On the Possibility Theory-Based Semantics for Logics of Preference *

Churn-Jung Liau
Institute of Information Science
Academia Sinica,Taipei, Taiwan
E-mail: liaucj@iis.sinica.edu.tw

Abstract

In this paper, we study the semantics for the logics of preference based on possibility theory. Possibility distributions representing the preference between worlds are associated with the possible world models for dynamic logics. Then the preference between actions are determined by comparing some measures of their consequences. We define different logics of preference by considering the comparisons of possibility measures and guaranteed possibility measures. Some properties of the proposed logics are studied and their relationships with deontic logics are also considered.
Key words: Possibility theory, deontic logic, logic of preference, dynamic logic.

1 Introduction

Deontic logic is the logic for reasoning about normative concepts. Deontic reasoning has been extensively exploited in ethics and legal philosophy since the ancient times. However, the first modern formal system for deontic logic is not established until the fifties[15]. Though the system is influential on the later work, there are arising many paradoxes when it is applied to practical deontic reasoning, so alternative systems have also been proposed since then for the resolution of paradoxes[2]. Among them, the Meyer's approach is one of the most interesting ones[11]. His system is based on the reduction to dynamic logic[8], so actions and propositions can be both represented and the distinction between "ought-to-do" and "ought-to-be" can be made clearly. This also clarifies much confusion caused by the inappropriate translation of practical deontic reasoning to formal systems.

On the other hand, while the different formal systems are mainly the consequence of foundational studies of deontic reasoning, the applications of deontic logics to computer science and artificial intelligence has received more and more attention recently[12]. The applications include automated legal reasoning, electronic commerce, system specification, and so on. Since Meyer's system is strongly based on dynamic logic and the latter is the logic for reasoning about computer program, it is in particular suitable for the potential application.

In addition to the reduction to dynamic logic, another important feature of Meyer's logic is the use of a special propositional atom V, meaning the violation of law (or something like sanction, punishment, etc.). This special atom is originally introduced by Anderson[1] for reducing deontic logic to alethic modal logic. By using the special atom, an action is forbidden if the execution of it will necessarily lead to states in which V holds, and it is permitted if not forbidden. Moreover, an action is obligatory if failing to executing it will result in violation of law.

Though Meyer's logic is successful in reasoning about normative actions, it is inadequate in the representation of action preference. However, the norms are usually relative and conflict with other ones, so we may have to make some decision choices between conflicting actions. For example, the violation of constitution is considered more serious than that of regulation laws, so we will try to obey the former instead of the latter provided that it is impossible to enforce both in the same time. Under the situations, we will need the capability of reasoning about action preference. In this paper, we will show that by extending Meyer's semantics with possibility theory

*This is an expanded and revised version of [10].

constructs, we can achieve the purpose. Intuitively, the possibility distribution will model the degrees of satisfaction of norms, and an action will be preferred to another one if it satisfies the norms to a higher degree than the other.

The rest of the paper is organized as follows. First, Meyer's deontic logic and possibility theory are reviewed in the next section. Then a logic for deontic degree is proposed and its semantics is presented. In section 4, we consider a logic of action preference. Its relationship with deontic logics will also be studied. Then an alternative logic based on the minimal semantics (neighborhood semantics) of modal logics[4] is considered. Finally, we discuss some related works and further generalization in the concluding section.

2 Review of Deontic Logic and Possibility Theory

2.1 Deontic logic as dynamic logic

The system of Meyer's logic is called PD_eL. The elementary symbols of the PD_eL language consist of

1. A set of propositional letters, $PV = \{p, q, r, \ldots\}$ and a special propositional letter V not in PV, and
2. a set of atomic actions, $A = \{a, b, c, \ldots\}$ and three distinguished action symbols $\emptyset$, $\mathbf{u}$, and $\mathbf{i}$[1].

The set of well-formed formulas(Φ) and the set of action expressions(Σ) are defined inductively in the following way.

1. Φ is the smallest set such that
   - $PV \cup \{V\} \subseteq \Phi$, and
   - if $\varphi, \psi \in \Phi$ and $\alpha \in \Sigma$, then $\neg\varphi, \varphi \vee \psi, [\alpha]\varphi \in \Phi$.
2. Σ is the smallest set such that
   - $A \cup \{\emptyset, \mathbf{u}, \mathbf{i}\} \subseteq \Sigma$, and
   - if $\alpha, \beta \in \Sigma$ and $\varphi \in \Phi$, then $\alpha;\beta, \alpha \cup \beta, \alpha\&\beta, \overline{\alpha}, \varphi \to \alpha/\beta \in \Sigma$.

The wff $\neg[\alpha]\neg\varphi$ is abbreviated as $\langle\alpha\rangle\varphi$ and the other classical connectives($\top, \bot, \wedge, \supset, \equiv$) are defined as usual. The wff $[\alpha]\varphi$ means that if action α is done, φ will hold. The action expressions $\alpha;\beta, \alpha \cup \beta, \alpha\&\beta$ denote the sequential composition, nondeterministic choice, and simultaneous execution of α and β respectively, whereas $\overline{\alpha}$ means the non-execution of α and $\varphi \to \alpha/\beta$ denotes that if φ holds then execute α else execute β.

Note that the language of PD_eL is not the traditional one for dynamic logic. First, the Kleene star α^* is not in Σ. This is because in the deontic reasoning domain, the repetition of some actions is not so usual as in computer program. On the other hand, the simultaneous execution and non-execution of actions are nonstandard in dynamic logic. In fact, the two constructs significantly complicate the formal semantics of PD_eL. Since the formal semantics is rather involved, we will only present the informal one based on the ordinary state transition semantics of dynamic logics.

A Kripke model for PD_eL is a quadruple $\langle W, \models, [\![\cdot]\!], opt\rangle$, where W is a set of possible worlds and opt is a nonempty subset of W, meaning the best elements of W, whereas $\models \subseteq W \times \Phi$ and $[\![\cdot]\!] : \Sigma \to \mathcal{P}(W \times W)$ define the truth relation and the action denotation function respectively. It is required that $\models$ and $[\![\cdot]\!]$ must satisfy the following constraints.

- For all $w \in W$, $\varphi, \psi \in \Phi$, and $\alpha \in \Sigma$,

  ($\models$0) $w \models V \Leftrightarrow w \notin opt$,

[1] The last symbol is added by me for later use.

($\models$1) $w \models \varphi \vee \psi \Leftrightarrow w \models \varphi$ or $w \models \psi$,

($\models$2) $w \models \neg\varphi \Leftrightarrow w \not\models \varphi$,

($\models$3) $w \models [\alpha]\varphi \Leftrightarrow \forall u \in \|\alpha\|(w), u \models \varphi$, where $\|\alpha\|(w)$ is defined as $\{u \mid (w, u) \in \|\alpha\|\}$.

- For all $w \in W$, $\alpha, \beta \in \Sigma$, and $\varphi \in \Phi$,

  ($\|\cdot\|1$): $\|\emptyset\| = \emptyset$, $\|\mathbf{u}\| = W \times W$, $\|\mathbf{i}\| = \{(w, w) \mid w \in W\}$.

