Fast Compressive Sensing Recovery with Transform-based Sampling

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***Abstract*—We present a fast compression sensing (CS) reconstruction algorithm with computation complexity $O(M^2)$, where M denotes the length of a measurement vector $Y = \phi X$ that is sampled from the signal X of length N via the sampling matrix ϕ with dimensionality $M \times N$. Our method has the following characteristics: (1) it is fast due to a closed-form solution is derived; (2) it is accurate because significant components of X can be reconstructed with higher priority via a sophisticated design of ϕ; (3) thanks to (2), our method can better reconstruct a less sparse signal than the existing methods under the same measurement rate $\frac{M}{N}$.**

I. Introduction

In the context of compressive sensing (CS) [1], the constraint of sparsity enables the possibility of sparse signal recovery from measurements (far) fewer than the original signal length. Moreover, the measurements generated from random projection of the original signal via a sampling matrix are equally weighted; *i.e.*, no one is more significant than the others. Thus, compressive sensing is inherently weaken in handling less sparse signals such as highly textured images. The problem here is that can we yield weighted measurements so that non-sparse or less sparse signals can be properly reconstructed than the existing CS recovery solutions?

In this paper, we present a sophisticated design of the sampling matrix ϕ that can directly capture "important" measurements. With these information, the quality original signal can be sparsely reconstructed based on the important (corresponding to low-frequency) components in some transformed domain. Thus, the qualities of reconstructed signals mimic those of JPEG compressed images.

II. Proposed Method

We start from the random projection, $Y = \phi X$, and observe that if important information of X can be sampled and stored in Y, then it is possible to reconstruct X with fewer important measurements.

For this, we introduce a linear operator T and impose it to random projection to obtain $T \circ Y = T \circ (\phi X)$, where $\circ$ stands for a linear operation. This equation is further derived[1] based on the principle of linear operations [2] as:

$$T \circ Y = T \circ (\phi X) = (T \circ \phi)(T \circ X). \tag{1}$$

Eq. (1) indicates that if T is a transform operator, then $T \circ X$ is a transformed vector in some transform domain. In particular, the positions at lower frequencies in $T \circ X$ indicate important transformed coefficients and $T \circ Y$ indicates important measurements since they are linear combinations of significant transformed coefficients. For simplicity, the operator $\circ$ will be omitted below.

In order to sample "important" transformed coefficients from TX and speed up recovery, we design a new sampling matrix, $(T\phi)^z$, by setting the last $N - M$ columns of $T\phi$ to be zeros. This implies that the non-zero columns of $(T\phi)^z$ form a full-rank matrix with rank M. Once $(T\phi)^z$ is built in the transform domain, it is inversely transformed back to the time/space domain and a sophisticated designed sampling matrix $\Phi = T^{-1}((T\phi)^z)$ is obtained.

[1]The proof is omitted here due to space limit.

Now, Φ is stored in the sensors for the purpose of compressive sensing. We have the following derivations:

$$Y = \Phi X \;=>\; TY = (T\Phi)(TX) = (T\phi)^z(TX). \tag{2}$$

Recall that the last $N - M$ columns of $(T\phi)^z$ are zeros. This means that we only sample the lower-frequency components in TX by discarding the remaining higher-frequency components. In order to speed up sparse signal recovery, let Φ^s denote the submatrix of dimensionality $M \times M$ by discarding the zero columns of $(T\phi)^z$, and let $(TX)^s$ denote the $M \times 1$ vector by discarding the last $N-M$ transformed coefficients. Therefore, we can derive:

$$TY = \Phi^s(TX)^s => (\Phi^s)^{-1}TY = (\Phi^s)^{-1}\Phi^s(TX)^s = (TX)^s. \tag{3}$$

It is evident that the signal X can be approximately and fast recovered if (i) Y is available via random projection in Eq. (2); (ii) Y is processed via Eq. (3); and (iii) $(\Phi^s)^{-1}TY$ is padded with $N - M$ zero values (to obtain TX) and inversely transformed via T^{-1}.

III. Analysis and Results

The principle of our method is to preserve the top K-lowest frequency components of TX. Here, T is chosen to be a DCT operator. Thus, we have $M = K$ and the computation complexity of recovery is in the order of $O(M^2)$; *i.e.*, only one inverse matrix operation and two DCT operations are required.

In this paper, a $1D$ DCT structure is exploited to design Φ. The original signal X can be approximately reconstructed from as many measurements as the number of coefficients sampled via Eq. (2). We provide recovery comparison of some CS algorithms [1] under different measurement rates (MRs) in Table I[2]. The exploitation of the simple structure inherent in the Haar wavelet is also studied in our framework.

TABLE I
Recovery Comparison of CS Algorithms for Barbara Image.

Methods	Metrics	MR (6.25%)	MR (12.5%)	MR (25.0%)
Our Method (DCT-based)	PSNR(dB)	22.20	23.78	26.27
	SSIM	0.59	0.67	0.81
Lasso	PSNR(dB)	16.82	20.31	23.91
	SSIM	0.33	0.51	0.71
OMP (Sparsify toolbox)	PSNR(dB)	17.62	19.86	22.53
	SSIM	0.34	0.48	0.65
Basis Pursuit	PSNR(dB)	16.82	20.31	23.91
	SSIM	0.33	0.51	0.71
StOMP (SparseLab toolbox)	PSNR(dB)	10.94	12.51	21.92
	SSIM	0.23	0.37	0.61

References

[1] http://dsp.rice.edu/cs
[2] N. Merhav and V. Bhaskaran, "A transform domain approach to spatial domain image," *HPL-94-116*, Technion City, Haifa 32000, Israel, 1994.

[2]Structural similarity (SSIM) indexing is also adopted for image quality evaluation, where $0 \leq$ SSIM ≤ 1. The bigger, the better.