Abstract

Data transmission over wireless networks is challenging due to the occurrence of burst errors, and packet loss caused by such errors seriously limits the maximum achievable throughput of wireless networks. To tailor efficient transmission schemes, it is essential to develop a wireless error model that can provide insight into the behavior of wireless transmissions. In this study, we investigate the wireless error model of Bluetooth networks. We study the FHSS feature of Bluetooth using both ordinary hopping kernels and Adaptive Frequency Hopping (AFH) kernels, and design analytical error models accordingly to capture the channel behavior of Bluetooth networks. We evaluate the proposed models by comparing the analytical results to the simulation results obtained by Markov Chain Monte Carlo (MCMC) algorithms. The results show that our analytical models can represent the channel behavior of Bluetooth networks in all test cases.

1 Introduction

Data transmission over wireless networks is challenging because the dynamic environment is error-prone; hence, mutual interference arising from simultaneous transmissions can be a significant problem in such networks. As a result, packet loss caused by co-channel interference imposes a serious limitation on the maximum achievable throughput in the wireless channel [12]. Bluetooth technology, which operates in the crowded unlicensed 2.4GHz ISM (Industrial-Scientific-Medical) frequency band, must coexist with various wireless and radio technologies, such as IEEE 802.11b/g standard, IEEE 802.15.4 standard, cordless telephones, and even microwave ovens.

Knowing the fundamental properties of wireless networks is the key to designing effective transmission schemes. A number of researchers have studied the behavior of wireless channels. Among them, Gilbert and Elliot proposed a Discrete Time Markov Chain (DTMC) model, called the Gilbert-Elliot model [10] [11], which consists of two states; a Good state and a Bad state. A two-by-two transition matrix is given to specify the state transition probabilities. For example, the probability of remaining in the bad state defined in the transition matrix reflects the level of burst errors.

Extensive models of wireless channels were proposed subsequently [16]. It is commonly assumed that the Markov Chain Model is more accurate as the number of states increases. However, the computational complexity of the model also increases exponentially, so the subsequent performance analysis becomes more difficult. To simplify the modeling, the lumpability property is used to aggregate chains [7]; nevertheless, not all the evaluated protocols have the desired characteristics [17]. A number of works have conducted trace-based error rate analyses with two-state Markov Chain modeling on heterogeneous wireless networks [21]. The results for the 2.4GHz ISM frequency band are encouraging [17] [4] [18] [22], and confirm the accuracy of the first-order Markov model on wireless fading channels [23].

In this paper, we study channel behavior under the mechanism of the Bluetooth Frequency Hopping Spread Spectrum (FHSS), and propose a Discrete Time Markov Chain (DTMC) model of the link in Bluetooth networks. Using the Markov Chain Monte Carlo (MCMC) method, we evaluate our proposed model using both ordinary and adaptive frequency hopping kernels, and demonstrate that it is accurate and representative of Bluetooth networks.

The remainder of this paper is organized as follows. Section II introduces the background of the Bluetooth technique. Section III analyzes the channel behavior of the Bluetooth link. Using Monte Carlo methods, we evaluate the proposed model in Section IV. We then present our conclusions in Section V.

2 Background and Related Work

Bluetooth is a short-range, low cost, and low power consumption radio technology that operates in the un-
licensed 2.4GHz Industrial-Scientific-Medical (ISM) frequency band. It employs the Frequency Hopping Spread Spectrum (FHSS) technique, and implements stop-and-wait Automatic Repeat reQuest (ARQ), Cyclic Redundancy Check (CRC), and Forward Error Correction (FEC) functions to ensure that the wireless links are reliable. As a result, it alleviates interference caused by other radio technologies, such as IEEE 802.11b/g [2], cordless phones, and microwave ovens.

The FHSS used in Bluetooth has 79 channels¹, each of which has 1MHz of bandwidth. The center frequencies of the 79 channels (in MHz) are

\[ f = 2402 + k; \quad k = 0 \ldots 78. \]  

(1)

The frequency hopping sequence is determined by a hopping kernel. In each round, the hopping kernel first selects a segment of 64 adjacent channels (note that the last channel, i.e., \( k=78 \), is adjacent to the first channel, i.e., \( k=0 \), as illustrated in Fig. 1) and then hops to 32 of them at random without repetition. Next, a different 32-hop sequence is chosen from another segment of 64 adjacent channels, and the process is repeated. In this way, we derive a pseudo-random sequence of frequency hopping slides as the hopping kernel passes through the 79 available channels.

