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A Two-State Markov-based Wireless Error Model for Bluetooth Networks

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Abstract Data transmission over wireless networks is challenging due to the occurrence of burst errors, and packet loss caused by such errors seriously limits the maximum achievable throughput of wireless networks. To tailor efficient transmission schemes, it is essential to develop a wireless error model that can provide insight into the behavior of wireless transmissions. In this study, we investigate the wireless error model of Bluetooth networks. We study the FHSS feature of Bluetooth using both ordinary hopping kernels and Adaptive Frequency Hopping (AFH) kernels, and design analytical error models accordingly to capture the channel behavior of Bluetooth networks. We evaluate the proposed models by comparing the analytical results to the simulation results obtained by Markov Chain Monte Carlo (MCMC) algorithms. The results show that our analytical models can represent the channel behavior of Bluetooth networks in all test cases.

Keywords Bluetooth · Error Model · Markov Chain Monte Carlo · Gilbert-Elliot model

1 Introduction

Data transmission over wireless networks is challenging because the dynamic environment is error-prone; hence, mutual interference arising from simultaneous transmissions can be a significant problem in such networks. As a result, packet loss caused by co-channel interference imposes a serious limitation on the maximum achievable throughput in the wireless channel [12], and knowing the fundamental properties of wireless networks is the key to designing effective transmission schemes.

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A number of researchers have studied the behavior of wireless channels. Among them, Gilbert and Elliot proposed a *Discrete Time Markov Chain* (DTMC) model, called the *Gilbert-Elliot model* [10, 11], which consists of two states (i.e., a *good* state and a *bad* state) and uses a two-by-two transition matrix to specify the state transition probabilities. As extensive models of wireless channels are proposed subsequently [16], it is commonly assumed that the *Markov Chain Model* is more accurate as the number of states increases. However, the computational complexity of the model also increases exponentially, so the subsequent performance analysis becomes more difficult.

To simplify the modeling, the lumpability property is used to aggregate chains [7]; nevertheless, not all the evaluated protocols have the desired characteristics [17]. For instance, Bluetooth technology employs the *Frequency Hopping Spread Spectrum* (FHSS) technique, which has 79 channels in the crowded unlicensed 2.4GHz ISM (Industrial-Scientific-Medical) frequency band. As a result, it is difficult to model wireless channels of Bluetooth networks, because the model has to consider both the frequency hopping mechanism of the Bluetooth technology and the wireless dynamics (i.e., the coexisting wireless and radio technologies, such as IEEE 802.11b/g standard, IEEE 802.15.4 standard, cordless telephones, and even microwave ovens [3, 8, 18, 19]).

In this paper, we study the channel behavior when the two FHSS kernels are used in the Bluetooth network, namely the ordinary and adaptive frequency hopping kernels. Using the analysis method, we consider the frequency hopping mechanisms of the two FHSS kernels, as well as the 79 Bluetooth channels. We propose two reduced DTMC models for Bluetooth networks for the two frequency hopping kernels respectively. Using the *Markov Chain Monte Carlo* (MCMC) method, we evaluate our proposed model using both ordinary and adaptive frequency hopping kernels, and demonstrate that it is accurate and representative of Bluetooth networks. Moreover, the precision and simplicity of the proposed models make them ideal for providing representative wireless error models of Bluetooth networks in future research.

The remainder of this paper is organized as follows. Section 2 introduces the background of the Bluetooth technique. Section 3 analyzes the channel behavior of the Bluetooth link. Using Monte Carlo methods, we evaluate the proposed model in Section 4. We then present our conclusions in Section 5.

2 Background and Related Work

Bluetooth is a short-range, low cost, and low power consumption radio technology that operates in the unlicensed 2.4GHz Industrial-Scientific-Medical (ISM) frequency band. It employs the Frequency Hopping Spread Spectrum (FHSS) technique, and implements stop-and-wait Automatic Repeat reQuest (ARQ), Cyclic Redundancy Check (CRC), and Forward Error Correction (FEC) functions to ensure that the wireless links are reliable. As a result, it alleviates interference caused by other radio technologies, such as IEEE 802.11b/g [2], IEEE 802.15.4 [4], cordless phones, and microwave ovens.

The FHSS used in Bluetooth has 79 channels[1], each of which has 1MHz of bandwidth. The center frequencies of the 79 channels (in MHz) are

$$f = 2402 + k\,;\, k = 0 \ldots 78. \tag{1}$$

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[1] In some countries, e.g. France, there are only 23 channels.

