"NEXT" HEURISTIC FOR ON-THE-FLY MODEL CHECKING

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ABSTRACT. We present a new heuristic for on-the-fly enumerative invariant verification. The heuristic is based on a construct for temporal scaling, called **next**, that compresses a sequence of transitions leading to a given target set into a single metatransition. First, we give an on-the-fly algorithm to search a process expression built using the constructs of hiding, parallel composition, and temporal scaling. Second, we show that as long the target set Θ of transitions includes all transitions that access variables shared with the environment, the process next Θ for P and P are equivalent according to the weak-simulation equivalence. As a result, to search the product of given processes, we can cluster processes into groups with as little communication among them as possible, and compose the groups only after applying appropriate hiding and temporal scaling operators. Applying this process recursively gives an expression that has multiple nested applications of **next**, and has potentially much fewer states than the original product. We report on an implementation, and show significant reductions for a tree-structured parity computer and a ring-structured leader-election protocol.

1. Introduction

Model checking [CE81, QS82, CK96, Hol97] has proved to be a useful technique for automatic debugging of high-level designs of hardware and protocols. Model checking requires search of the global state-space of the design, and since the number of global states increases exponentially with the size of the description, model checking tools must employ a variety of heuristics to battle this so-called *state explosion problem*. In this paper, we present a new heuristic for enumerative verification of invariant requirements. Before we describe our approach, let us briefly review two techniques, on-the-fly search of reachable state-space, as implemented in tools such as Spin [Hol97] and Murφ [DDHY92], and reduction techniques based on process equivalences, as implemented in tools such as *Concurrency workbench* [CPS93] and CADP [FGK$^+$96].

In on-the-fly model checking, the system is described as a collection of communicating processes. The global state-transition graph of the system is explored on demand starting from initial states using a systematic search algorithm. Only the states, and not the transitions, are stored in a global hash-table, and as soon as a violation of the requirement is encountered, a counter-example is reported to the user. A variety of heuristics have been shown to be effective in restricting the search to a subset of the global state-space. Sample heuristics are reductions using symmetries [ES93, CFJ93, ID96] and partial orders [Pel94, Val92, God90].

In the so-called compositional approaches, the system is described using a richer language (e.g. CCS [Mil80]) that supports operators such as parallel composition and hiding. Compositional techniques rely on the definition of a process equivalence (e.g. weak bisimulation) and algorithms to reduce a process with respect to this equivalence [KS90, PT87, CPS93, BFH90, BRdSV87]. Starting with a complex expression, subexpressions can then be replaced by equivalent smaller ones in a bottom-up fashion. The order in which processes are composed reflects the connectivity, and a judicious use of hiding allows greater reductions by making more transitions internal. For well-structured designs such as trees and rings, this approach can yield significant reductions in principle. However, it does not guarantee early detection of violations which is extremely important in typical applications of model checking. Furthermore, processing of subexpressions can be expensive. First, minimization with respect to an equivalence is typically $O(n \log n)$ or $O(n^2)$, where n is the number of states in the unreduced graph. While minimization can be replaced by less expensive heuristic reduction algorithms, as far as we know, all the tools explicitly build the state-transition graphs for the subexpressions. Second, a subexpression, when considered on its own, has lot more states than when considered in the context of the whole expression. Heuristic approaches to capture the context as interfaces to subexpressions during reduction have been proposed [KM97], but offer only a partial remedy to this problem.

In this paper, we propose an on-the-fly search algorithm that takes into account the compositional structure of the system. The reduction performed by the algorithm on subexpressions compresses internal transitions, and preserves the weak-simulation equivalence. While the basic reduction strategy is simple and well known, the novelty, and the technical challenge, lies in applying it on demand in a hierarchical manner.

We develop our search algorithm in the more general setting of explicitly structured transition relations. Our process model, in addition to the standard operations of hiding and parallel composition, employs a construct for hierarchically structuring transition relations. For a process P and a set Θ of target transitions, the process $\mathsf{next}\ \Theta\ \mathsf{for}\ P$ is obtained by compressing a sequence of transitions of P ending in a Θ-transition into a single metatransition. The **next** operator is a temporal scaling operator inspired by notions of multiform time in synchronous languages [BG88, Hal93], and was introduced in the language *reactive modules* [AH96]. The parallel composition and temporal scaling can be mixed freely giving complex processes. Note that applications of **next** can cause significant reductions in the global state-space by ruling out transient states. For instance, the expression $\mathsf{next}\ \Theta\ \mathsf{for}\ P \parallel \mathsf{next}\ \Sigma\ \mathsf{for}\ Q$ can have much fewer reachable states than $P \| Q$: in searching $P \| Q$ at every step either P or Q is allowed to take a step, while in searching $\mathsf{next}\ \Theta\ \mathsf{for}\ P \parallel \mathsf{next}\ \Sigma\ \mathsf{for}\ Q$, once

P is chosen, it is executed until it takes a Θ-transition, and once Q is scheduled, it is executed until it takes a Σ-transition.

The first challenge is to design an on-the-fly algorithm to search a process expression built using parallel composition and temporal scaling. Such an algorithm is presented in Section 3. It generates global states only on demand, and can do early reporting of violations of invariants. To avoid recomputation, it stores in the global hash-table, along with the global states, transient states generated during processing of **next** operator. It is guaranteed to visit every reachable state of the given expression, and visits no more, and typically much less, than that required to explore the flattened expression obtained by removing the applications of **next**. The running time is linear in the number of states (more precisely, the transitions) visited by the algorithm, and there is basically no overhead in applying the reduction.

