A Conservative Approach to Distributed Belief Fusion *

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Abstract - In this paper, we develop logics for merging beliefs of agents with different degrees of reliability. The logics are obtained by combining the multi-agent epistemic logic from [8] and multi-sources reasoning systems from [2]. Every ordering for the reliability of the agents is represented by a modal operator, so we can reason with the merging information under different situations. The approach is conservative in the sense that if an agent’s belief is in conflict with those of higher priorities, then his belief is completely discarded from the merged result. We consider two strategies for the conservative merging of beliefs. In the first one, if inconsistency occurs at some level, then all beliefs at the lower levels are discarded simultaneously, so it is called level cutting strategy. For the second one, only the level at which the inconsistency occurs is skipped, so it is called level skipping strategy. The formal semantics and axiomatic systems for these two strategies are presented.

Keywords: Epistemic logic, multi-sources reasoning, distributed belief, modal logic, multi-agent systems.

1 Introduction

Recently, there has been much attention on the infoglut problem in information retrieval research due to the rapid growth of internet information. If a keyword is input to a commonly-used search engine, it is not unusual to get back a list of thousands of web pages, so the real difficulty is not how to find information, but how to find useful information. To circumvent the problem, many software agents have been designed to do the information search works. The agents can search through the web and try to find and filter out information matching the user’s need. However, not all internet information sources are reliable. Some web sites are out-of-date, some news provide wrong information, and someone even intentionally spreads rumor or deceives by anonymity. Thus the main task of an information search agent will be how to merge so much information coming from different sources according to their degrees of reliability.

In [13], an agent is characterized by mental attitudes, such as knowledge, belief, obligation, and commitment. This view of agent, in accordance with the intentional stance proposed in [4], has been widely accepted as a convenient way for the analysis and description of complex systems[14]. From this viewpoint, each information provider can be considered as an agent and the information provided by the agent corresponds to his belief, so our problem is also that of merging beliefs from different agents.

The philosophical analysis of these mental attitudes has motivated the development of many non-classical logical systems[9]. In particular, the analysis of informational attitudes, such as knowledge and belief, has been a traditional concern of epistemology, a very important branch of philosophy since the ancient times. To answer the basic questions such as “What is knowledge?” “What can we know” and “What are the characteristic properties of knowledge?”, some formalism more rigorous than natural language is needed. This results in the development of the so-called epistemic logic[10]. This kind of logic has attracted much attention of researchers from diverse fields such as artificial intelligence(AI), economics, linguistics, and theoretical computer science. Among them, the AI researchers and computer scientist have elaborated some technically sophisticated formalisms and applied them to the analysis of distributed and multi-agent systems[8, 12].

Though the original epistemic logic in philosophy is mainly about the single-agent case, the application to AI and computer science put its emphasis on the interaction of agents, so multi-agent epistemic logic is strongly needed. One representative example of such logic is proposed by Fagin et al.[8]. In their logic, the knowledge of each agent is represented by a normal modal operator[1], so if no interactions between agents occur, this is not more than a multi-modal logic. However, the most novel feature of their logic is the consideration of common knowledge and distributed knowledge among a group of agents. While common knowledge is the facts that everyone knows, everyone knows that everyone knows, everyone knows that everyone knows that everyone knows, and so on, distributed knowledge is that can be deduced by pooling together the knowledge of everyone, so it is the latter that really concerns the fusion of knowledge among agents. However, the term “knowledge” is used in a wide sense in [8] to cover the cases of belief and infor-

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called level skipping strategy.

The rest of the paper is organized as follows. In the next section, the multi-agent epistemic logic and the multi-sources reasoning approach are reviewed. Then the logics for level cutting and skipping strategies are presented respectively in section 3 and 4. The syntax, semantics, and axiomatic systems of the logics will be given. Finally, related works and future perspectives are discussed in the conclusion section.

2 Logical Preliminary

In this section, we review the syntax, semantics and some notations for multi-agent epistemic logic and multi-sources reasoning.

2.1 Multi-agent epistemic logic

In [8], some variants of epistemic logic systems are presented. The most basic one with distributed belief is called K\(D_n^p\) by following the naming convention in [1], with \(n\) being the number of agents and \(D\) denoting the distributed belief operators. In the system, no properties except logical omniscience are imposed on the agents’ beliefs. Nevertheless, in the following, we will assume the belief of each individual agent is consistent though the collective ones of several agents may not be, so the system will be K\(D_n^{D}\) where the additional axiom \(D\) is added to K\(D_n^p\) for ensuring the consistency of each agent’s belief.\(^2\)

Assume we have \(n\) agents and a set \(\Phi_0\) of countably many atomic propositions, then the set of well-formed formulas(wff) for the logic K\(D_n^D\) is the least set containing \(\Phi_0\) and closed under the following formation rules:\(^3\)

1. if \(\varphi\) is a wff, so are \(\neg\varphi\), \(B_i\varphi\), and \(D_G\varphi\) for all \(1 \leq i, j \leq n\) and nonempty \(G \subseteq \{1, \ldots, n\}\), and
2. if \(\varphi\) and \(\psi\) are wffs, then \(\varphi \vee \psi\) is, too.