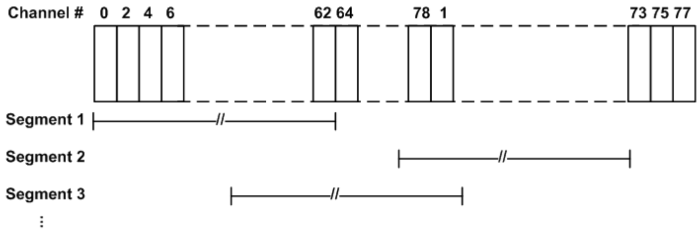


Fig. 1 An example of sequence selection in Bluetooth frequency hopping.

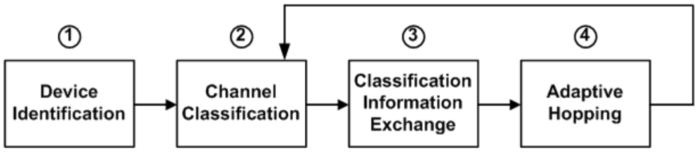


Fig. 2 The procedure of the Adaptive Frequency Hopping (AFH) scheme.

The frequency hopping sequence is determined by a hopping kernel. In each round, the hopping kernel first selects a segment of 64 adjacent channels (note that the last channel, i.e., k=78, is adjacent to the first channel, i.e., k=0, as illustrated in Figure 1) and then hops to 32 of them at random without repetition. Next, a different 32-hop sequence is chosen from another segment of 64 adjacent channels, and the process is repeated. In this way, we derive a pseudo-random sequence of frequency hopping slides as the hopping kernel passes through the 79 available channels.

In addition to the ordinary Bluetooth hopping kernel, several approaches have been proposed to further improve Bluetooth's throughput performance in the crowded 2.4GHz ISM frequency band. For instance, Golmie et al. proposed a Bluetooth Interference Aware Scheduling (BIAS) scheme, which determines the frequency hopping pattern based on a *Frequency Usage Table* [14]. Subsequently, the Adaptive Frequency Hopping (AFH) scheme [15], a BIAS-like approach, was proposed and included in the Bluetooth Spec v1.2 [1].

Figure 2 shows the procedure of the AFH scheme. In the AFH scheme, the master device first checks whether the slave devices support the AFH scheme in the *Device Identification* phase; if they do, the slave devices measure the quality of the 79 Bluetooth channels in the *Channel Classification* phase. The slave devices then send their measurement results to the master device so that its AFH hopping kernel can determine the appropriate hopping sequences.

More precisely, the AFH scheme classifies the 79 Bluetooth channels into two groups: *unused* and *used*. The former should not be used because they might have suffered from heavy interference, but the latter are suitable for transmission. The AFH scheme then employs a mapping function to uniformly map the *unused* channels to the *used* channels. As a result, the scheme can avoid the channels affected by heavy interference (the *unused* channels), and thereby improve data throughput.

Golmie [13] investigated the latency and throughput performance of Bluetooth networks, and proposed a dynamic scheduling algorithm that guarantees QoS while reducing the impact of interference. Kwok et al. [6] used 802.11b as the major interference source and customized the mapping function to achieve a lower collision rate and higher ISM spectrum utilization. In addition, Popovski et al. [20] subdivided frequencies into groups and assigned them to members of piconets in a round-robin manner to avoid self-interference. Omer et al. [5] advocated increasing the number of channels by considering overlapped hopping frequencies. Although the above works focus on the channel modeling techniques used in different wireless networks, to the best of our knowledge, an analytical study of the modeling of the FHSS and AFH mechanisms for Bluetooth networks is still lacking. In this paper, we seek to fill that research gap.

3 Analysis

In this section, we present the analysis of wireless error models for Bluetooth networks with the ordinary hopping kernel and the AFH hopping kernel respectively. For simplicity, we assume the wireless errors of the 79 channels in Bluetooth networks are independent. Moreover, a two-state Gilbert-Elliot model [10, 11] is used to capture the behavior of wireless channel errors for each channel. The Gilbert-Elliot model consists of two states, namely the *good* state and the *bad* state. Events originate from these states are denoted as *g* and *b* respectively. Four transition probabilities, $P_{gg}(i)$, $P_{gb}(i)$, $P_{bg}(i)$, and $P_{bb}(i)$, are then given to specify the state transition probabilities for the i-th channel. For example, for the i-th channel, $P_{gb}(i)$ defines the probability of the transition from the *good* state to the *bad* state, and $P_{bb}(i)$ defines the probability of remaining in the *bad* state. The Markov chain is ergodic with stationary probabilities $P_g(i)$ for the *good* state and $P_b(i)$ for the *bad* state. The values of $P_g(i)$ and $P_b(i)$ can be derived by Eq. 2 and 3, where $P_b(i)$ denotes the average packet error rate (PER) of the i-th channel [23].