Then, we proceed to identify restriction under which the answer to the invariant verification problem is independent of the application of **next**. Along the lines of well known results concerning internal transitions [BRdSV87], we establish that if Θ includes those transitions that read or write shared variables then the processes P and next Θ for P are equivalent according to the *weak-simulation* relation. Since weak-simulation is shown to be a congruence with repsect to all the operators in our language, we can substitute freely and repeatedly any subexpression with **next** applied to it. Thus, given a problem to search the composition of a collection of processes, we can heuristically build an expression that clusters the given processes to limit the communication variables across clusters, and apply hiding and temporal scaling to individual clusters before composing them. This process is applied recursively to maximize the reduction. The resulting expression is then searched using the algorithm of Section 3.

The heuristic is demonstrated on a couple of benchmark examples using a prototype implementation. The first example concerns a tree-structured system which computes the parity of inputs presented at the leaves. This example is ideally suited for our reduction: while the number of reachable states of the global system increases exponentially with the size of the tree, the number of reachable states of the reduced search increases almost linearly. The second example concerns a standard leader election algorithm in a ring network. While the reduction is less dramatic, it is still significant for both space and time. The experimental results also support our analysis that the running time is linear in the number of states visited.

The rest of the paper is organized as follows. Section 2 presents our process model. Section 3 gives the search algorithm for process expressions employing **next** and parallel composition. Section 4 presents the heuristic application of **next** and justifies its correctness using the theory of simulation equivalences. Section 5 gives experimental results. Finally, in Section 6, we conclude with comparison with other heuristics.

2. Process Model

We start with the definition of processes. Our process model uses interleaving semantics, and read-shared exclusive-write variables. The model is a simplification of *reactive modules* [AH96].

Let X be a set of typed variables. A *state* over X is a function mapping variables to their values. Let s be a state. Define $s[v/x]$ to be the state obtained from s by replacing the value of x by v. Similarly, let t be a state over X, we denote simultaneous substitution by $s[t/X]$.

A process is defined by the set of its variables, rules for initializing the variables, and rules for updating the variables. The variables of a process P are partitioned into three classes: *private* variables that cannot be read or written by other processes, *interface* variables that are written only by P, but can be read by other processes, and *external* variables that can only be read by P, and are written by other processes. Thus, interface and external variables are used for communication, and are called *observable* variables. The process controls its private and interface variables, and the environment controls the external variables. Once the variables are defined, the state space of the process is determined. A state is also partitioned into different components as the variables are, for

instance, controlled state and external state. The initialization specifies initial controlled states, and the transition relation specifies how to change the controlled state as a function of the current state.

Definition 1. *A* process P *is a tuple* (X, I, T) *where*

- $X = (X_p, X_i, X_e)$ *is the (typed) variable declaration.* X_p, X_i, X_e *represent the sets of* private variables, interface variables *and* external variables *respectively. Define the* controlled variables $X_c = X_p \cup X_i$ *and the* observable variables $X_o = X_i \cup X_e$*;*
- Q_c *is the set of* controlled states *over* X_c *and* Q_e *is the set of* external states *over* X_e*.* $Q = Q_c \times Q_e$ *is the set of states. We also define* Q_o *to be the set of* observable states *over* X_o*;*
- $I \subseteq Q_c$ *is the set of* initial states*;*
- $T \subseteq Q_c \times Q_e \times Q_c$ *is the* transition relation*. We use the notation* $q \xrightarrow{e} q'$ *for* $(q, e, q') \in T$*.*

■

From a state q, a *step* can be taken in two ways. If the process makes its move, it changes its controlled state according to the transition relation. If the environment makes its move, it can change the external state to any arbitrary value.

Definition 2. *Let* $P = ((X_p, X_i, X_e), I, T)$ *be a process, and* q, q' *be states. Then* q' *is a* successor *of* q*, written* $q \to q'$*, if*

- $q[X_e] = q'[X_e]$ *and* $q[X_c] \xrightarrow{q[X_e]} q'[X_c]$*; or*
- $q[X_c] = q'[X_c]$.

An execution *of* P *is a sequence of states in* Q^**,* $q_0 q_1 \cdots q_n$*, where* $q_0 \in I \times Q_e$ *and* $q_i \to q_{i+1}$ *for* $0 \leq i < n$.

The reachable set *of* P*, denoted* $\mathcal{R}(P)$*, contains states* q *such that there is an execution ending in* q*.* ■

In order to support structured descriptions, we would like to build complex processes from simpler ones. Three constructs, hide H in P, $P \| P'$ and next Θ for P for building new processes are defined in our model as follows. The hide operator makes interface variables inaccessible to other processes.

Definition 3. *Let* $P = ((X_p, X_i, X_e), I, T)$ *be a process and* $H \subseteq X_i$*. Define the process* hide H in P *to be* $((X_p \cup H, X_i \setminus H, X_e), I, T)$*.* ■

The parallel composition operator allows to combine two processes into a single one. The composition is defined only when the controlled variables of the two processes are disjoint. Intuitively, a step of $P \| Q$ is taken by either P or Q but not both.