  ($\|\cdot\|2$): $\|\alpha \cup \beta\| = \|\alpha\| \cup \|\beta\|$.

  ($\|\cdot\|3$): $\|\alpha; \beta\| = \|\alpha\| \circ \|\beta\|$.[2]

  ($\|\cdot\|4$): $\|\alpha \& \beta\| \subseteq \|\alpha\| \cap \|\beta\|$.

  ($\|\cdot\|5$): (negated actions)

  1. $\|\overline{\overline{\alpha}}\| = \|\alpha\|$
  2. $\|\alpha \cup \overline{\alpha}\| = \|\mathbf{u}\|$, $\|\alpha \& \overline{\alpha}\| = \|\emptyset\|$.
  3. $\|\overline{\alpha \cup \beta}\| = \|\overline{\alpha} \& \overline{\beta}\|$
  4. $\|\overline{\alpha \& \beta}\| = \|\overline{\alpha} \cup \overline{\beta}\|$
  5. $\|\overline{\alpha; \beta}\| = \|\overline{\alpha} \cup (\alpha; \overline{\beta})\|$
  6. $\|\overline{\varphi \to \alpha/\beta}\| = \|\varphi \to \overline{\alpha}/\overline{\beta}\|$

  ($\|\cdot\|6$): (conditional actions) $\|\varphi \to \alpha/\beta\|(w) = \begin{cases} \|\alpha\|(w), & \text{if } w \models \varphi, \\ \|\beta\|(w), & \text{if } w \not\models \varphi. \end{cases}$

As usual, the denotation of an action is just a state transition relation. However, unlike standard compound actions in traditional dynamic logics, the denotations of $\overline{\alpha}$ and $\alpha\&\beta$ are not functionally determined by those of their component actions. This makes it impossible to define the denotation function only for primitive actions and then extend it to all actions.

Then the deontic wffs are defined as abbreviations,

$$F_m\alpha = [\alpha]V,$$

$$P_m\alpha = \neg F_m\alpha = \langle\alpha\rangle\neg V,$$

$$O_m\alpha = F_m\overline{\alpha} = [\overline{\alpha}]V,$$

for all $\alpha \in \Sigma$. Here we use subscript m to indicate that the deontic operators are according to Meyer's definitions.

Let $S \cup \{\varphi\} \subseteq \Phi$, then φ is a PD_eL-consequence of S, denoted by $S \models_{PD_eL} \varphi$, iff for all PD_eL models and w, $w \models \psi$ for all $\psi \in S$ implies $w \models \varphi$.

To facilitate the comparison between PD_eL and the logics we will develop, we use a slight variant of it and still call it PD_eL. In the variant, we assume the deontic operators are primitive logical symbols and drop the propositional atom V, so the formation rules for the wffs are as follows: Φ is the smallest set such that

- $PV \subseteq \Phi$, and
- if $\varphi, \psi \in \Phi$ and $\alpha \in \Sigma$, then $\neg\varphi, \varphi \vee \psi, [\alpha]\varphi, O_m\alpha, P_m\alpha, F_m\alpha \in \Phi$.

As for the semantics, the clause ($\models$ 0) is replaced by three constraints for the deontic wffs:

($\models$0.1) $w \models F_m\alpha \Leftrightarrow \forall u \in \|\alpha\|(w), u \notin opt$,

($\models$0.2) $w \models P_m\alpha \Leftrightarrow \exists u \in \|\alpha\|(w), u \in opt$,

[2] i.e. the relational composition of $\|\alpha\|$ and $\|\beta\|$.

($\models$0.3) $w \models O_m\alpha \Leftrightarrow \forall u \in \llbracket\overline{\alpha}\rrbracket(w), u \notin opt$.

Although the variant is less expressive than the original PD_eL, we assume the propositional atom V is mainly used in the definition of deontic operators, so the modification will be inessential to PD_eL as a kind of deontic logic. The important point is that we keep the semantics unchanged. From now on, we will refer PD_eL only to this variant.

2.2 Possibility theory

Possibility theory is developed by Zadeh from fuzzy set theory[16]. Given a universe W, a *possibility distribution* on W is a function $\pi : W \to [0,1]$. A possibility distribution π on W is said to be finite-valued if the set $\{\pi(w) \mid w \in W\}$ is finite. For some technical reasons we will discuss later, we assume the possibility distributions used in this paper are finite-valued. In general, we require the pseudo-normalization condition is satisfied. That is, $\sup_{w\in W} \pi(w) > 0$ must hold. Obviously, π is a characteristic function of a nonempty fuzzy subset of W. Two measures on W can be derived from π. They are called possibility and necessity measures and denoted by Π and N respectively. Formally, $\Pi, N : 2^W \to [0,1]$ are defined as

$$\Pi(A) = \sup_{w\in A} \pi(w),$$

$$N(A) = 1 - \Pi(\overline{A}),$$

where $\overline{A}$ is the complement of A with respect to W. Another measure, called guaranteed possibility and denoted by Δ, is defined as

$$\Delta(A) = \inf_{w\in A} \pi(w),$$

in [7]. For convenience, we assume $\sup\emptyset = \inf\emptyset = 0$.

2.3 Two viewpoints of the semantics

What distinguishes the semantics of PD_eL from ordinary dynamic logic is the component *opt*. We can consider this component from two different viewpoints. In the first one, the possible worlds are divided into two levels by *opt*. The level 1 worlds are those in *opt* and the level 2 are those not. This means that the worlds in *opt* are preferred to those not. A natural generalization of the semantics from this viewpoint is to allow multiple levels of division of the possible worlds. This is essentially what we will do in the next two sections. As for the second viewpoint, the set *opt* can be considered as an obligation (or a norm) in the sense that a state is in *opt* iff it meets the requirement of the obligation. In this sense, PD_eL is a logic for single obligation, so we can generalize it into one for multiple obligations. However, in the logic for multiple obligations, it is inevitable that mutually conflicting obligations may exist in the same time and these obligations may be not equally important, so the further generalization is to allow multiple (and possibly conflicting) obligations with different degrees of importance in the semantics. This will be essentially the main topic of section 5. The main role the possibility distributions will play is to encode the degree of preference of possible worlds or the degree of importance of obligations.

3 A Graded Deontic Logic

While deontic logics are relevant to reasoning with norms, we may be also interested in the more general logic of value concepts. For example, in the decision-making context, we may select some actions to do according to our preference. Most research on the logics of preference is based on the analysis of probability and utility theory. However, since possibility theory models ordering relation in a natural way, it should be suitable to use the theory in the analysis of preference relations. In this section, we will first develop a kind of graded deontic logic, called

GD_eL, with semantics based on possibility theory. This logic, though does not involve with the comparison of action preference directly, may represent different degrees of permission, obligation and prohibition and will serve as a basis of prohairetic logics.