$$P_g(i) = \frac{1 - P_{bb}(i)}{1 - P_{bb}(i) + P_{gb}(i)}, \text{ and} \tag{2}$$

$$P_b(i) = \frac{P_{gb}(i)}{1 - P_{bb}(i) + P_{gb}(i)}. \tag{3}$$

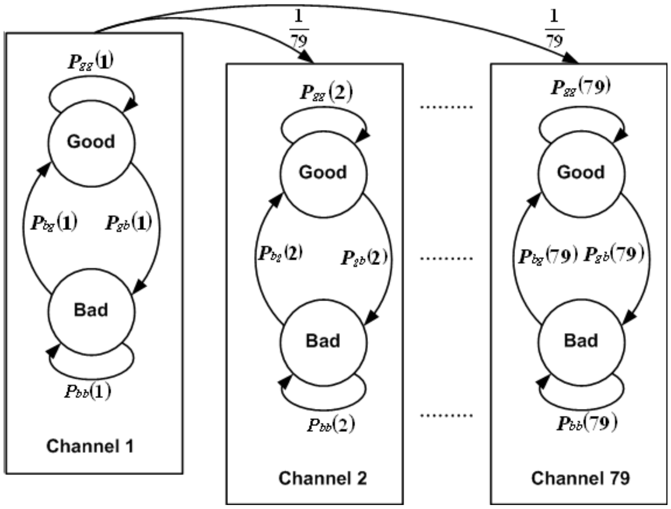
3.1 Wireless Error Model for the Ordinary Hopping Kernel

First, we consider the wireless error model for the ordinary Bluetooth hopping kernel, and the logical link of the Bluetooth network is modeled as follows.

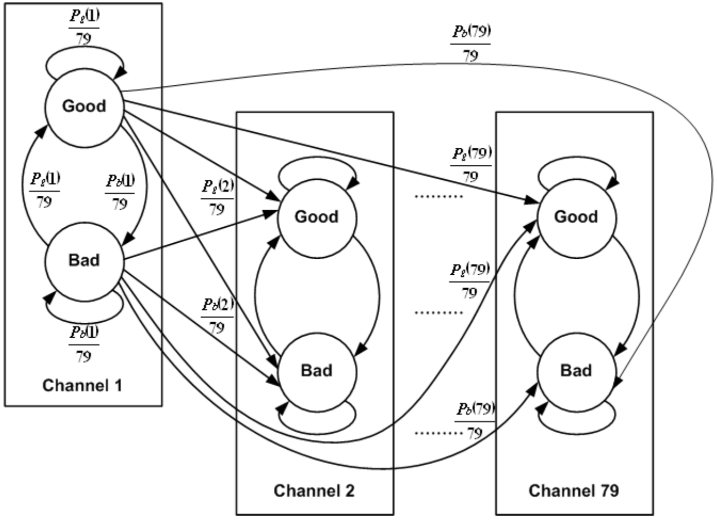
Figure 3(a) shows all the Markov chains of the Bluetooth network. Since the ordinary hopping kernel must hop through all the channels equally, the distribution of the hopping sequence is uniform. In other words, the probability that the kernel will hop to each channel in the next time slot is $\frac{1}{79}$. Moreover, in the next time slot, the probability of hopping to channel i in a good state and a bad state is $\frac{P_g(i)}{79}$ and $\frac{P_b(i)}{79}$, respectively. The hierarchical structure, shown in Figure. 3(b), illustrates how the logical link hops through the 79 *good* states and the 79 *bad* states.

Note that the structure in Figure 3(b) is reducible because the next state is not determined by the hopping behavior, but by the state of the channel to be hopped. Therefore, we can combine the states according to the derived $P_g(i)$ and $P_b(i)$ values. The probability of the hopped channels, using the ordinary Bluetooth hopping kernel, in the *good* state (P_g^O) and in the *bad* state (P_b^O) can thus be obtained by Eq. 4 and 5.