Definition 4. *Let* $P' = ((X'_p, X'_i, X'_e), I', T')$ *and* $P'' = ((X''_p, X''_i, X''_e), I'', T'')$ *be processes where* $X'_c \cap X''_c = \emptyset$*. The* composition *of* P' *and* P''*, denoted* $P' \| P''$*, is defined as follows.*

- $X_p = X'_p \cup X''_p$*;*
- $X_i = X'_i \cup X''_i$*;*
- $X_e = (X'_e \cup X''_e) \setminus X_i$*;*
- $I = I' \times I''$*;*
- $T \subseteq Q_c \times Q_e \times Q_c$ *where* $(q, e, r) \in T$ *if*
  - $q[X'_c] \xrightarrow{(e \cup q)[X'_e]} r[X'_c]$ *in* P' *and* $q[X''_c] = r[X''_c]$*; or*
  - $q[X''_c] \xrightarrow{(e \cup q)[X''_e]} r[X''_c]$ *in* P'' *and* $q[X'_c] = r[X'_c]$.

■

Finally, we introduce the next operator. The next operator is a *temporal* construct where new transitions are created by grouping a number of lower-level transitions. A transition in next Θ for P is a sequence of transitions in P where the last transition is in Θ. By applying the next operator, a sequence of small steps becomes a big step. The next operator is inspired by the notions of multi-form time in synchronous languages [Hal93], and was introduced in the language *reactive modules* [AH96].

Definition 5. *Let $P = (X, I, T)$ be a process and Θ a subset of T. Define the process* next Θ for P *to be (X, I, T') where $(q, e, r) \in T'$ if there are $q_0 = q, q_1, \ldots, q_n = r$ in Q_c such that $(q_i, e, q_{i+1}) \in T \setminus \Theta$ for $0 \leq i < n - 1$ and $(q_{n-1}, e, q_n) \in \Theta$.* ■

3. Search Algorithm

In this section, we'll develop an on-the-fly search algorithm to explore reachable states of processes formed using the applications of parallel composition, hiding, and next. The applications of **next** nested with parallel composition lead to hierarchically structured transitions. The search algorithm proceeds recursively in the structure of the process. In order to present the algorithm, we distinguish atomic processes from composite ones. A process is called a *composite process* if it uses composition, hide or next operator. Otherwise, it is called an *atomic process.*

Definition 6. *A* process expression *is defined recursively as follows.*

$$M \quad \equiv \quad P \mid M \| M \mid \text{hide } H \text{ in } M \mid \text{next } \Theta \text{ for } M$$

where P is an atomic process, H is a set of variables and Θ is a predicate on transitions.

We'll assume suitable syntax for atomic processes has been chosen. We also assume that any atomic process appears in a process expression at most once. The algorithm uses three pre-defined functions *AtomicGetNext(P, q)*, *GetInitialStates(M)*, and *BelongsTo(s, t, Θ)*.

- The function *BelongsTo(s, t, Θ)* tests whether the transition from state s to state t is in Θ.
- For an atomic process P and a state q in P, *AtomicGetNext(P, q)* returns the set of successors of q in P.
- The function *GetInitialStates(M)* returns the set of initial states of any process expression M.

Since initial states are preserved by hide and next constructs, they can be computed by ignoring these operators. The function *AtomicGetNext(P, q)* is used to handle the basic case and should be easy to implement as well.

Given a process expression, the algorithm traverses reachable states. Since initial states are reachable, it calls the depth-first search subroutine on initial states iteratively. The algorithm uses a global hash table to store states.

```
proc Main(M) ≡
  tbl := NewHashTable();
  foreach s ∈ GetInitialStates(M) do
    if s ∉ tbl then
                  Insert(tbl, s);
                  DFS(M, s)
    fi
  od.
```

The depth-first algorithm then uses the key function $GetNext(M, s)$ to compute the set of successors of s in process M.

```
proc DFS(M, s) ≡
  foreach s' ∈ GetNext(M, s) do
    DFS(M, s')
  od.
```

The function $GetNext(M, s)$ proceeds in a top-down fashion. Given a global state s, $GetNext(M, s)$ will return global states that are unvisited successors to s. It uses the pre-defined function $AtomicGetNext(P, s)$ for the atomic case. For the composition, it computes the successors of each component and returns their union. When it encounters a process defined by next operator, it generates the successors of the

```
funct GetNext(M, s) ≡
  if M ≡ P
    then result := ∅;
         foreach s' ∈ AtomicGetNext(P, s[P]) do
           if s[s'/X_P] ∉ tbl
             then Insert(result, s[s'/X_P]);
                  Insert(tbl, s[s'/X_P])
           fi
         od;
         return result
  elsif M ≡ hide H in M'
        then return GetNext(M', s)
  elsif M ≡ M1 ‖ M2
        then return GetNext(M1, s) ∪ GetNext(M2, s);
  elsif M ≡ next Θ for M'
        then result := ∅;
             foreach s' ∈ GetNext(M', s) do
               if BelongsTo(s, s', Θ)
                 then Insert(result, s')
                  else Insert(result, GetNext(M, s'))
               fi;
             od
             return result
  fi.
```

FIGURE 1. Algorithm GetNext(M, s)

current state and tests the transition predicate. If a target transition is found, the function adds new states to its returned result. Otherwise, it generates the successors of the new state repeatedly until the transition predicate is satisfied.

To illustrate how the algorithm works, consider the processes P and Q shown in figure 2. Let P' and Q' denote next Θ for P and next Σ for Q respectively, where Θ and Σ consist of thick transitions (i.e. $\Theta = \{(0,1),(3,4)\}$ and $\Sigma = \{(B,C)\}$). The product $P \| Q$ is also shown in the figure, and has 15 states. The product $P' \| Q'$ has much fewer states, namely, 6, shown as filled circles. This illustrates that the application of **next** rules out many transient states, and can potentially save space. The challenge is to explore the expression in a top-down manner computing metatransitions only on demand.