As usual, other classical Boolean connectives \(\land\)(and), \(\supset\)(implication), \(\equiv\)(equivalence), \(\top\)(tautology), and \(\bot\)(contradiction) can be defined as abbreviations.

The intuitive meaning of \(B_i\varphi\) is “The agent \(i\) believes \(\varphi\).”, whereas that for \(D_G\varphi\) is “The group of agents \(G\) has distributed belief \(\varphi\).”. The possible-worlds semantics provides a general framework for the modeling of knowledge and belief[8]. In the semantics, an agent’s belief state corresponds to the extent to which he can determine what world he is in. In a given world, the belief state determines the set of worlds that the agent considers possible. Then an agent is said to believe a fact \(\varphi\) if \(\varphi\) is true in all worlds in this set. Since the distributed belief of a group is the result of pooling together the individual beliefs of its members, this can

\(^1\)More precisely, the logic for belief is called doxastic logic. However, here we will use the three terms knowledge, belief, and information interchangeably, so epistemic logic is assumed to cover all these notions.

\(^2\)Though it is well accepted that K\(D45p\) is more appropriate for modeling of belief with positive and negative introspection (axioms 4 and 5), we adopt the K\(D_n^D\) system for emphasizing the agents may represent databases and their beliefs may be just the facts stored in the databases and their consequences.

\(^3\)In [8], the modal operators are denoted by \(K_i\) instead of \(B_i\).
be achieved by intersecting the sets of worlds that each agent in the groups considers possible.

Formally, a KD\textsubscript{n} model is a tuple \((W, (B_i)_{1 \leq i \leq n}, V)\), where

- \(W\) is a set of possible worlds,
- \(B_i \subseteq W \times W\) is a serial binary relation on \(W\) for \(1 \leq i \leq n\), \(^4\)
- \(V : \Phi_0 \rightarrow 2^W\) is a truth assignment mapping each atomic proposition to the set of worlds in which it is true.

In the following, we will use some standard notations for binary relations. If \(R \subseteq A \times B\) is a binary relation between \(A\) and \(B\), we will write \(R(a, b)\) for \((a, b) \in R\) and \(R(a)\) for the subset \(\{ b \in B \mid R(a, b) \}\). Thus for any \(w \in W\), \(B_i(w)\) is a subset of \(W\). Informally, \(B_i(w)\) is the set of worlds that agent \(i\) considers possible under \(w\) according to his belief. The informal intuition is reflected in the definition of satisfaction relation. Let \(M = (W, (B_i)_{1 \leq i \leq n}, V)\) be a KD\textsubscript{n} model and \(\Phi\) be the set of wffs, then the satisfaction relation \(\models_M \subseteq W \times \Phi\) is defined by the following inductive rules (we will use the infix notation for the relation and omit the subscript \(M\) for convenience):

1. \(w \models p\) iff \(w \in V(p)\) for any \(p \in \Phi_0\),
2. \(w \models \neg \phi\) iff \(w \not\models \phi\),
3. \(w \models \phi \lor \psi\) iff \(w \models \phi\) or \(w \models \psi\),
4. \(w \models B_i \phi\) iff for all \(u \in B_i(w)\), \(u \models \phi\),
5. \(w \models D_i \phi\) iff for all \(u \in \bigcap_{i \in G} B_i(w)\), \(u \models \phi\).

The notion of validity is defined from the satisfaction relation. A wff \(\phi\) is valid in \(M\), denoted by \(\models_M \phi\), if for every \(w \in W\), \(w \models_M \phi\), and valid in a class of models \(\mathcal{M}\), written as \(\models_\mathcal{M} \phi\), if for all \(M \in \mathcal{M}\), \(\models_M \phi\).

### 2.2 Multi-sources reasoning

The context of multi-sources reasoning is the merging of \(n\) databases. To encode the degrees of reliability of these databases, the total ordering on the subsets of \(\{1, \ldots, n\}\) is used. Let \(\mathcal{T}O_n\) denote the set of all possible total orders on the subsets of \(\{1, \ldots, n\}\). \(\Phi_0\) denote a finite set of atomic propositions and \(\mathcal{L}(\Phi_0)\) be the classical propositional language formed from \(\Phi_0\), then the set of wffs for logic FU\textsubscript{n} (originally called FUSION in [2]) is the least set containing \(\Phi_0\) and \([O] \phi : \phi \in \mathcal{L}(\Phi_0), \phi \in \mathcal{T}O_n\) and being closed under Boolean connectives. If \(O\) is the ordering \(i_1 > i_2 > \cdots > i_m\) for some \(\{i_1, \ldots, i_m\} \subseteq \{1, \ldots, n\}\), then the wff \([O] \phi\) means that \(\phi\) holds after merging the databases \(i_1, \ldots, i_m\) according to the specified ordering. In this case, \(O > i_{m+1}\) denotes \(i_1 > i_2 > \cdots > i_m > i_{m+1}\). Furthermore, the set \(\{i_1, i_2, \ldots, i_m\}\) is called the domain of \(O\) and is denoted by \(\delta(O)\).