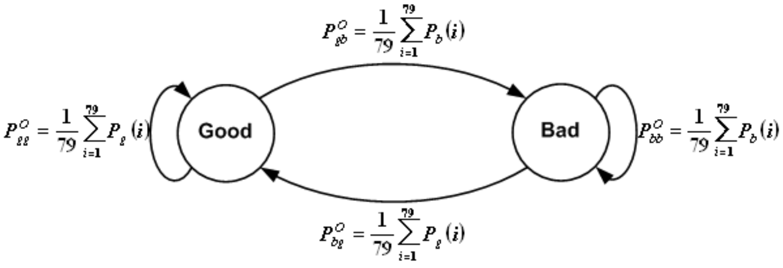
$$P_g^O = \frac{1}{79}\sum_{i=1}^{79} P_g(i) \tag{4}$$



(a) A Bluetooth link hops between channels.



(b) The hierarchical structure of 79 good states and 79 bad states.



(c) The modeling of a Bluetooth link.

Fig. 3 The Markov chain model of the ordinary Bluetooth hopping kernel.

$$P_b^O = 1 - P_g^O = \frac{1}{79}\sum_{i=1}^{79} P_b(i) \qquad (5)$$

Additionally, since the 79 Bluetooth channels are independent, the state of the current channel is not connected to the state of the channel in the next time slot. Therefore, if P_{gg}^O, P_{gb}^O, P_{bg}^O and P_{bb}^O are the transition probabilities of the Bluetooth link between two consecutive time slots, we can apply *Bayes' Theorem* and obtain $P_{gg}^O = P_{bg}^O = P_g^O$ and $P_{gb}^O = P_{bb}^O = P_b^O$. Figure 3(c) shows the reduced model. Note it is also a Gilbert-Elliot model.

3.2 Wireless Error Model for the AFH Hopping Kernel

Next, we consider the wireless error model for cases where the AFH scheme is implemented. The AFH kernel classifies the 79 Bluetooth channels into *used* and *unused* channels during the *Channel Classification* phase. From the previous analysis, we know that the expected number of *used* channels can by derived by $N_{good} = \sum_{i=1}^{79} P_g(i)$, where $P_g(i)$ is the probability that the i-th channel will be marked as *used*. The IEEE 802.15.2 standard [3] specifies two operating modes: $N_{good} \geq N_{min}$ (i.e., *Mode L*) and $N_{good} < N_{min}$ (i.e., *Mode H*)[2]. Suppose $\delta(i)$ is a function that indicates whether the i-th channel is *used* or *unused*, as shown by Eq. 6. We describe the two operating modes below.

$$\delta(i) = \begin{cases} 0 \text{ if the } i\text{-th channel is } \textit{unused}, \\ 1 \text{ if the } i\text{-th channel is } \textit{used}. \end{cases} \qquad (6)$$

3.2.1 Mode L

This mode is used when N_{good} is equal to or larger than N_{min}. A mapping function is then employed by AFH to uniformly map *unused* channels to the *used* channels. Therefore, the classified N_{good} channels will be the reduced hopping set. Let P_g^L denote the probability that the hopped channels will be in the $good$ state in *Mode L*, we can then obtain P_g^L by

$$P_g^L = \frac{1}{N_{good}}\sum_{i=1}^{79} P_g(i) \times \delta(i). \qquad (7)$$

Moreover, since the 79 Bluetooth channels are independent, the state of the current channel is not connected to the state of the channel in the next time slot. Therefore, the wireless error model for Bluetooth of *Mode L* can be reduced to a two-state Gilbert-Elliot model with $P_{gg}^L = P_{bg}^L = P_g^L$ and $P_{gb}^L = P_{bb}^L = P_b^L = 1 - P_g^L$.

3.2.2 Mode H

This mode is used when N_{good} is less than N_{min}. The hopping sequence is divided into R_g consecutive good slots and R_b consecutive bad slots alternately. Although the values of R_g and R_b are determined by the traffic type required by the application [5], to preserve the frequency diversity, $R_g + R_b$ must not be less than N_{min} [22]. All *used* channels are

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[2] According to [1], N_{min} should be set to 20.

uniformly mapped into the good slots, and *unused* channels are uniformly mapped into the bad slots.

Therefore, under the AFH mechanism, the probability that the hopped channels will be in the $good$ state, P_g^H, can be obtained by

$$P_g^H = \frac{R_g}{R_g + R_b} \times \frac{\sum_{i=1}^{79} P_g(i)\delta(i)}{N_{good}} + \frac{R_b}{R_g + R_b} \times \frac{\sum_{i=1}^{79} P_g(i)(1-\delta(i))}{79 - N_{good}}. \tag{8}$$

Similarly, by applying Bayes' Theorem, the wireless error model can be also reduced to a two-state Gilbert-Elliot model with $P_{gg}^H = P_{bg}^H = P_g^H$ and $P_{gb}^H = P_{bb}^H = P_b^H = 1 - P_g^H$.