In addition to the ordinary Bluetooth hopping kernel, several approaches have been proposed to further improve Bluetooth’s throughput performance in the crowded 2.4GHz ISM frequency band. For instance, Golmie et al. proposed a Bluetooth Interference Aware Scheduling (BIAS) scheme, which determines the frequency hopping pattern based on a Frequency Usage Table [14]. Subsequently, the Adaptive Frequency Hopping (AFH) scheme [15], a BIAS-like approach, was proposed and included in the Bluetooth Spec v1.2 [1].

In the AFH scheme, the master device first checks whether the slave devices support the AFH scheme in the Device Identification phase; if they do, the slave devices measure the quality of the 79 Bluetooth channels in the Channel Classification phase. The slave devices then send their measurement results to the master device so that its AFH hopping kernel can determine the appropriate hopping sequences.

More precisely, the AFH scheme classifies the 79 Bluetooth channels into two groups: unused and used. The former should not be used because they might have suffered from heavy interference, but the latter are suitable for transmission. The AFH scheme then employs a mapping function to uniformly map the unused channels to the used channels. As a result, the scheme can avoid the channels affected by heavy interference (the unused channels), and thereby improve data throughput.

Golmie [13] investigated the latency and throughput performance of Bluetooth networks, and proposed a dynamic scheduling algorithm that guarantees QoS while reducing the impact of interference. Kwok et al. [6] used 802.11b as the major interference source and customized the mapping function to achieve a lower collision rate and higher ISM spectrum utilization. In addition, Popovski et al. [19] subdivided frequencies into groups and assigned them to members of piconets in a round-robin manner to avoid self-interference. Omer et al. [5] advocated increasing the number of channels by considering overlapped hopping frequencies. Although the above works focus on the channel modeling techniques used in different wireless networks, to the best of our knowledge, an analytical study of the modeling of the FHSS and AFH mechanisms for Bluetooth networks is still lacking. In this paper, we seek to fill that research gap.

3 Analysis

3.1 Wireless Error Model for the Ordinary Hopping Kernel

First, we consider the wireless error model for the ordinary Bluetooth hopping kernel, i.e., without the AFH scheme. When considering the hopping behavior of Bluetooth, we assume the 79 Gilbert-Elliot channels [10] [11] are i.i.d. (identical and independently distributed). For each channel, a two-state Gilbert-Elliot model is used to capture the behavior of wireless channel errors. Suppose \( P_{gb}^{(i)} \), \( P_{bg}^{(i)} \), \( P_{bb}^{(i)} \), and \( P_{gb}^{(i)} \) are the state transition probabilities of the \( i \)-th channel. Moreover, the Markov chain is ergodic with stationary probabilities \( P_{g}^{(i)} = \frac{1-P_{bb}^{(i)}}{1-P_{bb}^{(i)}+P_{gb}^{(i)}} \) for the good state and \( P_{b}^{(i)} = \frac{P_{gb}^{(i)}}{1-P_{bb}^{(i)}+P_{gb}^{(i)}} \) for the bad state, where \( P_{b}^{(i)} \) is denotes the average packet error rate (PER).

The logical link of the Bluetooth network is modeled as follows. Fig.2(a) shows all the Markov chains of the network. Since the hopping kernel must hop through all the channels equally, the distribution of the hopping sequence is

Figure 1. An example of sequence selection in Bluetooth frequency hopping.

¹In some countries, e.g. France, there are only 23 channels.
uniform. In other words, the probability that the kernel will hop to each channel in the next time slot is $\frac{1}{79}$. Moreover, in the next time slot, the probability of hopping to channel $i$ in a good state and a bad state is $P_g^{(i)}$ and $P_b^{(i)}$, respectively. The hierarchical structure, shown in Fig. 2(b), illustrates how the logical link hops through the 79 good states and the 79 bad states.