Suppose the algorithm is exploring reachable states of $P' \| Q'$. First, it visits the initial state $0A$. After the first invocation of $GetNext(P' \| Q', 0A)$, it returns the states $1A$, $4A$ and $0C$ and the hash table contains these states as well as light gray states such as 2A and 3A. It is crucial that the transient states such as 2A are stored in the hash-table to avoid recomputation.

Next, the algorithm searches from $1A$. It ignores successor state $2A$ since it is in the hash table, returns state $1C$ and puts state $1B$ in the hash table. Then, it tries to explore state $4A$. It in turn returns state $4C$ and puts state $4B$ in the table. When the algorithm traverses state $0C$, it adds states $2C$ and $3C$ to the hash table. However, since states $1C$ and $4C$ have been visited, it'll ignore them. At the end, the algorithm returns black states, puts shaded states in the hash table. Note that the algorithm never visits the states 2B and 3B, and thus, does less work than required to search $P \| Q$.

To analyze the algorithm, given a process expression M, define $Strip(M)$ to be the process expression obtained by removing all hide and next operators in M. More formally, we have

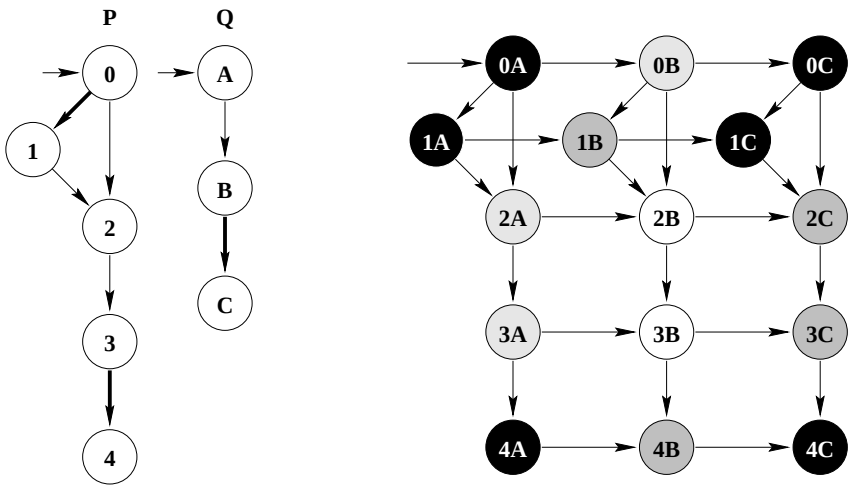

FIGURE 2. Traversal Example

Definition 7. *Let M be a process expression. Define $Strip(M)$ recursively as follows.*

$$\begin{aligned} Strip(P) &= P \\ Strip(\text{hide } H \text{ in } M) &= Strip(M) \\ Strip(\text{next } \Theta \text{ for } M) &= Strip(M) \\ Strip(M_1 \| M_2) &= Strip(M_1) \| Strip(M_2) \end{aligned}$$

where P is an atomic process, H is a set of variables and Θ is a set of transitions.

For any process expression M, consider two reachable sets $\mathcal{R}(M)$ and $\mathcal{R}(Strip(M))$. Clearly, for each state $s \in \mathcal{R}(M)$, $s \in \mathcal{R}(Strip(M))$ because $Strip(M)$ contains all transitions of atomic processes and hence each transition in M is a sequence of transitions in $Strip(M)$. On the other hand, if M contains the next construct, some of the intermediate states may be reachable in $Strip(M)$ but not M. Hence $\mathcal{R}(M)$ and $\mathcal{R}(Strip(M))$ are the two extremes of the state spaces one can hope to search. Since we're interested in checking invariants of the process, we'd like the algorithm to visit all reachable states of M. Besides, the number of states visited by the algorithm shouldn't exceed those of $Strip(M)$. In figure 2, one can see that $\mathcal{R}(P' \| Q')$, *tbl* and $\mathcal{R}(Strip(P' \| Q'))$ are black states, shaded states and all states respectively. The following theorem thus shows the correctness and the space requirements of our algorithm.

Theorem 1. *Let M be an arbitrary process expression. Then at the end of the procedure $Main(M)$,*

- $\mathcal{R}(M) \subseteq$ *tbl;*
- *tbl* $\subseteq \mathcal{R}(Strip(M))$.

The proof of the correctness of the algorithm is deferred to the full paper.

Since the hash table is used, each state in *tbl* is processed precisely once. The cost of processing a state is proportional to its outdegree, and this gives a bound on the running time of the algorithm. Thus, the saving in space comes at no extra overhead in time.

It is straightforward to modify the search algorithm to check invariants of process expressions. However, it should be noted that our algorithm cannot be used to check path properties. For instance, in the example of Figure 2, the global product $P' \| Q'$ has a transition from 1A to 4A, but this transition will not be discovered by the algorithm.

4. Applying Next as a heuristic

Suppose we wish to check an invariant of the process expression $M = P \| Q$. Can we find transition sets Θ and Σ, and search instead, the process expression $M' = \mathsf{next}\ \Theta\ \mathsf{for}\ P \,\|\, \mathsf{next}\ \Sigma\ \mathsf{for}\ Q$. As seen already, searching M' can be more efficient than searching M, and we can use the algorithm from previous section. In this section, we would like to find conditions under which searching M' suffices to solve the original problem.