Let $K_1 = \frac{\sum_{i=1}^{79} P_g(i)\delta(i)}{N_{good}}$ and $K_2 = \frac{\sum_{i=1}^{79} P_g(i)(1-\delta(i))}{79-N_{good}}$. Since P_g must be larger/smaller than 0.5 for every *used*/*unused* channel, we know that $K_1 = K_2 + \epsilon$ ($\epsilon > 0$), and obtain

$$\begin{aligned} P_g^H &= \frac{R_g}{R_g + R_b} \times (K_2 + \epsilon) + \frac{R_b}{R_g + R_b} \times K_2 \\ &= K_2 + \frac{R_g}{R_g + R_b} \times \epsilon. \end{aligned} \tag{9}$$

To maximize the value of P_g^H, it is preferred to have the value of R_g as large as possible, while keeping the value of $R_g + R_b$ as small as possible. Since $R_g \leq N_{good}$ and $R_g + R_b \geq N_{min}$, we can derive the optimal values of R_g and R_b by

$$R_g = N_{good}, \textit{ and} \tag{10}$$

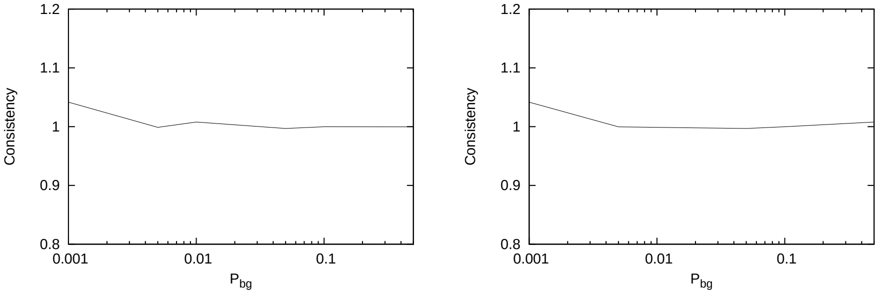
$$R_b = N_{min} - R_g; \tag{11}$$

We then rewrite Eq. 8 as

$$P_g^H = \frac{N_{good}}{N_{min}} \times \frac{\sum_{i=1}^{79} P_g(i)\delta(i)}{N_{good}} + \frac{N_{min} - N_{good}}{N_{min}} \times \frac{\sum_{i=1}^{79} P_g(i)(1-\delta(i))}{79 - N_{good}}. \tag{12}$$

4 Evaluation

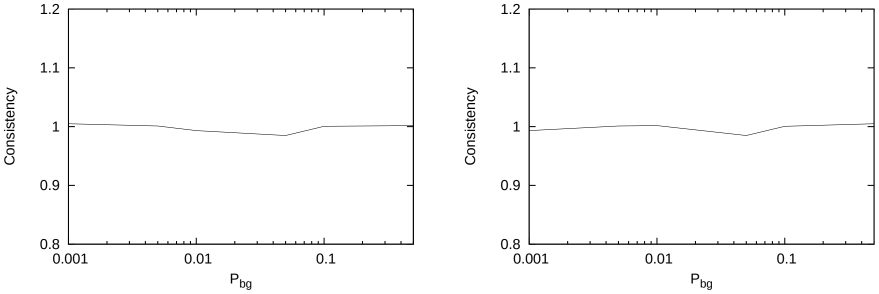
In this section, we evaluate the proposed Bluetooth wireless error models using the Markov Chain Monte Carlo (MCMC) method. The Bluetooth Frequency Hopping Selection Kernel [21] in the Matlab environment is used for simulations. Based on the kernel, we implement a packet level burst error channel model and record the state sequence of 20,000 consecutively hopped channels in each simulation run. We design the *consistency* metric to compare the analytical and simulation results, and the *consistency* estimate is obtained by taking the ratio of the number of *good* states in the simulation results over that in our analytical results. All the results presented in this section are based on the average performance of 200 runs.



(a) $P_{gg} = 0.6$

(b) $P_{gg} = 0.8$

Fig. 4 The consistency results using different P_{bg} values in homogeneous scenarios (i.e., the 79 channels have the same Gilbert-Elliot model parameters) with the ordinary Bluetooth hopping kernel.