Let \(\text{Lit}(\Phi_0)\) denote the set of literals in \(\mathcal{L}(\Phi_0)\).\(^5\) In the context of multi-sources reasoning, assume \(DB_1, \ldots, DB_n\) are \(n\) databases, where each \(DB_i\) is a finite satisfiable subset of \(\text{Lit}(\Phi_0)\), then the informal semantics for the merging databases can be given according to two attitudes. For the suspicious attitude, only the case of \(n = 2\) is given in [2], where the definition of \(DB_{1>2}\) is defined by

\[
DB_{1>2} = \begin{cases} 
DB_1 \cup DB_2 & \text{if } DB_1 \cup DB_2 \text{ is consistent,} \\
DB_1 & \text{otherwise.}
\end{cases}
\]

On the other hand, for the trusting attitude, the definition of \(DB_O\) is given in the following recursive formula

\[
DB_{O>1} = DB_O \cup \{ l \in DB_i : \overline{1} \not\in DB_O \},
\]

where \(\overline{1}\) is the complementary of \(1\). The intended meaning of \([O] \phi\) is \(DB_O \models_{\text{CL}} \phi\), where \(\text{CL}\) denotes classical propositional reasoning.

Thus an FU\textsubscript{n} model is a tuple \((W, (R_i)_{1 \leq i \leq n}, V)\), where \(W\) and \(V\) are as defined in KD\textsubscript{n} models, and each \(R_i\) is a serial binary relation on \(W\).\(^6\) The clause for satisfaction of the formula \([O] \phi\) is then

\[w \models [O] \phi\] iff for all \(u \in R_O(w), u \models \phi\),

where \(R_O\) is defined from \(R_i\), according to two attitudes. For the suspicious attitude,

\[
R_{1>2}(w) = \begin{cases} 
R_1(w) & \text{if } R_1(w) \cap R_2(w) = \emptyset, \\
R_1(w) \cap R_2(w) & \text{otherwise,}
\end{cases}
\]

for all \(w \in W\). For the trusting attitude, we need some auxiliary notations. Let \(f : 2^W \times 2^W \rightarrow 2^{\text{Lit}(\Phi_0)}\) be defined as

\[f(S, T) = \{ l \in \text{Lit}(\Phi_0) : \forall w \in S(w \models l) \land \exists w \in T(w \models l) \},\]

i.e., \([S, T] = \{ l \in \text{Lit}(\Phi_0) : \forall w \in S(w \models l) \land \exists w \in T(w \models l) \}\). Then for any \(w \in W\),

\[R_{O>1}(w) = R_O(w) \cap \{ u \in W : u \models \bigvee f(R_i(w), R_O(w)) \} \}

Note that if each \(R_i(w)\) denotes the set of possible worlds in which the literals in \(DB_i\) are all true, then \(f(R_i(w), R_O(w))\) is just the set \(\{ l \in DB_i : l \not\in DB_O \}\), so \(R_{O>1}(w)\) is exactly the set of possible worlds satisfying all literals in \(DB_{O>1}\).

A severe restriction of FU\textsubscript{n} is the background databases \(DB_i\)’s can contain only literals which is not the case in general practice. Though from the semantic viewpoint, there is no essential difficulty to lift the restriction, it seems hard to obtain an axiomatic system for the logic when the databases contain general formulas. For example, it is suggested in [2] that according to the trusting attitude, the merged database \(DB_{O>1}\) is equivalent to \(DB_1 \odot DB_2\), the result of updating \(DB_1\) with \(DB_O\) by belief update operator in [11] and since the operator can be applied to knowledge base containing any forms of wffs, the restriction to literal forms is automatically removed. However, since the operator is a meta-level one, it is unclear how

\(^4\)A relation \(R\) on \(W\) is serial if \(\forall w \exists u R(w, u)\).

\(^5\)A literal is an atom or a negated atom.

\(^6\)In [2], it is assumed that each \(R_i\) is an equivalence relation. However, since nested modalities are not allowed in FU\textsubscript{n}, the difference is inessential.
the postulates of the update operator can be recast in the FUₙ logic axioms. On the other hand, for the suspicious semantics, the merged database in fact contains the distributed belief of the two databases if they are consistent. However, since distributed belief operator is not in the language of FUₙ, the formula [O]φ can only be characterized by the modal operators [i] for i ∈ δ(O). Nevertheless, unless φ is a literal, it seems difficult (if not impossible) to define D₁φ in terms of B₁ and B₂. Thus, a natural solution to merge general databases in the suspicious semantics is to add the distributed belief operators into the language of FUₙ. This is exactly what we will do in the following.