Notice that application of **next** compresses a sequence of transitions into a single transition. Intuitively, as long as the compressed transitions are *local*, and do not access shared variables, such a transformation should be transparent to the remaining processes. The fact that sequences of invisible transitions can be compressed preserving weak equivalences is well known in theory and tools based on standard process algebras (see, for instance, [BRdSV87]). We adopt this concept to our framework in the sequel. In particular, we show that under appropriate restrictions, $\mathsf{next}\ \Theta\ \mathsf{for}\ P$ is equivalent to P under the definition of Milner's *weak-simulation equivalence*. This allows us, then, to apply **next** to subexpressions repeatedly in a recursive manner.

We start with the definition of observable transition. If a transition doesn't change the values of interface variables, it is called *unobservable*. From other processes' point of view, unobservable transitions are not interesting. This motivates the following definition of observable transition relation.

Definition 8. *Let $P = ((X_p, X_i, X_e), I, T)$ be a process, $q, q' \in Q_c$ and $e \in Q_e$. Then $q \xrightarrow{e}_w q'$ is an* observable transition *if there are $q_0 = q, q_1, \ldots, q_n = q' \in Q_c$ such that*

- *for all $0 \leq i < n$. $q_i[X_i] = q_0[X_i]$;*
- *for all $0 \leq i < n$. $q_i \xrightarrow{e} q_{i+1}$.*

■

Now we define the simulation equivalence.

Definition 9. *Let $P = ((X_p, X_i, X_e), I, T)$ and $P' = ((X'_p, X_i, X_e), I', T')$ be two processes, $q \in Q_c$, $q' \in Q'_c$ and $e \in Q_e$. A* (weak) simulation *relation $\preceq \subseteq Q_c \times Q'_c \times Q_e$ satisfies*

$q \preceq_e q'$ *implies* $q[X_i] = q'[X_i]$ *and*

for all $r \in Q_c$, if $q \xrightarrow{e} r$ then there exists $r' \in Q'_c$ such that $q' \xrightarrow{e}_w r'$ and $r \preceq_e r'$.

P is (weakly) simulated by *P' if there is a simulation $\preceq$ such that for all $q \in I$, $e \in Q_e$ there exists $q' \in I'$ with $q \preceq_e q'$. $P \cong P'$ if $P \preceq P'$ and $P' \preceq P$.* ■

Intuitively, if two processes are simulated by each other, one can perform the other's visible behaviors and vice versa. More specifically, if one is able to change its interface variables under certain environment, the other is able to do the same. Therefore, if an observable state is reachable for one process, it is reachable in the other too. Observe that if P and Q are simulation-equivalent, then to check invariant concerning observable states of P, it suffices to check that invariant of Q.

Now we proceed to establish simulation-equivalence between a process and its next-abstraction provided the compressed transitions do not access shared variables.

Definition 10. *Let $P = ((X_p, X_i, X_e), I, T)$ be a process, $q, q' \in Q_c$. Then $(q, e, q') \in T$ is called a* write-visible transition *if $q[X_i] \neq q'[X_i]$. $(q, e, q') \in T$ is called a* read-visible transition *if there exists $e' \in Q_e$, $(q, e', q') \notin T$.*

A transition in T is called visible *if it is write-visible or read-visible. The set of visible transitions in T is denoted by T_v.* ■

The next technical lemma is needed when we prove the theorem.

Lemma 1. *Let $P = (X, I, T)$ be a process and $T_v \subseteq \Theta \subseteq T$. Define the relation $S \subseteq Q_c \times Q_c \times Q_e$ as follows.*

$$S \;=\; \{\ (q, q', e) \mid \exists q_0, \ldots, q_n . q_0 = q' \wedge q_n = q \wedge \forall 0 \leq i < n . (q_i, e, q_{i+1}) \in T \setminus \Theta\ \}.$$

Then for any $q \in \mathcal{R}(P)$, there exists $q' \in \mathcal{R}(\mathsf{next}\ \Theta\ \mathsf{for}\ P)$ such that $(q, q', q[X_e]) \in S$.

Proof: The only interesting case is when $q \in \mathcal{R}(P) \setminus \mathcal{R}(\mathsf{next}\ \Theta\ \mathsf{for}\ P)$. Consider any such q, there are $r_0, \ldots, r_m \in \mathcal{R}(P)$ such that $r_0 \in I \times Q_e$, $r_m = q$ and $\forall 0 \leq i < n, r_i \rightarrow r_{i+1}$. Since $I \times Q_e \subseteq \mathcal{R}(\mathsf{next}\ \Theta\ \mathsf{for}\ P)$, there exists $j = 0 \ldots n-1$ such that $r_j \in \mathcal{R}(\mathsf{next}\ \Theta\ \mathsf{for}\ P)$ but $r_k \notin \mathcal{R}(\mathsf{next}\ \Theta\ \mathsf{for}\ P)$ for all $j < k \leq m$. It suffices to show $(r_k[X_c], r_j[X_c], r_k[X_e]) \in S$ for all $j < k \leq m$. We prove the claim by induction on $k - j$.

Basic step $k - j = 1$. Since $r_j \rightarrow r_{j+1}$, either $r_j[X_e] = r_{j+1}[X_e]$ and $r_j[X_c] \xrightarrow{r_j[X_e]} r_{j+1}[X_c]$, or $r_j[X_c] = r_{j+1}[X_c]$. The claim holds in the former case where $(r_j[X_c], r_j[X_e], r_{j+1}[X_c]) \in T \setminus \Theta$ and $r_{j+1}[X_e] = r_j[X_e]$. The latter follows trivially.