The alphabet of GD_eL is that of PD_eL without Meyer's deontic operators but with the addition of six classes of graded deontic operators O_g^c, P_g^c, F_g^c, $O_g^{>c}$, $P_g^{>c}$, and $F_g^{>c}$ for all $c \in [0, 1]$. The formation rule for wffs of GD_eL (Φ_1) will be

- $PV \subseteq \Phi_1$, and
- if $\varphi, \psi \in \Phi_1$ and $\alpha \in \Sigma_1$, then $\neg\varphi, \varphi \vee \psi, [\alpha]\varphi, \Box\alpha \in \Phi_1$, where $\Box = O_g^c, P_g^c, F_g^c, O_g^{>c}, P_g^{>c}$, or $F_g^{>c}$ for all $c \in [0, 1]$

The set of action expressions for GD_eL (Σ_1) is simultaneously defined with Φ_1 by the same rules as in PD_eL.

The intuitive meaning of these graded deontic formulas is to represent the degree of obligation, permission and prohibition respectively. For example, $P_g^c\alpha$ means that doing α is permitted at least to the degree c.

A model of GD_eL will be a quadruple $\langle W, \models, \llbracket \cdot \rrbracket, \pi \rangle$, where W and $\llbracket \cdot \rrbracket$ are defined as above, π is a possibility distribution on W, and $\models$ is as above except that the constraint ($\models$0) is replaced by

($\models$4) $w \models P_g^c\alpha$ (resp. $P_g^{>c}\alpha$) iff $\Pi(\llbracket\alpha\rrbracket(w)) \geq c$ (resp. $> c$).

($\models$5) $w \models F_g^c\alpha$ (resp. $F_g^{>c}\alpha$) iff $w \models \neg P_g^c\alpha$ (resp. $w \models \neg P_g^{>c}\alpha$).

($\models$6) $w \models O_g^c\alpha$ (resp. $O_g^{>c}\alpha$) iff $w \models F_g^c\ \overline{\alpha}$ (resp. $F_g^{>c}\ \overline{\alpha}$).

For any world w, the possibility value $\pi(w)$ denotes the degree of "goodness" of w, so the graded deontic formulas reflect the degree of goodness of the states to which some action will lead. The GD_eL consequence relation $\models_{GD_eL}$ is defined analogous to $\models_{PD_eL}$. Note that according to the semantics, $F_g^c\alpha$ is true means that α is permitted to the degree less than c, so the smaller the value of c is, the stronger the prohibition is. Since obligation of α is defined as prohibition of $\overline{\alpha}$, analogously, the larger the value of c is, the weaker the obligation is.

A GD_eL model can be viewed as a generalization of a PD_eL one since the characteristic function of the subset *opt* in the latter can be seen as a (crisp) possibility distribution. Hence we can expect that it is more expressive than PD_eL in the representation of deontic formulas. The following result verifies the expectation.

Let us fix any $c > 0$. Define a translation mapping $\tau_1 : \Phi \cup \Sigma \to \Phi_1 \cup \Sigma_1$ such that τ_1 satisfies the following conditions:

1. (basic condition): $\tau_1(p) = p$ if $p \in PV$ and $\tau_1(a) = a$ if $a \in A \cup \{\emptyset, \mathbf{u}, \mathbf{i}\}$,
2. (action morphism):
   - $\tau_1(\alpha; \beta) = \tau_1(\alpha); \tau_1(\beta)$,
   - $\tau_1(\alpha \cup \beta) = \tau_1(\alpha) \cup \tau_1(\beta)$,
   - $\tau_1(\alpha\&\beta) = \tau_1(\alpha)\&\tau_1(\beta)$,
   - $\tau_1(\overline{\alpha}) = \overline{\tau_1(\alpha)}$,
   - $\tau_1(\varphi \to \alpha/\beta) = \tau_1(\varphi) \to \tau_1(\alpha)/\tau_1(\beta)$
3. (classical morphism):
   - $\tau_1(\neg\varphi) = \neg\tau_1(\varphi)$
   - $\tau_1(\varphi \vee \psi) = \tau_1(\varphi) \vee \tau_1(\psi)$
   - $\tau_1([\alpha]\varphi) = [\tau_1(\alpha)]\tau_1(\varphi)$

4. (deontic translation): $\tau_1(\Box_m \alpha) = \Box_g^c \alpha$ for any $\alpha \in \Sigma$ and $\Box = O, P$, or F,

Theorem 1 *If $S \cup \{\varphi\} \subseteq \Phi$, then $S \models_{PD_eL} \varphi$ iff $\tau_1(S) \models_{GD_eL} \tau_1(\varphi)$,*

Proof: The proof of the theorem relies on the following lemma.

Lemma 1 *1. For each PD_eL model $M = \langle W, \models, \llbracket \cdot \rrbracket, opt \rangle$, we can find a GD_eL model $M' = \langle W, \models', \llbracket \cdot \rrbracket', \pi \rangle$ such that for all $\varphi \in \Phi$, $\alpha \in \Sigma$, and $w \in W$, $w \models \varphi$ iff $w \models' \tau_1(\varphi)$ and $\llbracket \alpha \rrbracket = \llbracket \tau_1(\alpha) \rrbracket'$.*

2. For each GD_eL model $M = \langle W, \models, \llbracket \cdot \rrbracket, \pi \rangle$, we can find a PD_eL model $M' = \langle W, \models', \llbracket \cdot \rrbracket', opt \rangle$ such that for all $\varphi \in \Phi$, $\alpha \in \Sigma$, and $w \in W$, $w \models' \varphi$ iff $w \models \tau_1(\varphi)$ and $\llbracket \alpha \rrbracket' = \llbracket \tau_1(\alpha) \rrbracket$.

Proof: First, we note that τ_1 is a 1-1 mapping, so for each $\varphi \in \tau_1(\Phi)$ and $\alpha \in \tau_1(\Sigma)$, the inverse mapping τ_1^{-1} is well-defined.

1. Given M, we define M' by
   - $\models'$: for $\varphi \in \tau_1(\Phi)$, $w \models' \varphi$ iff $w \models \tau_1^{-1}(\varphi)$, and for other wffs in Φ_1, $\models'$ can be defined arbitrarily subject to the constraints ($\models 1$)–($\models 6$).
   - $\llbracket \cdot \rrbracket'$: for $\alpha \in \tau_1(\Sigma)$, $\llbracket \alpha \rrbracket' = \llbracket \tau_1^{-1}(\alpha) \rrbracket$, and for other expressions in Σ_1, $\llbracket \cdot \rrbracket'$ can also be defined arbitrarily subject to the constraints ($\llbracket \cdot \rrbracket 1$)–($\llbracket \cdot \rrbracket 6$).
   - π: $\pi(w) = 1$ if $w \in opt$, otherwise, $\pi(w) = 0$

   Obviously, if M' is indeed a GD_eL model, then it satisfies our requirement. To verify that M' is a GD_eL model, we must show that for all expressions in $\Phi_1 \cup \Sigma_1$, the constraints ($\models$1)–($\models$6) and ($\llbracket \cdot \rrbracket$1)–($\llbracket \cdot \rrbracket$6) are satisfied. However, since for the expressions outside $\tau_1(\Phi \cup \Sigma)$, we have the choice of freedom, it suffice to do verification for the set $\tau_1(\Phi \cup \Sigma)$. The verification work is quite routine and we only consider a typical case with deontic formulas. If $\varphi = \tau_1(P_m \alpha) = P_g^c \tau_1(\alpha)$, then by definition of M', $w \models' \varphi$ iff $w \models P_m \alpha$ iff $\exists u \in \llbracket \alpha \rrbracket(w), u \in opt$ iff $\Pi(\llbracket \tau_1(\alpha) \rrbracket'(w)) = 1 \geq c$ (by the definition of π and $\llbracket \cdot \rrbracket'$), so ($\models$4) is satisfied.