3 Level Cutting Strategy

To unify the notations from multi-agent epistemic logic and multi-sources reasoning, we will use the language DBFⁿ (for distributed belief fusion and cutting strategy) defined as follows. The wffs of DBFⁿ are the least set containing Φ₀ and being closed under Boolean connectives and the following rule:

- if φ is a wff, so are [G]φ and [O]φ for any nonempty G ⊆ {1,...,n} and O ∈ TOₙ.

When G is a singleton {i} and O is the unique total order on {i}, we will use [i]φ to denote both [G]φ and [O]φ. Thus [i]φ and [G]φ correspond respectively to Bᵢφ and Dᵢφ in KDⁿ, so DBFⁿ is an extension of the multi-agent epistemic logic with distributed belief operators. On the other hand, [O]φ and [i]φ are precisely those in FUₙ, so DBFⁿ is also a generalization of multi-sources reasoning system. However, note that nested modalities are not allowed in FUₙ, whereas this is not restricted in DBFⁿ any more. Thus, for example, we can include a wff [j]φ in a database DBᵢ which means that DBᵢ has the information that φ is in j.

For the semantics, a DBFⁿ model is just a FUₙ model (W, (ℛᵢ)₁≤i≤n, V). The clauses for the satisfaction of wffs are defined exactly as in FUₙ model in addition to a clause for the [G] operator which is the one for distributed knowledge in KDⁿ. However, the relation ℛᵢ is now defined in an inductive way:

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for any w ∈ W. Let O = (i₁ > i₂ > ... > iₘ) and define G_j = {i₁, i₂, ..., i_j} for 1 ≤ j ≤ m and assume k is the largest j such that ∩ᵢ∈G_k ℛᵢ(w) ≠ Ø, then we have

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In other words, the beliefs from the agents after the level k are completely discarded in the merged result. The rationale behind this is if belief in level k+1 is not acceptable, neither any belief in a less reliable level, so this is a very conservative attitude to belief fusion.

The notion of validity in DBFⁿ is defined just as that for KDⁿ. The notation ⊨DBFⁿ φ denote that φ is valid in all DBFⁿ model and the subscript is usually omitted if there is no confusion. The valid wffs of DBFⁿ can be captured by the axiomatic system in Fig 1.

1. Axioms:

P: all tautologies of the propositional calculus
G₁: ([G]φ ∧ [G](φ ⊢ ψ)) ⊢ [G]ψ
G₂: ¬[i]⊥
G₃: [G₁]φ ⊢ [G₂]φ if G₁ ⊆ G₂
O₁: ¬[δ(O > i)]⊥ ⊢ ([O > i]φ ≡ [δ(O > i)]φ)
O₂: [δ(O > i)]⊥ ⊢ ([O > i]φ ≡ [O]φ)

2. Rules of Inference:

R1(Modus ponens, MP):

\[ \varphi, \varphi \supset \psi \vdash \psi \]

R2(Generalization, Gen):

\[ \varphi \vdash [G]φ \]

Figure 1: The axiomatic system for DBFⁿ

The axioms G1-G3 and rule R2 are those for KDⁿ. G1 and rule R2 are properties of knowledge for perfect reasoners. They also are the causes of the notorious logical omniscience problem. However, it is appropriate to describe implicit information in this way. G₂ is the requirement that the belief of each individual agent is consistent. G₃ is a characteristic property of distributed knowledge. The larger the subgroup, the more knowledge it possesses. In [8], another axiom related distributed knowledge and individual ones is added. That is,

\[ D_{[i]}φ ≡ B_{i}φ, \]

however, we do not need this because we identify [i]φ and [[i]]φ which respectively correspond to Bᵢφ and Dᵢφ in KDⁿ. The two axioms O₁ and O₂ define the merged belief in terms of distributed belief in a recursive way. O₁ is the case when ∩ᵢ∈G_k ℛᵢ(w) ≠ Ø, whereas O₂ is the opposite case.

The derivability in the system is defined as follows. Let Σ ∪ {φ} be a subset of wffs, then φ is derivable from Σ in the system DBFⁿ, written as Σ ⊢DBFⁿ φ, if there is a finite sequence φ₁,...,φₘ such that every φᵢ is an instance of an axiom schema, a wff in Σ, or obtainable from earlier φᵢ’s by application of an inference rule. When Σ = Ø, we simply write ⊢ DBFⁿ φ. We will drop the subscript when no confusion occurs. We have the soundness and completeness results for the system DBFⁿ.

Theorem 1 For any wff of DBFⁿ, φ ⊨ φ iff ⊢ φ.

Some basic theorems can be derived from the system.
Proposition 1 For any \( O = (i_1 > i_2 > \cdots > i_m) \) and \( G_j = \{i_1, i_2, \ldots, i_j\} (1 \leq j \leq m) \), we have:

1. \( \vdash (\neg[G_j]\perp \land [G_{j+1}]\perp) \supset ([O] \varphi \equiv [G_j] \varphi) \).
2. \( \vdash ([O] \varphi \land [O] (\varphi \supset \psi)) \supset [O] \psi \).
3. \( \vdash \neg[O] \perp \).
4. \( \varphi \equiv [O] \varphi \).