Inductive step $k - j = \ell + 1$. By induction hypothesis, $(r_\ell[X_c], r_j[X_c], r_\ell[X_e]) \in S$. There are $s_j = r_j[X_c], \ldots, s_\ell = r_\ell[X_c] \in Q_c$ such that $(s_i, r_\ell[X_e], s_{i+1}) \in T \setminus \Theta$ for all $j \leq i < \ell$. Since Θ contains all read-visible transitions, $(s_i, r_{\ell+1}[X_e], s_{i+1}) \in T \setminus \Theta$ for all $j \leq i < \ell$.

Now consider the last step $r_\ell \rightarrow r_{\ell+1}$. If $(r_\ell[X_c], r_\ell[X_e], r_{\ell+1}[X_c]) \in T \setminus \Theta$, $(r_\ell[X_c], r_{\ell+1}[X_e], r_{\ell+1}[X_c]) \in T \setminus \Theta$ for Θ contains all read-visible transitions. Hence $(r_{\ell+1}[X_c], r_j[X_c], r_{\ell+1}[X_e]) \in S$. The other case also holds since $r_{\ell+1}[X_c] = r_\ell[X_c]$. ■

Theorem 2. *Let $P = (X, I, T)$ be a process and $T_v \subseteq \Theta \subseteq T$. Then $P \cong \mathsf{next}\ \Theta\ \mathsf{for}\ P$.*

Proof: Assume $\mathsf{next}\ \Theta\ \mathsf{for}\ P = (X, I, T')$.

We first establish $\mathsf{next}\ \Theta\ \mathsf{for}\ P \preceq P$. Let $q \in Q_c$ and $e \in Q_e$. Define the relation

$$R \quad = \quad \{\ (q, q, e) \mid q \in Q_c\ \}.$$

Suppose $r \in Q_c$ such that $q \xrightarrow{e} r$ in $\mathsf{next}\ \Theta\ \mathsf{for}\ P$. By definition, there are states $q_0 = q, q_1, \ldots, q_n = r$ in Q_c such that $(q_i, e, q_{i+1}) \in T \setminus \Theta$ for $0 \leq i < n-1$ and $(q_{n-1}, e, q_n) \in \Theta$. Since Θ contains all visible transitions, transitions in $T \setminus \Theta$ are not write-visible. Thus, $q_i[X_i] = q_{i+1}[X_i]$ for all $0 \leq i < n-1$. We have $q_0[X_i] = q_i[X_i]$ for all $0 \leq i < n$. Hence, $q \xrightarrow{e}_w r$ in P and $(r, r, e) \in R$ since $r \in Q_c$. R is a simulation relation.

Next, let $q \in Q_c$ and $e \in Q_e$. Define the relation S as in lemma 1. We'll show S is indeed a simulation.

Assume $q' \in Q_c$ and $(q, q', e) \in S$. Suppose $r \in Q_c$ and $q \xrightarrow{e} r$ in P. Consider the following cases:

- $(q, e, r) \in T \setminus \Theta$. By the definition of S and $(q, q', e) \in S$, there exist $q_0 = q', q_1, \ldots, q_n = q$ such that $(q_i, e, q_{i+1}) \in T \setminus \Theta$ for all $0 \leq i < n$. Since $(q, e, r) \in T \setminus \Theta$, $(r, q', e) \in S$. Choose $r' = q'$. $q' \xrightarrow{e}_w r'$ and $(r, r', e) \in S$ as required.
- $(q, e, r) \in \Theta$. By the definition of S, there exist $q_0 = q', q_1, \ldots, q_n = q$ such that $(q_i, e, q_{i+1}) \in T \setminus \Theta$ for all $0 \leq i < n$. If $q' = q$, take $r' = r$. Then $(r, r', e) \in S$ and $q' \xrightarrow{e} r'$ in $\mathsf{next}\ \Theta\ \mathsf{for}\ P$. The result follows.

  Otherwise, since $(q, e, r) \in \Theta$, $(q', e, r) \in T'$. Therefore, choose $r' = r$. We have $q' \xrightarrow{e}_w r'$ in $\mathsf{next}\ \Theta\ \mathsf{for}\ P$ and $(r, r', e) \in S$ as required.

In all cases, S is indeed a simulation. This proves the theorem. ■

Given a process P, we now define its state-saving version $\mathsf{NEXT}\ P$ to be $\mathsf{next}\ T_v\ \mathsf{for}\ P$. By theorem 2, $P \cong \mathsf{NEXT}\ P$. To be able to replace a process subexpression by application of NEXT freely and repeatedly, we must establish that the simulation equivalence is a congruence for the operators in our language:

Theorem 3. *Let P, P' be processes and $P \preceq P'$. Then*

- *$P \| Q \preceq P' \| Q$ for any Q;*
- *$\mathsf{hide}\ H\ \mathsf{in}\ P \preceq \mathsf{hide}\ H\ \mathsf{in}\ P'$;*
- *$\mathsf{NEXT}\ P \preceq \mathsf{NEXT}\ P'$.*

Proof: (Sketch)

Since $P \preceq P'$, for any $q \in I$, $e \in Q_e$, there exists $q' \in Q_c$ such that

- $q[X_i] = q'[X_i]$; and
- for all $r \in Q_c$, $q \xrightarrow{e} r$ implies there exists $r' \in Q'_c$ such that $q' \xrightarrow{e}_w r'$ and $r \preceq r'$.