2. From M, the M' is defined by
   - $\models'$: $w \models' \varphi$ iff $w \models \tau_1(\varphi)$
   - $\llbracket \cdot \rrbracket'$: $\llbracket \alpha \rrbracket' = \llbracket \tau_1(\alpha) \rrbracket$
   - $opt = \{w \mid \pi(w) \geq c\}$

   To verify that M' is a PD_eL model, we consider another typical case. If $\varphi = F_m \alpha$, then by definition, $w \models' \varphi$ iff $w \models F_g^c \tau_1(\alpha)$ iff $\Pi(\llbracket \tau_1(\alpha) \rrbracket(w)) < c$ iff for all $u \in \llbracket \alpha \rrbracket'(w)$, $\pi(u) < c$, i.e. $u \notin opt$, so ($\models$ 0.1) is satisfied. ■

Now, to prove the main theorem, assume $S \not\models_{PD_eL} \varphi$, then there exists a model M and $w \in W$ such that $w \models \psi$ for all $\psi \in S$ but $w \not\models \varphi$. By the lemma, we can also find a GD_eL model which is the witness of $\tau_1(S) \not\models_{GD_eL} \tau_1(\varphi)$. The converse can be proved analogously, so this completes the proof. ■

We remark that if in the definition of τ_1, we replace $\Box_g^c$ with $\Box_g^{>c}$ in the deontic translation condition for any $c < 1$, the result still holds. Therefore, each graded deontic operator behaves just like its classical counterpart. So what is the advantage of employing GD_eL? Why are infinitely many deontic operators behaving in the same way not just redundancy? The secret lies on the interaction of the class of graded deontic operators. If it is known that $P_g^c \alpha$ and $F_g^d \beta$ both hold where $d \leq c$, then we can reason that the degree of permission of α is higher than that of β, so we can compare the degree of permission, obligation and prohibition of different actions in GD_eL. This will be helpful under the decision-making environment.

4 A Logic of Action Preference

Though GD_eL can represent the degree of permission, obligation, and prohibition of actions directly, the comparison of preference between actions is not made explicit in the language. To exploit the full generality of our semantics, we can develop logics that include the comparative constructs directly.

To compare the preference between two actions, we will compare the resultant subsets of worlds after doing them. Now, given a possibility distribution on W and two subsets of W, A and B, there are at least four ways to compare them. Namely, we can define

$$A >_1 B \Leftrightarrow \Pi(A) > \Pi(B)$$

$$A >_2 B \Leftrightarrow \Delta(A) > \Delta(B)$$

$$A >_3 B \Leftrightarrow \Delta(A) > \Pi(B)$$

$$A >_4 B \Leftrightarrow \Pi(A) > \Delta(B),$$

and we have $A >_3 B$ implies $A >_1 B$ and $A >_2 B$ which in turn imply the weakest $A >_4 B$. In this section, we will explore a logic of action preference with binary connectives based on the orderings $>_1$ and $>_2$. The resultant logic will be called LAP.

The basic symbols of LAP are those of PD_eL without Meyer's deontic operators, but with two binary connectives $\succ_1$ and $\succ_2$. Let Φ_2 and Σ_2 denote respectively the set of wffs and action expressions of LAP, the formation rules for wffs are

- $PV \subseteq \Phi_2$, and
- if $\varphi, \psi \in \Phi_2$ and $\alpha \in \Sigma_2$, then $\neg\varphi, \varphi \vee \psi, [\alpha]\varphi \in \Phi_2$,
- If α and $\beta \in \Sigma_2$, then $\alpha \succ_1 \beta, \alpha \succ_2 \beta \in \Phi_2$,

and the formation rules for action expressions remain unchanged. The wff $\alpha \succ_i \beta$ is also written as $\beta \prec_i \alpha$ for $i = 1, 2$.

The semantic models are those for GD_eL with ($\models 4$)-($\models 6$) being replaced by

($\models$7) $w \models \alpha \succ_1 \beta$ iff $[\![\alpha]\!](w) >_1 [\![\beta]\!](w)$

($\models$8) $w \models \alpha \succ_2 \beta$ iff $[\![\alpha]\!](w) >_2 [\![\beta]\!](w)$

Let $\models_{LAP}$ denote the semantic consequence relation of LAP defined in a way analogous to PD_eL.

According to the semantics, $\alpha \succ_1 \beta$ denotes a comparison of actions based on the optimistic view, so α is preferred to β if the execution of α may lead to better worlds than β, whereas $\alpha \succ_2 \beta$ is a comparison based on the pessimistic view since α is preferred to β only when α produces better results than β under the worst condition.

The basic properties of LAP are as follows. Most of these properties easily follows from possibility theory. For simplicity, we omit the subscript LAP from $\models_{LAP}$ in the following propositions.

Proposition 1 *1.* $\models (\alpha_1 \cup \alpha_2 \succ_1 \beta) \equiv (\alpha_1 \succ_1 \beta) \vee (\alpha_2 \succ_1 \beta)$

2. $\models (\alpha_1 \cup \alpha_2 \prec_1 \beta) \equiv (\alpha_1 \prec_1 \beta) \wedge (\alpha_2 \prec_1 \beta)$

3. $\models (\alpha_1 \& \alpha_2 \succ_1 \beta) \supset (\alpha_1 \succ_1 \beta) \wedge (\alpha_2 \succ_1 \beta)$

4. $\models \neg(\alpha \& \beta \succ_1 \beta)$

5. $\models \neg(\alpha \succ_1 \alpha \cup \beta)$

6. $\models \alpha \succ_1 \beta \supset \neg(\overline{\alpha} \succ_1 \overline{\beta})$

7. $\models (\alpha;\beta_1 \succ_1 \alpha;\beta_2) \supset \langle\alpha\rangle(\beta_1 \succ_1 \beta_2)$

8. $\models (\varphi \to \alpha_1/\alpha_2 \succ_1 \beta) \equiv (\varphi \supset \alpha_1 \succ_1 \beta) \wedge (\neg\varphi \supset \alpha_2 \succ_1 \beta)$

Proposition 2 1. $\models (\alpha_1 \cup \alpha_2 \succ_2 \beta) \equiv (\alpha_1 \succ_2 \beta) \wedge (\alpha_2 \succ_2 \beta)$

2. $\models (\alpha_1 \cup \alpha_2 \prec_2 \beta) \equiv (\alpha_1 \prec_2 \beta) \vee (\alpha_2 \prec_2 \beta)$

3. $\neg(\alpha \succ_2 \alpha \& \beta)$

4. $\models (\alpha;\beta_1 \succ_2 \alpha;\beta_2) \supset \langle\alpha\rangle(\beta_1 \succ_2 \beta_2)$

5. $\models (\varphi \to \alpha_1/\alpha_2 \succ_2 \beta) \equiv (\varphi \supset \alpha_1 \succ_2 \beta) \wedge (\neg\varphi \supset \alpha_2 \succ_2 \beta)$

The action preference wffs can be combined with the conditional action wffs to denote a decision choice action. More specifically, define

$$\alpha \oplus_1 \beta = (\alpha \succ_1 \beta) \to \alpha/(\alpha \prec_1 \beta \to \beta/(\alpha \cup \beta)).$$

Then doing $\alpha \oplus_1 \beta$ will mean doing α or β selectively according to the optimistic preference relation $\succ_1$. The same definition can be carried out for $\succ_2$.