Proposition 1.1 shows that any total order can be cut into a head and a tail according to some consistency level, and the merged belief according to the ordering is just the distributed belief of the agents from the head part. Proposition 1.2 and 1.4 show that merged belief inherits the properties of the distributed one since the former is equivalent to the latter for the head part of the ordering. Furthermore, Proposition 1.3 shows that belief fusion keeps consistency.

4 Level Skipping Strategy

Though level cutting strategy is useful in practice, it is sometimes too conservative from the viewpoint of information fusion. A less conservative strategy is to skip only the level causing inconsistency and continue to consider the next level. This strategy is easily obtained by modifying the inductive definition of \( R_{O>i} \) as follows.

\[
R_{O>i}(w) = \begin{cases} 
R_O(w) & \text{if } R_O(w) \cap R_i(w) = \emptyset, \\
R_O(w) \cap R_i(w) & \text{otherwise}, 
\end{cases}
\]

for any \( w \in W \).

According to the definition, \( [O > i] \varphi \) will be equivalent to the distributed fusion of \( [O] \varphi \) and \([i] \varphi \) when the belief of \( i \) is consistent with the merged belief of \( O \), so to axiomatize reasoning under the strategy, we must view \( O \) as a virtual agent and consider the distributed belief between \( O \) and \( i \). However, to get a bit more general, we will consider the distributed belief among a group of virtual agents. Thus, we define the wffs of the logic \( D BF^n \) (for skipping strategy) as the least set containing \( \Phi_0 \) and being closed under Boolean connectives and the following rule:

- if \( \varphi \) is a wff, so are \([O] \varphi \) for any nonempty \( O \subseteq T \).

When \( \Omega \) is a singleton \( \{O\} \), we will write \([O] \varphi \) instead of \( [\{O\}] \varphi \). If \( \Omega = \{O_1, \ldots, O_m\} \) is such that \( \delta_i(O_i) = 1 \) for all \( i \)'s, then \( \Omega \) is the distributed belief operator among ordinary agents. Therefore, the language is more general than that of \( D BF^n \).

For the semantics, a \( D BF^n \) model is still a \( D BF^n \) model, however, the satisfaction clauses for \([O] \) and \([O] \) operators are replaced by the following

\[
w \models [O] \varphi \text{ iff for all } u \in R_O(w), u \models \varphi,
\]

where \( R_O(w) = \bigcap_{O \subseteq \Omega} R_O(w) \) and \( R_O \) is defined inductively at the beginning of the section. Given this language and semantics, the valid wffs of \( D BF^n \) is capture by the axiomatic system in Fig 2. The axioms V1-V3 and rule R2' correspond to G1-G3 and R2 for distributed belief, but now for virtual agents instead of ordinary agents. Nevertheless, since an ordinary agent is a special case of the virtual one, these in fact also cover G1-G3 and R2. O1' and O2' are axioms for describing the level skipping strategy and correspond exactly to the inductive definition of \( R_{O>i} \). We can still have the soundness and completeness theorem.

Theorem 2 For any wff of \( D BF^n \), \( \models \varphi \text{ iff } \vdash \varphi \).

Since operator \([O] \) is a special case of \([\Omega] \), the properties 1.2 and 1.4 hold trivially for \( D BF^n \). The property 1.3 can be easily proved by using V2, O1' and O2'. However, it is unclear whether a counterpart of properties 1.1 can be given.

5 Related Works

In the preceding section, it has been emphasized the strong dependence of the present work on multi-agent epistemic logic and multi-sources reasoning. Formally, the relationship between them can be described by the following translation mappings. Let \( L_1 \) and \( L_2 \) be two logics based on the same set of propositional symbols \( \Phi_0 \), then a translation mapping \( \tau : L_1 \rightarrow L_2 \) assigns to each wff of \( L_1 \) one wff of \( L_2 \) and must satisfy the following homomorphism condition on Boolean connectives:

1. \( \tau(p) = p \) for all \( p \in \Phi_0 \).
2. \( \tau(\neg \varphi) = \neg \tau(\varphi) \).
3. \( \tau(\varphi \lor \psi) = \tau(\varphi) \lor \tau(\psi) \).

Let us consider the following two translation mappings.