Let $s \in I$ and $e \in Q_e$ in NEXT P. Suppose $s \xrightarrow{e} t$ in NEXT P. There are $s_0 = s, s_1, \ldots, s_n = t \in Q_c$ such that for $0 \leq i < n-1$ $(s_i, e, s_{i+1}) \in T \setminus T_v$ and $(s_{n-1}, e, s_n) \in T_v$. Since $P \preceq P'$, there are $s'_i \in Q'_c$ such that $s_i \preceq_e s'_i$ and $s'_i \xrightarrow{e}_w s'_{i+1}$ for each i. We analyze $s'_i \xrightarrow{e}_w s'_{i+1}$ by cases.

- $0 \leq i < n-1$. Consider intermediate states between s'_i and s'_{i+1}, they have the same values on interface variables as s'_i. In addition, $s'_{i+1}[X_i] = s_{i+1}[X_i] = s_i[X_i] = s'_i[X_i]$ since $s_i \preceq_e s'_i$, $(s_i, e, s_{i+1}) \in T \setminus T_v$ and $s_{i+1} \preceq_e s'_{i+1}$. Therefore, every intermediate steps are not write-visible.
- $i = n-1$. Using the previous argument, one can prove all intermediate steps but last in $s'_{n-1} \xrightarrow{e}_w s'_n$ are not write-visible. Now since $(s_{n-1}, e, s_n) \in T_v$, it is write-visible or read-visible. If it is write-visible, $s'_{n-1}[X_i] = s_{n-1}[X_i] \neq s_n[X_i] = s'_n[X_i]$. And $s'_0 \xrightarrow{e}_w s'_n$ in NEXT P' with $s_n \preceq_e s'_n$. We're done.

  Otherwise, $s'_n[X_i] = s'_0[X_i]$. Simply choose $t' = s' = s'_0$. Then $s' \xrightarrow{e}_w t'$ and $t \preceq_e t'$.

■

Using the search algorithm shown in Section 3, one can check invariants φ on observable variables more efficiently as follows. Consider any process expression M, we construct a new process M' by substituting subexpressions E in M by NEXT E. This process can be applied repeatedly. Then every observable state of M will be visited by $Main(M')$. Thus, invariant verification of M reduces to the invariant verification of M'.

Consider a process expression $M \equiv M_1 \| \cdots \| M_n$ where each M_i is an atomic process P. We want to construct in a heuristic fashion an expression M' such that M' involves applications of **hide** and NEXT to subexpressions of M with $Strip(M') = M$. By the previous sections, any substitution will yield the same result but the memory requirement may differ by cases. How to find a suitable partition of M to apply this technique becomes an interesting problem on its own.

In principle, one would like to divide the system into parts so that the communication between any two parts is minimal. When a system is built with certain interconnection structures (like trees, rings, or grids etc), these structures should be exploited in verification. Intuitively, one can reduce the number of states by applying the next construct to subtrees in a tree-like structure. One can use the technique recursively on each subtrees and obtain even more efficient result.

Furthermore, hiding unnecessary interface variables is also important since our space-saving version can't ignore write-visible transitions. If one can hide interface variables properly, the number of write-visible transitions reduces as well as the number of states.

5. Implementation and Experiments

We've done two experiments to test our theory. In our experiments, we use a syntax similar to Mocha to specify modules. The experiments were done using a prototype implementation in O'Caml. Details of the experiment, along with the code for the examples, is available to the readers at `www.cis.upenn.edu/~bywang/concur99.html`.

The first example models a parity-computer with a binary tree structure. The system consists of several Clients modules as leaves and Join modules as nodes. The interconnection between these modules and Root modules is shown in Figure 3 for 4-client configuration.

The interface channels of various modules are shown in Figure 4.

Instead of searching

Root ‖ Join ‖ Join0 ‖ Join1 ‖ Client00 ‖ Client01 ‖ Client10 ‖ Client11

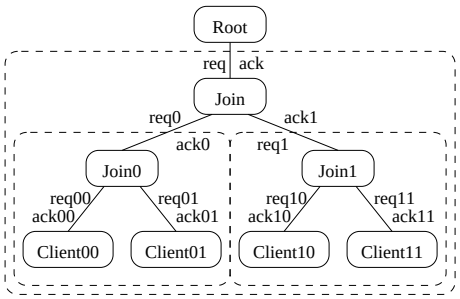

FIGURE 3. Tree-structured Parity Computer

FIGURE 4. Interface and External Variables of Modules in Parity Tree

Num. of clients	Without next		With next	
	States	Time	States	Time
3	202	0.37s	189	0.36s
4	1748	1.05s	515	0.39s
5	6994	4.46s	2017	0.77s
6	69614	55.22s	2633	0.94s
7	320094	324.73s	3531	1.32s
8	2782634	4449.12s	4751	1.79s

FIGURE 5. Reduction for the parity tree computer

we can naturally cluster modules as shown in Figure 3. The expression we search is

$$\text{Root} \parallel \text{NEXT hide [Join} \parallel \begin{array}{l} \text{NEXT hide (Join0} \parallel \text{Client00} \parallel \text{Client01)} \\ \text{NEXT hide (Join1} \parallel \text{Client10} \parallel \text{Client11)} \end{array} \text{]}.$$

For readability the argument variables to **hide** are not shown above, but should be obvious from context: only the variables communicating with the parent need to be observable at each level of the subtree. Notice that for the top-level expression, a step is either a step of the module Root, or it is a sequence of steps of the tree rooted at *Join* until either *req* is updated or *ack* is read. A step of the tree rooted at *Join* is either a step of the module *Join*, or a sequence of steps of the subtree rooted at *Join0* until *req0* or *ack0* is accessed, or a sequence of steps of the subtree rooted at *Join1* until *req1* or *ack1* is accessed. Since each *Join* node communicates with parent only after it has received and processed requests from its children, we get excellent reduction. In the table of Figure 5, we show the number of states visited, together with the CPU time required by the algorithm for different number of clients. Observe that while the number of states grows exponentially without any reduction, our heuristic scales almost linearly.