We can also embody the unary deontic operators of PD_eL by using the special actions $\mathbf{u}$ and $\emptyset$ or the comparison between an action and its negation. Define the following translation mappings, $\tau_2, \tau_3, \tau_4 : \Phi \cup \Sigma \to \Phi_2 \cup \Sigma_2$ such that they all satisfy basic condition, action morphism, classical morphism, and

$$\tau_2(P_m\alpha) = \neg(\mathbf{u} \succ_1 \tau_2(\alpha)), \tau_2(F_m\alpha) = \mathbf{u} \succ_1 \tau_2(\alpha), \tau_2(O_m\alpha) = \mathbf{u} \succ_1 \overline{\tau_2(\alpha)}$$

$$\tau_3(P_m\alpha) = \tau_3(\alpha) \succ_1 \emptyset, \tau_3(F_m\alpha) = \neg(\tau_3(\alpha) \succ_1 \emptyset), \tau_3(O_m\alpha) = \neg(\overline{\tau_3(\alpha)} \succ_1 \emptyset)$$

and

$$\tau_4(P_m\alpha) = \neg(\overline{\tau_4(\alpha)} \succ_1 \tau_4(\alpha)), \tau_4(F_m\alpha) = \overline{\tau_4(\alpha)} \succ_1 \tau_4(\alpha), \tau_4(O_m\alpha) = \tau_4(\alpha) \succ_1 \overline{\tau_4(\alpha)},$$

then we have the following result.

Theorem 2 *If $S \cup \varphi \subseteq \Phi$ then $S \models_{PD_eL} \varphi$ iff $\tau_i(S) \models_{LAP} \tau_i(\varphi)$ for $i = 2, 3, 4$.*

Proof: The key step of the proof still depends on the model transformation between these two logics. However, strictly speaking, the translation mappings τ_i's are not 1-1 any more. What we have is a weaker result. For $\varphi, \psi \in \Phi$, we said that φ and ψ are logically equivalent if for any PD_eL model M and $w \in W$, $w \models \varphi$ iff $w \models \psi$, and for $\alpha, \beta \in \Sigma$, we said that they are logically equivalent if for any PD_eL model M, $\|\alpha\| = \|\beta\|$. Then, we can show that if $\tau_i(\varphi) = \tau_i(\psi)$(resp. $\tau_i(\alpha) = \tau_i(\beta)$), then φ and ψ (resp. α and β) are logically equivalent. Thus, for the current purpose, we can again define the inverse mapping of τ_i by letting $\tau_i^{-1}(\varphi)$ be an arbitrary element in the equivalence class that is mapped to φ by τ_i. So, the model transformation process is just like that in the last proof. The main difference lies on the transformation between *opt* and π.

1. For the direction from a PD_eL model $M = \langle W, \models, \|\cdot\|, opt\rangle$ to a LAP model $M' = \langle W, \models', \|\cdot\|', \pi\rangle$, we just let π be the characteristic function of *opt* as above. To show that M' is indeed a LAP model when $\models'$ and $\|\cdot\|'$ are defined according to different τ_i's, we consider the typical cases.
   (a) If $\varphi = \neg(\mathbf{u} \succ_1 \tau_2(\alpha)) = \tau_2(P_m\alpha)$, then $w \models' \varphi$ iff there exists $u \in \|\alpha\|(w)$ such that $u \in opt$ iff $\Pi(\|\tau_2(\alpha)\|'(w)) = 1 \geq \Pi(\|\mathbf{u}\|'(w))$, so ($\models$7) is satisfied for this type of wffs.
   (b) If $\varphi = \tau_3(\alpha) \succ_1 \emptyset = \tau_3(P_m\alpha)$ then $w \models' \varphi$ iff there exists $u \in \|\alpha\|(w)$ such that $u \in opt$ iff $\Pi(\|\tau_3(\alpha)\|'(w)) = 1 > \Pi(\|\emptyset\|'(w))$, so ($\models$7) is satisfied for this type of wffs.

(c) If $\varphi = \neg(\overline{\tau_4(\alpha)} \succ_1 \tau_4(\alpha)) = \tau_4(P_m\alpha)$ then $w \models' \varphi$ iff there exists $u \in \llbracket\alpha\rrbracket(w)$ such that $u \in opt$ iff $\Pi(\llbracket\tau_4(\alpha)\rrbracket'(w)) \geq \Pi(\llbracket\overline{\tau_4(\alpha)}\rrbracket'(w))$ since opt is nonempty and by ($\llbracket\cdot\rrbracket$5.1) and ($\llbracket\cdot\rrbracket$1), $\llbracket\tau_4(\alpha)\rrbracket'(w) \cup \llbracket\overline{\tau_4(\alpha)}\rrbracket'(w) = W$.

2. For the other direction from a LAP model $M = \langle W, \models, \llbracket\cdot\rrbracket, \pi\rangle$ to a PD_eL model $M' = \langle W, \models', \llbracket\cdot\rrbracket', opt\rangle$, we consider the three cases separately.

   (a) For τ_2, let $opt = \{w \mid \pi(w) = \Pi(W)\}$. Since our possibility distribution π is finite-valued, opt is nonempty. This is the technical reason why we restrict our attention to finite-valued possibility distributions [3]. Then we can show that $w \models P_m\alpha$ iff $\Pi(\llbracket\tau_2(\alpha)\rrbracket(w)) \geq \Pi(W)$ iff there exists $u \in \llbracket\alpha\rrbracket'(w)$ such that $\pi(u) = \Pi(W)$, i.e. $u \in opt$ since π is finite-valued.

   (b) For τ_3, let $opt = \{w \mid \pi(w) \neq 0\}$. opt is again nonempty since π is pseudo-normal. We can prove that $w \models P_m\alpha$ iff $\Pi(\llbracket\tau_3(\alpha)\rrbracket(w)) > 0$ iff there exists $u \in \llbracket\alpha\rrbracket'(w)$ such that $\pi(u) > 0$, i.e. $u \in opt$.

   (c) For τ_4, we can use either one of the above two transformations. For example, let $opt = \{w \mid \pi(w) = \Pi(W)\}$, then $w \models P_m\alpha$ iff $\Pi(\llbracket\tau_2(\alpha)\rrbracket(w) \geq \Pi(\llbracket\overline{\tau_2(\alpha)}\rrbracket(w)$ iff $\Pi(\llbracket\tau_2(\alpha)\rrbracket(w) = \Pi(W)$ iff there exists $u \in \llbracket\alpha\rrbracket'(w)$ such that $\pi(u) = \Pi(W)$, i.e. $u \in opt$ since π is finite-valued.