1. Axioms:
   
   \[
   P: \text{all tautologies of the propositional calculus}
   \]
   
   \[
   V1: ([\Omega] \varphi \land [\Omega] (\varphi \supset \psi)) \supset [\Omega] \psi
   \]
   
   \[
   V2: \neg[i] \perp
   \]
   
   \[
   V3: [\Omega_1] \varphi \supset [\Omega_2] \varphi \text{ if } \Omega_1 \subset \Omega_2
   \]
   
   \[
   O1': \neg[\{O, i\}] \perp \supset ([O > i] \varphi \equiv \{[O, i]\}] \varphi
   \]
   
   \[
   O2': [\{O, i\}] \perp \supset ([O > i] \varphi \equiv [O] \varphi)
   \]

2. Rules of Inference:
   
   \[
   \frac{\varphi \varphi \supset \psi}{\psi}
   \]
   
   \[
   \frac{\varphi}{[\Omega] \varphi}
   \]

Figure 2: The axiomatic system for \( D BF^n \)
1. \( \tau_1 : KD_n^D \rightarrow DBF_n^c \), and
2. \( \tau_2 : KD_n^D \rightarrow DBF_n^a \),

such that when \( \tau = \tau_1 \) or \( \tau_2 \), \( \tau \) must satisfy
1. \( \tau(B, \varphi) = [i] \tau(\varphi) \), and
2. \( \tau(DG\varphi) = [G] \tau(\varphi) \).

Then the following proposition holds.

**Proposition 2** 1. For any wff \( \varphi \) in \( KD_n^D \), \( \vdash_{KD_n^D} \varphi \) iff \( \models_{DBF_2} \tau_1(\varphi) \) iff \( \models_{DBF_2} \tau_2(\varphi) \),
2. for any wff \( \varphi \) in \( FU_2 \), \( \models_{FU_2} \varphi \) iff \( \models_{DBF_2} \varphi \) iff \( \models_{DBF_2} \varphi \), where \( \models_{FU_2} \) is defined with respect to suspicious semantics.

In addition to these obvious results, below we will show that \( DBF_n^a \) can simulate the trusting attitude semantics for multi-sources reasoning in the particular case when all databases contain only finite set of literals. Furthermore, it is also shown that the inconsistency handling in possibilistic logic can be modeled in the \( DBF_n^a \).

### 5.1 Simulation of trusting attitude

In section 2.2, it is assumed that \( \Phi_0 \) is finite and each \( DB_i \) is a finite satisfiable subset of \( Lit(\Phi_0) \). Let \( CLS(\Phi_0) \) be the set of clauses in \( \mathcal{L}(\Phi_0) \). Then in \( FU_n \), each \( DB_i \) is characterized by a wff

\[
\psi_i = \bigwedge\{[i]l : l \in DB_i\} \land \bigwedge\{-[i]l : c \in CLS(\Phi_0), DB \not\models CL c\},
\]

and the reasoning problem is to decide whether the following holds:

\[
\models_{FU_n} \bigwedge_{i=1}^{n} \psi_i \supset [O] \varphi,
\]

for some given \( O \) and \( \varphi \in \mathcal{L}(\Phi_0) \). In this subsection, we will show that under the same assumption, the reasoning problem in \( FU_n \) can be simulated in \( DBF_{nm}^a \) where \( m = |\Phi_0| \). The approach is to split each database into \( m \) agents such that for the \( i \)th database, the \( (i,j) \)th agent has the belief \( i_j \) if \( i_j = p_j \) or \( \neg p_j \in DB_i \). The ordering among agents is determined by that among databases. That is, if \( DB_i \) has higher reliability than \( DB_j \), then all \( m \) agents for \( DB_i \) have higher reliability than all those for \( DB_j \). However, the ordering between agents for the same database is irrelevant since we assume each database is individually satisfiable, so we can use an arbitrary but fixed ordering for them.

Let \( \Phi_0 = \{p_1, \ldots, p_m\} \) and assume \( n \times m \) agents \( (i,j) \) for \( 1 \leq i \leq n, 1 \leq j \leq m \) in the logic \( DBF_{nm}^a \).

For each \( DB_i \) and \( 1 \leq j \leq m \), define

\[
DB_{ij} = \begin{cases} 
\{l_j\} & \text{if } l_j = p_j \text{ or } \neg p_j \in DB_i, \\
\emptyset & \text{otherwise},
\end{cases}
\]

Then we can define \( \psi_{ij} \) as in (1) by replacing \([i] \) with \([i,j] \) when \( DB_{ij} \) is nonempty. For each total order \( O \) on \( TO_n \), define \( O' \) on \( TO_{nm} \) such that

\[
\delta(O') = \{(i,j) : i \in \delta(O), DB_{ij} \neq \emptyset\}
\]

and \( (i_1,j_1) > (i_2,j_2) \) in \( O' \) iff \( i_1 > i_2 \) in \( O \) or \( i_1 = i_2 \) and \( j_1 > j_2 \) by natural number ordering. Then we can have the following result.

**Proposition 3** For any \( \varphi \in L(\Phi_0) \) and \( O \in TO_n \),

\[
\models_{FU_n} \bigwedge_{i=1}^{n} \psi_i \supset [O] \varphi
\]

iff

\[
\models_{DBF_{nm}^a} \bigwedge_{DB_{ij} \neq \emptyset} \psi_{ij} \supset [O'] \varphi
\]

where \( \models_{FU_n} \) is according with trusting attitude semantics.

