i[\ell] \geq 0,\ \forall i. \quad (18b)$$

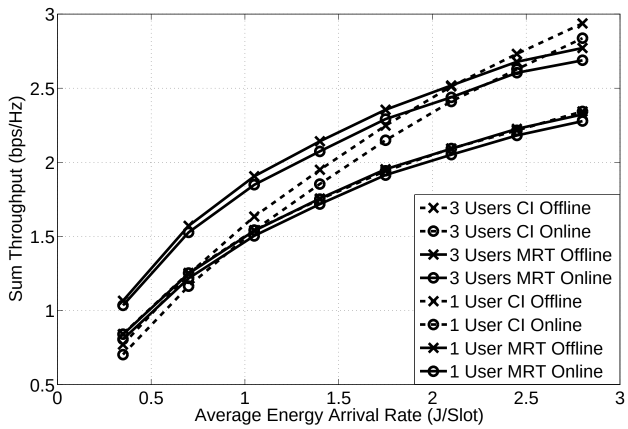

Fig. 2. Performance of the CI and MRT beamforming and with different energy arrival rate.

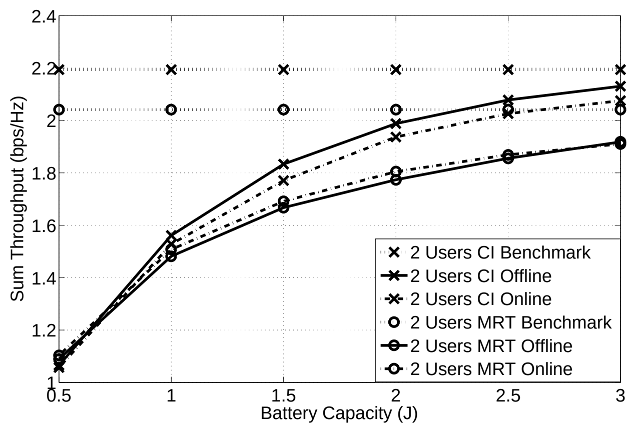

Fig. 3. Performance of the CI and MRT beamforming under different battery capacity limitation.

The solution is given by water-filling in the case of CI beamforming and is obtained using GP approximations in the case of MRT beamforming, as described in previous chapters.

VI. Numerical Comparison

In this section, we compare the performance of the proposed power control policies for both CI and MRT beamforming. The results of the experiments were obtained by averaging over 1000 randomly generated realizations of channel fading and energy arrival. The channel vectors are assumed to be i.i.d. with entries that are Gaussian with mean zero and variance $\sigma_h^2 = 1$. The noise variance is $\sigma_n^2 = 1$. The number of antennas at the BS is $M = 4$ and the deadline is $T = 10$ for all cases. The termination threshold for the GP approximation case η equals to 10^{-2}. The energy arrival in each time slot is assumed to be Gamma distributed, i.e., $\epsilon[t] \sim \mathcal{G}(\alpha, \theta)$, since it can be used to approximate many positive continuous random variables [12]. We set $\alpha = 2$ and scale θ to obtain different mean average rate $\bar{\epsilon} = \alpha\theta$ J/slot.

In Fig. 2, the average sum throughputs of the power control policies are shown for varying energy arrival rates. Here, the maximum battery capacity is set as $B_{max} = 10$ J. We can see that the performance of online policies are close to the offline schemes, and the sum throughput can be increased with higher energy arrival rate. In the single user case, CI beamforming and MRT beamforming are exactly the same, and with more users, the performance can be increased. We can observe that CI beamforming outperforms MRT beamforming with higher energy rate since there is no inter-user interference in CI beamforming case.

In Fig. 3, the average sum throughput for the different schemes are shown for varying values of B_{max}. The average energy arrival rate is 1.5 J/slot. The performance is compared with the benchmark scenario where no battery capacity constraint is imposed, i.e. $B_{max} = \infty$. One can observe that the throughput increases with B_{max} since, with larger battery storage capacity, more energy can be accumulated over time and the energy-sharing with later time slots is less restricted.

VII. Conclusion

In this work, we derived both offline and online power control policies for a multiantenna BS supported by renewable energy. Two multiuser beamforming schemes were considered, i.e., the CI and the MRT beamforming schemes. The power control policies were derived with the goal of maximizing the sum throughput by a deadline subject to energy causality and battery storage constraints. The power control for the CI beamforming case was obtained using the directional water-filling algorithm whereas that for the MRT beamforming case was obtained using GP approximations. The effectiveness of the proposed schemes were shown through numerical simulations.

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