The remaining part of the proof is completely the same as that for the last theorem. ■

The proof of the theorem shows how we can transform a multiple level preference model into a two-level one. The different translations of deontic formulas in LAP reflect the different ways we use in the transformation. In the first translation, we let optimal worlds be those with maximal degree of preference, while in the second one, we let optimal worlds be those not minimal in the preference ordering. As for the third one, we can in fact take a threshold value, and let any worlds with degree of preference beyond the threshold be optimal worlds.

The result also shows that LAP is indeed more expressive than PD_eL in deontic reasoning. Since the optimistic action preference connective $\succ_1$ is used in the definition of Meyer's deontic operators, it is expected that the same thing can be done for $\succ_2$. For example, we can define

$$O_2\alpha = \alpha \succ_2 \overline{\alpha},$$

$$F_2\alpha = \overline{\alpha} \succ_2 \alpha,$$

$$P_2\alpha = \neg(\overline{\alpha} \succ_2 \alpha),$$

by the pessimistic preference connective.

Furthermore, we can also define some derived modal operators by using the special action $\mathbf{i}$. Let

$$I_1\alpha = \alpha \succ_1 \mathbf{i}, D_1\alpha = \overline{\alpha} \succ_1 \mathbf{i},$$

$$I_2\alpha = \alpha \succ_2 \mathbf{i}, D_2\alpha = \overline{\alpha} \succ_2 \mathbf{i},$$

then $I_1\alpha$ (resp. $I_2\alpha$) means it is strongly (resp. weakly) inclined to do α, whereas $D_1\alpha$ (resp. $D_2\alpha$) means it is strongly (resp. weakly) declined to do α. These derived operators should be useful for the representation and reasoning of agents' mental attitudes in the agent-oriented programming[14].

Before proceding to a more general logic, we would like to consider an illustrative example. The example is originally given in the decision theory context by Savage[**?**] and recently cited in [**?**].

Example 1 (Savage's omelette example) The problem is about the decision of adding an egg to a 5-egg omelette. The egg may be good or rotten and we have three available acts.

[3] A way to lift the restriction is to provide a finite model property for our logics. That is, for any wff φ, if we have a model and a world w such that $w \models \varphi$, then we have a finite model and w' in it such that $w' \models \varphi$. Although we can not prove the finite model property for these logics in the mean time, we strongly believe it holds.

1. a_1: break the egg in the omelette.

2. a_2: break it apart in a cup.

3. a_3: throw it away.

The possible outcomes of doing a_1 are s_1: 6-egg omelette (when the egg is good) or s_2: a wated omelette (when the egg is rotten), those for doing a_2 are s_3: 6-egg omelette and a cup to wash or s_4: 5-egg omelette and a cup to wash, and those for doing a_3 are s_5: 5-egg omelette and a wasted egg or s_6: 5-egg omelette. Let s_0 denote the initial state, then by our notation, this means that

1. $[\![a_1]\!](s_0) = \{s_1, s_2\}$,

2. $[\![a_2]\!](s_0) = \{s_3, s_4\}$,

3. $[\![a_3]\!](s_0) = \{s_5, s_6\}$.

To denote the preference between possible outcomes of these acts, we use a possibility distribution π such that $\pi(s_1) = 1$, $\pi(s_2) = 0$, $\pi(s_3) = 0.8$, $\pi(s_4) = 0.4$, $\pi(s_5) = 0.2$, $\pi(s_6) = 0.6$. Then according to the semantics of GD_eL, in state s_0, we have $P_g^1 a_1$, $P_g^{0.8} a_2$, $P_g^{0.6} a_3$, $F_g^{0.9} a_2$, and $F_g^{0.7} a_3$, but we do not have $F_g^c a_1$ for any c. This means that a_1 is fully permitted and not forbidden at all, whereas a_2 is almost completely permitted and slightly forbidden and a_3 is moderately permitted and forbidden. On the other hand, since in this case, all a_i's are mutually exclusive (i.e., no two a_i's can be done simultaneously), we have $\overline{a_1} = a_2 \cup a_3$, $\overline{a_2} = a_1 \cup a_3$, and $\overline{a_3} = a_1 \cup a_2$, so we have $O_g^{0.9} a_1$, but do not have $O_g^c a_2$ or $O_g^c a_3$ for any c. This means that only a_1 is slightly obligatory, whereas a_2 and a_3 are not obligatory at all although they are permitted to some degree.

Let us turn to the semantics of LAP, then in s_0, we have $a_1 \succ_1 a_2 \succ_1 a_3$ and $a_2 \succ_1 a_1 \succ_1 a_3$. This means that from an optimistic view, a_1 is the preferred action, whereas from a pessimistic one, the best choice is a_2. In both cases, a_3 is the worst choice. Furthermore, if we adopt the translation mappings τ_2 or τ_4, then we have $O_m a_1$, $F_m a_2$, and $F_m a_3$, so only a_1 is permitted (it is even obligatory), and a_2 and a_3 are forbidden. However, if we use the translation τ_3, then all a_i's are permitted and none of them are obligatory or forbidden. ■

5 A Logic for Action under Conflicting Obligations

Recently, a logic for conflicting obligation has been proposed by Brown[3], in which the preference ordering is set between *obligations* instead of worlds, where an obligation is represented by a subset of possible worlds. Since the intersection of two sets of possible worlds may be empty, the conflicting obligations can be represented in the framework naturally. Then four unary deontic operators and a dyadic comparative operator are defined. Their semantics based on the combination of quantifiers on the set of obligation and traditional deontic operators O and P. For example, one of them means for all obligations φ is obligatory. The formal semantic models of Brown's logic are the so-called minimal models in modal logic[4].

In this section, we will propose an alternative logic for action under conflicting obligations ($LACO$) by assimilating Brown's idea into Meyer's framework.

The language is just that of PD_eL (without Meyer's deontic operators) with six classes of unary deontic operators, $\langle c\rangle_O$, $\langle c\rangle_O^+$, $\langle c\rangle_P$, $\langle c\rangle_P^+$, $\langle c\rangle_F$, and $\langle c\rangle_F^+$ for all $c \in [0,1]$, and six binary connectives, $\succ_i^j$ ($i = 1,2, j = P, O, F$). Let Φ_3 and Σ_3 denote the set of wffs and action expressions for LACO respectively, then the formation rules for the wffs are

- $PV \subseteq \Phi_3$, and
- if $\varphi, \psi \in \Phi_3$ and $\alpha \in \Sigma_3$, then $\neg\varphi, \varphi \vee \psi, [\alpha]\varphi, \langle c\rangle_O\alpha, \langle c\rangle_O^+\alpha, \langle c\rangle_P\alpha, \langle c\rangle_P^+\alpha, \langle c\rangle_F\alpha, \langle c\rangle_F^+\alpha \in \Phi_3$,

- If α and $\beta \in \Sigma_3$, then $\alpha \succ_i^j \beta (i = 1, 2, j = P, O, F) \in \Phi_3$,

A model of $LACO$ is then a quadruple $\langle W, \models, \| \cdot \|, \pi \rangle$, where W and $\| \cdot \|$ are defined as above. However, π is now a possibility distribution on $\mathcal{P}(W)$ such that $\pi(\emptyset) = 0$, and $\models$ satisfies the constraints ($\models 1$)–($\models 3$) and the following:

($\models 9$): $w \models \langle c \rangle_P \alpha$(resp. $\langle c \rangle_P^+ \alpha$) iff $\Pi(\{S \mid S \cap \|\alpha\|(w) \neq \emptyset\}) \geq c$(resp. $> c$)

($\models 10$): $w \models \langle c \rangle_O \alpha$(resp. $\langle c \rangle_O^+ \alpha$) iff $\Pi(\{S \mid S \cap \|\overline{\alpha}\|(w) = \emptyset\}) \geq c$(resp. $> c$)

($\models 11$): $w \models \langle c \rangle_F \alpha$(resp. $\langle c \rangle_F^+ \alpha$) iff $\Pi(\{S \mid S \cap \|\alpha\|(w) = \emptyset\}) \geq c$(resp. $> c$)

($\models 12$): $w \models \alpha \succ_i^P \beta$ iff $\{S \mid S \cap \|\alpha\|(w) \neq \emptyset\} >_i \{S \mid S \cap \|\beta\|(w) \neq \emptyset\} (i = 1, 2)$

($\models 13$): $w \models \alpha \succ_i^F \beta$ iff $\{S \mid S \cap \|\alpha\|(w) = \emptyset\} >_i \{S \mid S \cap \|\beta\|(w) = \emptyset\} (i = 1, 2)$

($\models 14$): $w \models \alpha \succ_i^O \beta$ iff $\{S \mid S \cap \|\overline{\alpha}\|(w) = \emptyset\} >_i \{S \mid S \cap \|\overline{\beta}\|(w) = \emptyset\} (i = 1, 2)$

In the semantics above, the set $\{S \mid S \cap \|\alpha\|(w) \neq \emptyset\}$ include all obligations that permit the execution of α in w, if we consider the possibility distribution π encode the degree of importance of these obligations, then $\Pi(\{S \mid S \cap \|\alpha\|(w) \neq \emptyset\})$ is just the degree of importance of the most important obligations that permit the execution of α. Thus $\langle c \rangle_P \alpha$ means that α is permitted according to an obligation with degree of importance at least c. Similarly, $\langle c \rangle_O \alpha$ denotes that α is obligatory according to an obligation with degree of importance at least c, while $\langle c \rangle_F \alpha$ means that α is forbidden according to an obligation with degree of importance at least c. Moreover, $\alpha \succ_1^P \beta$ denote α is permitted by some obligation that is more important than those permitting β, and $\alpha \succ_2^P \beta$ means that all obligations permitting α are more important than those permitting β. The similar interpretations can be given to $\alpha \succ_i^O \beta$ and $\alpha \succ_i^F \beta$.

Obviously, a LACO model is a generalization of a PD_eL one if we consider the *opt* component in the latter as a single obligation. However, it is in fact also a generalization of a GD_eL and LAP model. To see that, let us first prove a basic property in possibility theory.

Lemma 2 *1. For each possibility distribution π on W, we can find a π' on 2^W such that for each $U \subseteq W$*

$$\Pi(U) = \Pi'(\{S \mid S \cap U \neq \emptyset\})$$

2. For each possibility distribution π on 2^W,we can find a π' on W such that for each $U \subseteq W$

$$\Pi'(U) = \Pi(\{S \mid S \cap U \neq \emptyset\})$$

Proof:

1. The π' is defined by

$$\pi'(S) = \begin{cases} \pi(w), & \text{if } S = \{w\}, \\ 0, & \text{otherwise.} \end{cases}$$

for all $S \subseteq W$

2. The π' is defined by

$$\pi'(w) = \Pi(\{S \mid w \in S\}),$$

for each $w \in W$.

■

Thus give a GD_eL or LAP model, we can easily find a corresponding $LACO$ model by changing π to π', and vice verse. By using the approach in the preceding sections, we can also prove the similar translation results between GD_eL (resp. LAP) and $LACO$. Let $\tau_5 : \Phi_1 \cup \Sigma_1 \to \Phi_3 \cup \Sigma_3$ and $\tau_6 : \Phi_2 \cup \Sigma_2 \to \Phi_3 \cup \Sigma_3$ satisfy the basic condition, classical morphism, action morphism, and

$$\tau_5(P_g^c\alpha) = \langle c\rangle_P \tau_5(\alpha), \tau_5(P_g^{>c}\alpha) = \langle c\rangle_P^+ \tau_5(\alpha),$$

$$\tau_5(F_g^c\alpha) = \neg\tau_5(P_g^c\alpha), \tau_5(F_g^{>c}\alpha) = \neg\tau_5(P_g^{>c}\alpha), \tau_5(O_g^c\alpha) = \neg\tau_5(P_g^c\overline{\alpha}), \tau_5(O_g^{>c}\alpha) = \neg\tau_5(P_g^{>c}\overline{\alpha}),$$

$$\tau_6(\alpha \succ_i \beta) = \tau_6(\alpha) \succ_i^P \tau_6(\beta).$$

Then, we have

Theorem 3 *1. If $S \cup \{\varphi\} \subseteq \Phi_1$, then $S \models_{GD_eL} \varphi$ iff $\tau_5(S) \models_{LACO} \tau_5(\varphi)$.*

2. If $S \cup \{\varphi\} \subseteq \Phi_2$, then $S \models_{LAP} \varphi$ iff $\tau_6(S) \models_{LACO} \tau_6(\varphi)$.

Note that in the translation mapping τ_5, the graded deontic wffs $F_g^c\alpha$ is mapped to a wff of the form $\neg\langle c\rangle_P\alpha'$ instead of $\langle c\rangle_F\alpha'$. The former means that for all obligations with degree of importance at least c, α' is forbidden, while the latter only says that there exists such an obligation, so if there are indeed some obligations with degree of importance at least c(i.e. $\Pi(2^W) \geq c$), the former implies the latter. The same remark is applied to wffs of the form $F_g^{>c}\alpha$, $O_g^c\alpha$, and $O_g^{>c}\alpha$. This means that in $LACO$, in addition to the graded deontic wffs in GD_eL, we can also express some weaker notions of partial prohibition and obligation. Furthermore, we have four additional binary connectives $\succ_i^O$ and $\succ_i^F$ $(i = 1, 2)$ that are not available in LAP. Thus, $LACO$ indeed improves the expressive power of GD_eL and LAP further.

6 Concluding Remarks

In this paper, the possibility theory and Meyer's semantics for deontic logics are combined to provide a semantics for action preference. We generalize Meyer's model step by step to provide the semantic basis for three logics. The logic GD_eL can reason about graded normative behavior. In the logic LAP, we can compare the preference between actions and do decision choice according to the comparison. The third logic, LACO, is based on the semantics of conflicting obligations. Our main theorems show that PD_eL is less expressive than either GD_eL or LAP in deontic reasoning, which in turn are less expressive than the most general logic, $LACO$. In what follows, we consider some related work and possible further research.

6.1 Related Works

The deontic operators and preference connectives considered in this paper are all applied to actions. However, there have been volumes of works on the deontic and preference logics of propositions[2, 13]. This kind of logics can also be considered in the present framework.