### 5.2 Inconsistency handling in possibilistic logic

At the first glance, it seems that the level cutting strategy is too conservative to be useful. However, it turns out we can model the inconsistency handling technique of possibilistic logic in the strategy. Possibilistic logic (PL) is proposed by Dubois and Prade for uncertainty reasoning[7, 5, 6]. The semantic basis of PL is the possibility theory developed by Zadeh from fuzzy set theory[15]. Given a universe \( W \), a possibility distribution on \( W \) is a function \( \pi : W \rightarrow [0,1] \). Obviously, \( \pi \) is a characteristic function of a fuzzy subset of \( W \). Two measures on \( W \) can be derived from \( \pi \). They are called possibility and necessity measures and denoted by \( \Pi \) and \( N \) respectively. Formally, \( \Pi, N : 2^W \rightarrow [0,1] \) are defined as

\[
\Pi(A) = \sup_{w \in A} \pi(w),
\]

\[
N(A) = 1 - \Pi(\overline{A}),
\]

where \( \overline{A} \) is the complement of \( A \) with respect to \( W \).

In [6], a fragment for necessity-valued formula in PL, called PL1, is introduced. Each wff of PL1 is of the form \( (\varphi, \alpha) \), where \( \varphi \in L(\Phi_0) \) and \( \alpha \in (0,1] \) is a real number. Here, we still assume that \( \Phi_0 \) is finite. The number \( \alpha \) is called the valuation or weight of the formula. \( (\varphi, \alpha) \) expresses that \( \varphi \) is certain at least to degree \( \alpha \). Formally, a model for PL1 is given by a possibility distribution \( \pi \) on the set \( W \) of classical truth assignments for \( L(\Phi_0) \). For any \( \varphi \in L(\Phi_0) \), we can define \( |\varphi| \) as the set of truth assignments satisfying \( \varphi \). Then, by identifying \( \varphi \) and its truth set \(|\varphi|\), a PL1 model \( \pi \) satisfies \( (\varphi, \alpha) \), denoted by \( \pi \models (\varphi, \alpha) \), if \( N(\varphi) \geq \alpha \). Let \( \Sigma = \{(\varphi_i, \alpha_i) : 1 \leq i \leq m\} \) be a finite set of PL1 wffs, then \( \Sigma \models_{PL1} (\varphi, \alpha) \) if for each \( \pi, \pi \models (\varphi_i, \alpha_i) \) for all \( 1 \leq i \leq m \) implies \( \pi \models (\varphi, \alpha) \). It is shown that the consequence relation in PL1 can be determined completely by the least specific model satisfying \( \Sigma \). That is, if \( \pi_\Sigma : W \rightarrow [0,1] \) is defined by

\[
\pi_\Sigma(w) = \min\{1 - \alpha_i \mid w \models \neg \varphi_i, 1 \leq i \leq m\},
\]
where \( \min \emptyset = 1 \), then \( \Sigma \models_{\text{PL1}} (\varphi, \alpha) \) iff \( \pi_\Sigma \models (\varphi, \alpha) \).

A special feature of PL1 is its capability to cope with partial inconsistency. For \( \Sigma \) defined as above, let \( \Sigma^* \) denote the set of classical formulas \( \{ \varphi \mid 1 \leq i \leq m \} \). Then the set \( \Sigma \) is said to be partially inconsistent when \( \Sigma^* \) is classically inconsistent. It can be easily shown that \( \Sigma \) is partially inconsistent iff \( \sup_{\omega \in W} \pi_{\Sigma}(\omega) < 1 \). Thus \( \sup_{\omega \in W} \pi_{\Sigma}(\omega) \) is called the consistency degree of \( \Sigma \), denoted by \( \text{Cons}(\Sigma) \), and \( 1 - \text{Cons}(\Sigma) \) is called the inconsistency degree of \( \Sigma \), denoted by \( \text{Incons}(\Sigma) \). When \( \Sigma \) is partially inconsistent, it can be shown that \( \Sigma \models_{\text{PL1}} (\bot, \text{Incons}(\Sigma)) \), so for any classical wff \( \varphi \), \( (\varphi, \text{Incons}(\Sigma)) \) is a trivial logical consequence of \( \Sigma \). On the contrary, if \( \Sigma \models_{\text{PL1}} (\varphi, \alpha) \) for some \( \alpha > \text{Incons}(\Sigma) \), then \( \varphi \) is called a nontrivial consequence of \( \Sigma \).