Note that the structure in Fig. 2(b) is reducible because the next state is not determined by the hopping behavior, but by the state of the channel to be hopped. Therefore, we can combine the states according to the derived $P_g^{(i)}$ and $P_b^{(i)}$ values. The probability of the hopped channels in the good state ($P_g'$) can thus be obtained by

$$P_g' = \frac{1}{79} \sum_{i=1}^{79} P_g^{(i)}. \quad (2)$$

Additionally, since the 79 Bluetooth channels are independent, the state of the current channel is not connected to the state of the channel in the next time slot. Therefore, if $P_{gg}'$, $P_{gb}'$, $P_{bg}'$ and $P_{bb}'$ are the transition probabilities of the Bluetooth link between two consecutive time slots, we can apply Bayes' Theorem and obtain $P_{gg}' = P_{gg} = P_g'$ and $P_{gb}' = P_{bb}' = P_b'$. Fig. 2(c) shows the reduced model. Note it is also a Gilbert-Elliot model.

### 3.2 Wireless Error Model for the AFH Hopping Kernel

Next, we consider the wireless error model for cases where the AFH scheme is implemented. The AFH kernel classifies the 79 Bluetooth channels into used and unused channels during the Channel Classification phase. From the previous analysis, we know that the expected number of used channels can by derived by

$$N_{good} = \sum_{i=1}^{79} P_g^{(i)},$$

where $P_g^{(i)}$ is the probability that the $i$-th channel will be marked as used. The IEEE 802.15.2 standard [3] specifies two operating modes: $N_{good} \geq N_{min}$ (i.e., Mode L) and $N_{good} < N_{min}$ (i.e., Mode H). Suppose $\delta(i)$ is a function that indicates whether the $i$-th channel is used or unused, as shown by Eq. 3. We describe the two operating modes below.

$$\delta(i) = \begin{cases} 0 & \text{if the } i\text{-th channel is unused}, \\ 1 & \text{if the } i\text{-th channel is used}. \end{cases} \quad (3)$$

#### 3.2.1 Mode L

This mode is used when $N_{good}$ is equal to or larger than $N_{min}$. A mapping function is then employed by AFH to uniformly map unused channels to the used channels. Therefore, the classified $N_{good}$ channels will be the reduced hopping set. The probability that the channels will be in the good state is derived by

$$P_g' = \frac{1}{N_{good}} \sum_{i=1}^{79} P_g^{(i)} \times \delta(i). \quad (4)$$

2The probability of the hopped channels in bad state ($P_b'$) can be obtained by $P_b' = 1 - P_g'$.

3According to [1], $N_{min}$ should be set to 20.
3.2.2 Mode H

This mode is used when \( N_{\text{good}} \) is less than \( N_{\text{min}} \). The hopping sequence is divided into \( R_g \) consecutive good slots and \( R_b \) consecutive bad slots alternately. Although the values of \( R_g \) and \( R_b \) are determined by the traffic type required by the application [5], to preserve the frequency diversity, \( R_g + R_b \) must not be less than \( N_{\text{min}} \) [20]. All used channels are uniformly mapped into the good slots, and unused channels are uniformly mapped into the bad slots.

Therefore, under the AFH mechanism, \( P'_{\text{gg}} \) can be obtained by

\[
P'_{\text{gg}} = \frac{R_g}{R_g + R_b} \times \frac{\sum_{i=1}^{79} P_g(i) \delta(i)}{N_{\text{good}}} + \frac{R_b}{R_g + R_b} \times \frac{\sum_{i=1}^{79} P_g(i)(1 - \delta(i))}{79 - N_{\text{good}}}. \tag{5}
\]

4 Evaluation

In this section, we evaluate the proposed Bluetooth wireless error models using the Markov Chain Monte Carlo (MCMC) method. The Bluetooth Frequency Hopping Selection Kernel [9] in the Matlab environment is used for simulations. Based on the kernel, we implement a packet level burst error channel model and record the binary hopping sequence. All the results presented in this section are based on the average performance of 200 simulation runs.