Our second example concerns the standard leader election protocol in an asynchronous ring of cells. The protocol used here is taken from [Lyn96], and has been used as a benchmark to study how model checking algorithms scale. Figure 6 shows four cells connected together and the partition used for applications of **next**. The results are reported in the table of Figure 7. While not as spectacular as the parity-tree-computer, observe the significant reduction in space and time.

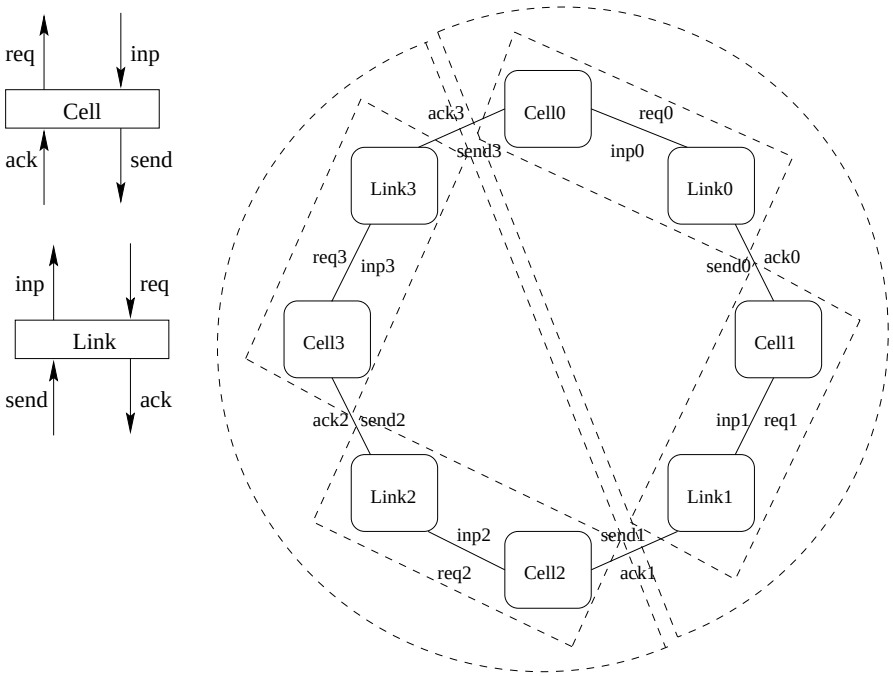

FIGURE 6. Leader Module

Num. of clients	Without next		With next	
	States	Time	States	Time
3	10847	7.64s	4401	1.71s
4	195017	210.15s	26430	10.79s
5	2235421	4351.20s	230569	115.21s

FIGURE 7. Reduction for the leader election in ring

6. CONCLUSIONS

We have presented a new heuristic for model checking based hierarchically structured transition relations. The heuristic potentially combines advantages of the on-the-fly search with early detection of violations and compositional reduction techniques based on process equivalence.

While the construct **next** has been studied previously in the context of symbolic search [AHR98], this is the first study that establishes conditions under which **next** construct can be introduced without changing the answer to the original verification problem. The transformation of Section 3 can be used for improving efficiency of symbolic model checking as well. The reduction offered by our method differs with examples. The reduction is maximum when processes can be clustered so that when one cluster communicates with another, it is only in a small number of different internal states.

Partial-order reduction has been successfully combined with on-the-fly search [Pel94, Val92, God90]. It may appear that our reduction is a simple form of partial-order reduction, but on more scrutiny, the two techniques are incomparable, and we believe them to be compatible. Partial-order reduction does not exploit the hierarchical structure, but uses independence among transitions to restrict the number of interleavings. It is easy to construct examples in which our method works better than the partial-order reduction method, and vice versa. On the examples reported in this paper, the software SPIN gives much more impressive reductions on the leader election protocol, but our method performs better for the parity computer. The modeling languages of SPIN and MOCHA are quite different to make any meaningful comparison in the absolute number of states for each configuration, so the comparison is

based on the rates of growth with the scaling parameter. More information about the comparison can be found along with the code of our examples (`www.cis.upenn.edu/~bywang/concur99.html`).

Another popular heuristic for improving on-the-fly search is reductions using symmetry [ES93, CFJ93, ID96]. Again, our method is incomparable to and compatible with symmetry reduction. In particular, note that in our examples, if we change the individual clients so that they behave slighly differently from each other, or the different join nodes compute different functions, there will be no significant impact on the reduction obtained.

The process-algebraic tools such as *Concurrency workbench* [CPS93] and CADP [FGK+96] implement the reductions based on weak-simulation (or even the less expensive reduction to compress internal transitions). We have already discussed our motivation to explore on-the-fly algorithms that implement the reduction in a top-down manner. In the final version, we plan to include experimental data to compare the two approaches.

In this paper, we have used **next** only to compress invisible transitions. However, our search algorithm can be used effectively in other contexts. First, **next** can be supplied an argument that contains only a subset of the visible transitions. This would perform a more efficient search at a coarser granularity of atomicity. Note that, while this is similar in spirit to the *atomic* construct in the language of SPIN to cluster a sequence of atomic transitions, it gives a high-level and hierarchical construct. Second, **next** can be used to imeplement the so-called "run-to-completion" semantics in which transitions are executed until there are no more enabled transitions.

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