To model the nontrivial deduction of PL1, we assume that the weights of the wffs are drawn from a finite subset \( V \) of \([0, 1]\). If \( |V| = n \), then define a mapping \( L : V \rightarrow \{1, \ldots, n\} \). Let \( \Sigma = \{(\varphi_i, \alpha_i) : 1 \leq i \leq m\} \) be given. Without loss of generality, we can assume all \( \alpha_i \)'s in \( \Sigma \) are distinct. Let \( \mathcal{O}_\Sigma \) be the ordering on \( TV_\alpha \) such that \( \delta(\mathcal{O}_\Sigma) = \{L(\alpha_i) : 1 \leq i \leq m\} \) and \( L(\alpha) > L(\beta) \) in \( \mathcal{O}_\Sigma \) if \( \alpha > \beta \) in real number. For any classical truth assignment \( \omega \), define the characteristic formula of \( \omega \) as

\[
\chi_\omega = \bigwedge_{i=1}^m (L(\alpha_i) \varphi_i \land \bigwedge_{\omega \models \varphi_i} \neg[L(\alpha_i)] \neg \chi_\omega). 
\]

Then the translation from PL1 to DBF_\Sigma is defined as

\[
\text{Tr}(\Sigma) = \bigwedge_{i=1}^m (L(\alpha_i) \varphi_i \land \bigwedge_{\omega \models \varphi_i} \neg[L(\alpha_i)] \neg \chi_\omega),
\]

then

**Proposition 4** Let \( \Sigma \) be defined as above such that \( \forall \varphi_i \neg \varphi_i \), for \( 1 \leq i \leq m \), then for any classical wff \( \varphi \), \( \varphi \) is a nontrivial consequence of \( \Sigma \) iff \( \models_{\text{DBF}_\Sigma} \text{Tr}(\Sigma) \supset [\mathcal{O}_\Sigma] \varphi \).

Though the representation of a set of PL1 wffs in DBF_\Sigma seems cumbersome, this is the price we have to pay for using a modal logic framework. As in the specification of databases in (1), we have to explicitly represent not only what is believed but also what is not. However, the advantage we gain here is that we can merge beliefs on modal level as well as those on objective level.

Two provisos for the proposition deserve further attention here. The first is the explicit requirement of \( \forall \varphi_i \neg \varphi_i \), for \( 1 \leq i \leq m \). Due to axiom G2, if the requirement is violated, then \( \models_{\text{DBF}_\Sigma} \text{Tr}(\Sigma) \supset [\mathcal{O}_\Sigma] \varphi \) holds trivially for any \( \varphi \) and in particular when \( \varphi = \bot \). However, for PL1, it is impossible that \( \bot \) is a nontrivial consequence. The second, a less explicit one, is based on the hypothetical nature of multi-sources reasoning. In PL1, for a given set \( \Sigma \), the lower bound of necessity valuation for each \( \varphi_i \) is fixed, so by PL1 deduction, we can only infer the nontrivial consequences of \( \Sigma \). However, from \( \text{Tr}(\Sigma) \), it is possible to infer

\[ [O] \varphi \text{ for different } O \text{'s in addition of } O_{\Sigma}. \] This is equivalent to simultaneously do inferences on the sets \([\{(\varphi, \alpha_i(i)) \mid 1 \leq i \leq m\}]\) for any permutation \( \sigma \) on \( \{1, \ldots, m\} \).

### 6 Conclusions

In this paper, we propose two logical systems for conservative information fusion. The systems are conservative in the sense that if an information source is in conflict with other more reliable ones, then the information from that source is totally discarded. The two systems correspond to two different strategies of discarding the information sources. For level skipping strategy, if an information source is to be discarded, then all others less reliable than it are also discarded without further examination. On the other hand, for level skipping strategy, only the level under conflict is skipped, and the next level will be considered independent of those discarded before it. Thus, level skipping strategy is relatively less conservative than the level cutting one and indeed, we can simulate the trusting attitude multi-sources reasoning in [2] by level skipping strategy. Though it seems that level cutting strategy is extremely conservative, it is shown that the inconsistency handling in possibilistic logic can be modeled by the strategy. Furthermore, it is also shown that our systems are both the generalizations of multi-agent epistemic logic and multi-sources reasoning.

In the exposition above, for simplicity, we do not distinguish belief and information. However, in a genuine agent systems, an agent’s belief may be different with the information he sends to or receives from other agents. Thus, in general, we should have a set of modal operators \([j]\), such that \([j] \varphi \) means that agent \( i \) receives the information \( \varphi \) from \( j \). In particular, \([i] \varphi \) may represent the observation of agent \( i \) himself, which should be the most reliable information source for \( i \). Then agent \( i \) may form his belief by fusing the information he received from different agents according to the degrees of trust he has on other agents. The fusion may be represented by the operators \([O]_{i}\). If we consider \([j] \varphi \) as the communication of message \( \varphi \) from \( j \) to \( i \), then we have a general framework for reasoning about agent’s belief and communication. In such a framework, we can discuss the problems like deception of agent. For example, \([O]_{i} \varphi \) and \([i] \varphi \) may mean agent \( i \) deceives to agent \( j \) by telling \( j \) the negation of what he believes.

In a recent paper, it is shown that the multi-sources reasoning of [2] can be applied to deontic logic under conflicting regulations[3]. Essentially, this is to merge conflicting regulations according to the priorities of them analogously to the fusion of information. However, inherited from the restriction of FU_\Sigma, it is also required that each regulation to be merged must be a set of deontic literals. Now, by the systems developed here, it is expected that the general forms of regulations can also be merged in a conservative way.
References