4.1 Homogeneous Channels

For simplicity, in the first set of simulations, we assumed that the 79 channels were homogeneous and had the same Gilbert-Elliot model parameters. More precisely, the value of \( P_{gg} \) was set to 0.8 and the value of \( P_{bb} \) varied in a range 0.001 to 0.5. In each simulation run, we randomly selected half of the 79 channels, and set their initial states to good. Using the above channel configurations, we performed MCMC simulations and recorded the state sequence of 20,000 consecutively hopped channels. Fig. 3 shows the simulation results for both Bluetooth hopping kernels. The consistency estimate is obtained by taking the ratio of the number of good states in the simulation results over that in our analytical results.

The results in Fig.3 show that the consistency estimates are approximately equal to 1 and very stable, regardless of the number of channels in the first channel group and the number of Bluetooth hopping kernels used. Once again, the results confirm that the proposed analytical model can capture the channel behavior of Bluetooth networks accurately.

4.2 Semi-Homogeneous Channels

Next, we evaluated the proposed analytical model in the scenario where the 79 channels were separated into two groups. Specifically, channels in the same group were configured with the same Gilbert-Elliot model parameters, which were different to those used for the other group of channels. In the simulation, we set \( P_{bb} \) to 0.5 for one group and 0.9 for the other. Similar to the first set of simulations, \( P_{gg} \) was fixed at 0.8, the initial state of each channel was randomly generated, and the state sequence of 20,000 consecutively hopped channels was recorded in each simulation run.

From the results, which are shown in Fig.4, we observe that the consistency estimates are approximately equal to 1 and very stable, regardless of the number of channels in the first channel group and the number of Bluetooth hopping kernels used. Once again, the results confirm that the proposed analytical model can capture the channel behavior of Bluetooth networks accurately.

4.3 Heterogeneous Channels

Here, we evaluated the proposed model when the 79 channels are heterogeneous. More specifically, for each channel, we randomly generated the values of \( P_{gg} \) and \( P_{bb} \), in the range \( [P_{min}^{gg}, 1] \) and \( [P_{min}^{bb}, 1] \) respectively (of course, \( 0 < P_{min}^{gg}, P_{min}^{bb} < 1 \)). Similar to the previous evaluation, the state sequence of 20,000 consecutively hopped channels was recorded in each simulation run, and the results were derived by averaging the results of 200 simulations. We measured the Kullback-Leibler Distance (KL-distance), which is the relative entropy of two probability mass functions [8], between the simulation results and our analytical results. Fig.5 shows the K-L distance for different \( P_{min}^{gg} \) and \( P_{min}^{bb} \) values. The results show that the K-L distance is very small in almost all test cases. Note that, in Fig.5(b), the K-L distance is slightly larger when \( P_{min}^{bb} \) is approximately 1 and \( P_{min}^{bb} \) is approximately 0 (i.e., the Bluetooth network is exceptionally error-prone). This is because the more error-prone the network is, the greater the likelihood that the MCMC simulations and the proposed analytical model will enter different AFH modes. As a result, the accumulated entropy (K-L distance) is larger than that in other settings.

We then observed the distribution of the burst length of good states with the focus on bursty channels. We set the values of both \( P_{min}^{gg} \) and \( P_{min}^{bb} \) to 0.8. The burst length distribution is shown in Fig.6. The results confirm our intuition that the frequency value decreases as the burst length increases. Moreover, the results indicate that when the AFH scheme is used, it is more likely to have a burst of good hopped states than the ordinary Bluetooth hopping kernel.
Finally, the results again verify the accuracy of the proposed analytical model, since the frequency distribution of the analytical results is very close to that of the simulation results in all test cases.

5 Conclusion

We have investigated the fundamental properties of the FHSS mechanism used in Bluetooth, and proposed two wireless error models to represent, respectively, the channel behavior of Bluetooth networks when the ordinary hopping kernel and the AFH kernel is implemented. Using Markov Chain Monte Carlo (MCMC) simulations, we compare the simulation results and our analytical results in homogeneous, semi-homogeneous, and heterogeneous channel scenarios. The results demonstrate that the proposed models can represent the channel behavior of Bluetooth networks accurately in all test cases. The precision and simplicity of the proposed models make them ideal for providing representative wireless error models of Bluetooth networks in future research.

References

Figure 5. The K-L distance between the simulation results and the analytical results.

Figure 6. Comparison of the frequency distribution of various burst good state lengths using ordinary hopping kernels and AFH kernels.