First, for GD_eL, wffs of the form $\Box_g^c\varphi$ and $\Box_g^{>c}\varphi$ can be added for $\Box = O, F, P$ and $\varphi \in \Phi$. In the semantics, replace $[\![\alpha]\!](w)$ by $|\varphi| =_{def} \{w \mid w \models \varphi\}$ and $\overline{\alpha}$ by $\neg\varphi$. Then when φ is a non-modal formula, $P_g^c\varphi$ and $O_g^c\varphi$ are just the wffs $(\varphi\ \Pi\ c)$ and $(\varphi\ N\ 1 - c)$ in possibilistic logic[6]. This also gives possibilistic logic a deontic interpretation.

Second, for LAP, we can add wffs of the form $\varphi \succ_1 \psi$ and interpret it as $|\varphi| >_1 |\psi|$ for any wffs φ and ψ. Again, this type of wffs are just equivalent to those of another well-known logics, called qualitative possibility logic (QPL), developed by Farinas del Cerro and Herzig[5]. It has been known that QPL has close relationship with conditional logic and nonmonotonic consequence relation[10], so the extension of the present framework to covering conditional and defeasible obligation will deserve further consideration.

6.2 Further generalization

In the above-mentioned semantics, a fixed possibility distribution is associated with a model, so the logics are of static preference. However, there is no essential difficulty in associating different possibility distributions with every possible world. Then we can get a variable preference relation. We believe that this step is necessary in developing dyadic deontic logics.

Second, in the present framework, the denotation of each action is a binary transition relation on the set of possible worlds. That is, we do not consider the action uncertainty. Though $[\![\alpha]\!](w)$ denote the all possibilities of doing α in w, it is not known which world is more possible than others. In the further research, we can interpret $[\![\alpha]\!]$ as a fuzzy binary relation so that $[\![\alpha]\!](w)$ is a possibility distribution or assign $[\![\alpha]\!](w)$ as a probability measure on W directly, then the expected payoff of doing α in w can be estimated as

$$\bigoplus_{u \in W} [\![\alpha]\!](w)(u) \otimes \pi(u),$$

where $\otimes$ and $\oplus$ are a kind of t-norms and co-t-norms respectively. Then the preference between two action can be compared via their expected payoff. This will also provide a kind of decision-theoretic semantics for our logic.

Third, the use of possibility theory forces our preference relation on possible worlds (or sets of possible worlds in the case of LACO) to be a connected ordering. This results in that we have only "don't care" incomparability (i.e. $\neg(w > u) \wedge \neg(u > w)$) but do not have "don't know" incomparability. This is indeed a limitation of our semantics, however, it can be easily overcome by allowing partial ordering between worlds. But the semantics then have to be written in qualitative terms. For example, $w \models \alpha \succ_1 \beta$ iff there exists $u \in [\![\alpha]\!](w)$ such that for all $v \in [\![\beta]\!](w)$, $u > v$, where $>$ is a partial ordering between worlds.

Forth, our system is only for single agent. To model multiagent environment, we must associate a preference relation for each agent in the model, then some actions, such as "request" and "commit", that will change the agents' preference can be considered in the framework and it can serve as a semantic basis of agent-oriented programming[14]. Moreover, the interference of different agents will change the choice of decision makers just like the situations analyzed by game theoreticians. These problems will all be considered in our long-term goal to the integration of different mental attitudes in a logical framework.

Finally, because the paper mainly concentrate on the semantic comparison of different logics, the proof-theoretical aspect of these logics are completely ignored. In fact, Meyer has provided an axiomatic system for PD_eL. Since we have shown that the graded deontic operators in GD_eL behave just like Meyer's ones, we can obtain an axiomatic system for GD_eL by replacing classical deontic operators with graded ones and adding some bookkeeping axioms like $P_g^c\alpha \supset P_g^d\alpha$ if $c > d$. As for the axiomatic system for LAP, we believe the properties given in propositions 1 and 2 will provide a basis. The further development of these properties into an axiomatic system and the study of axiomatic system for $LACO$ are all interesting topics for future research.

References

[1] A.R. Anderson. "A reduction of deontic logic to alethic modal logic". *Mind*, 67:100–103, 1958.

[2] L. Åqvist. "Deontic logic". In D.M. Gabbay and F. Guenthner, editors, *Handbook of Philosophical Logic, Vol II: Extensions of Classical Logic*, pages 605–714. D. Reidel Publishing Company, 1984.

[3] M.A. Brown. "A logic of comparative obligation". In A.J. Jones and M. Sergot, editors, *Proc. of the 2nd International Workshop on Deontic Logic in Computer Science*, pages 37–55. Oslo, Tano, 1994.

[4] B.F. Chellas. *Modal Logic : An Introduction.* Cambridge University Press, 1980.

[5] L. Farinas del Cerro and A. Herzig. "A modal analysis of possibility theory". In J. Kelemen Ph. Jorrand, editor, *Fundamentals of Artificial Intelligence Research (FAIR'91)*, LNAI 535, pages 11–18. Springer-Verlag, 1991.

[6] D. Dubois, J. Lang, and H. Prade. "Possibilistic logic". In D.M. Gabbay, C.J. Hogger, and J.A. Robinson, editors, *Handbook of Logic in Artificial Intelligence and Logic Programming, Vol 3 : Nonmonotonic Reasoning and Uncertain Reasoning*, pages 439–513. Clarendon Press - Oxford, 1994.

[7] D. Dubois and H. Prade. "Possibility theory as a basis for preference propagation in automated reasoning". In *Proc. of IEEE International Conference on Fuzzy Systems*, pages 821–832, 1992.

[8] D. Harel. "Dynamic logic". In D.M. Gabbay and F. Guenthner, editors, *Handbook of Philosophical Logic, Vol II: Extensions of Classical Logic*, pages 497–604. D. Reidel Publishing Company, 1984.

[9] C.-J. Liau. "A semantics for logics of preference based on possibility Theory". In *Proc. of 7th International Fuzzy Systems Association World Congress*, pages 243–248, 1997.

[10] C.-J. Liau and I-.P Lin. "Possibilistic Reasoning—A Mini-survey and Uniform Semantics". *Artificial Intelligence*, 88:163–193, 1996.

[11] J.-J. Ch. Meyer. "A different approach to deontic logic: Deontic logic viewed as a variant of dynamic logic". *Notre Dame J. of Formal Logic*, 29(1):109–136, 1988.

[12] J.-J. Ch. Meyer and R.J. Wieringa. *Deontic Logic in Computer Science: Normative System Specification*. John Wiley & Sons Ltd., 1993.

[13] N.J. Moutafakis. *The Logics of Preference: A Study of Prohairetic Logics in Twentieth Century Philosophy*. D. Reidel Publishing Comp., 1987.

[14] Y. Shoham. "Agent-oriented programming". *Artificial Intelligence*, 60(1):51–92, 1993.

[15] G.H. von Wright. "Deontic logic". *Mind*, 60:1–15, 1951.

[16] L.A. Zadeh. "Fuzzy sets as a basis for a theory of possibility". *Fuzzy Sets and Systems*, 1(1):3–28, 1978.