(a) $P_{gg} = 0.6$

(b) $P_{gg} = 0.8$

Fig. 5 The consistency results using different P_{bg} values in homogeneous scenarios (i.e., the 79 channels have the same Gilbert-Elliot model parameters) with the AFH hopping kernel.

4.1 Homogeneous Channels

For simplicity, in the first set of simulations, we assumed that the 79 channels were homogeneous and had the same Gilbert-Elliot model parameters. More precisely, we varied the value of P_{bg} varied in a range 0.001 to 0.5, and set the value of P_{gg} to 0.6 and 0.8 respectively. In each simulation run, we randomly selected half of the 79 channels, and set their initial states to *good*. Using MCMC simulation method, Figures 4 and 5 show the simulation results for both Bluetooth hopping kernels.

The results in Figures 4 and 5 show that the consistency estimates are approximately equal to 1 with fluctuations in a very small range (less than 3% when the ordinary kernel is used, and less than 1% when the AFH kernel is used). In other words, the results indicate that our proposed analytical model can accurately represent the behavior of homogeneous channels for both ordinary hopping kernels and AFH kernels.

4.2 Semi-Homogeneous Channels

Net, we evaluated the proposed analytical model in the scenario where the 79 channels were separated into two groups. Specifically, channels in the same group were configured with the same Gilbert-Elliot model parameters, which were different to those used for the

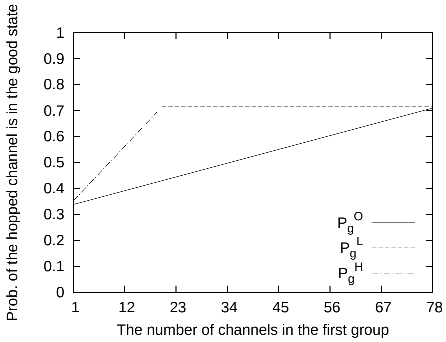


Fig. 6 Comparison of P_g^O, P_g^L, and P_g^H with different numbers of channels in the first group of channels in semi-homogeneous scenarios.

other group of channels. In the simulation, we set P_{bb} to 0.5 for one group and 0.9 for the other. Moreover, the value of P_{gg} was set to 0.8 for both groups of channels, and the initial state of each channel was randomly generated[3]. For the AFH hopping kernel, we suppose $N_{min} = 20$, $\delta(i) = 1$ (i.e., the i-th channel is *used*) when $P_g(i) \geq 0.5$, and $\delta(i) = 0$ (i.e., the i-th channel is *unused*) otherwise. Figure 6 compares the probabilities of the hopped channels in the *good* state when the ordinary and AFH hopping kernels are used respectively.

From the results, we observe that P_g^O decreases linearly as the number of channels in the first group (in which $P_{bb} = 0.5$) decreases. The reason is simply because the ordinary hopping kernel selects the hopping channel in a pseudo-random basis. As a result, it is influenced a lot by the condition (i.e., packet error rate) of each channel. In contrast, we also find that P_g^L remains consistent regardless of the number of channels in the first group, and P_g^L and P_g^H outperform P_g^O for all cases. The results confirm that the AFH hopping kernel is superior to the ordinary Bluetooth hopping kernel because the AFH hopping kernel favors *used* channels (i.e., $P_g(i) \geq 0.5$) in determining the hopping channel. More precisely, when the number of *used* channels (i.e., N_{good}) is larger than N_{min}, the AFH hopping kernel operates in *Mode L*, and it hops only to the *used* channels; whereas when $N_{good} < N_{min}$, the AFH hopping kernel operates in *Mode H*, and it hops to all the *used* channels and a subset of the *unused* channels. As a result, the AFH hopping kernel is more likely to hop to good channels than the ordinary Bluetooth hopping kernel.

In addition, we performed MCMC simulations and recorded the state sequence of 20,000 consecutively hopped channels using the above channel configurations. Figure 7 shows the simulation results for both Bluetooth hopping kernels. From the results, we observe that the consistency estimates are approximately equal to 1 and very stable, regardless of the number of channels in the first channel group and the number of Bluetooth hopping kernels used. Once again, the results confirm that the proposed analytical model can capture the channel behavior of Bluetooth networks accurately.

4.3 Heterogeneous Channels

Here, we evaluated the proposed model when the 79 channels are heterogeneous. More specifically, for each channel, we randomly generated the values of P_{gg} and P_{bb}, in the

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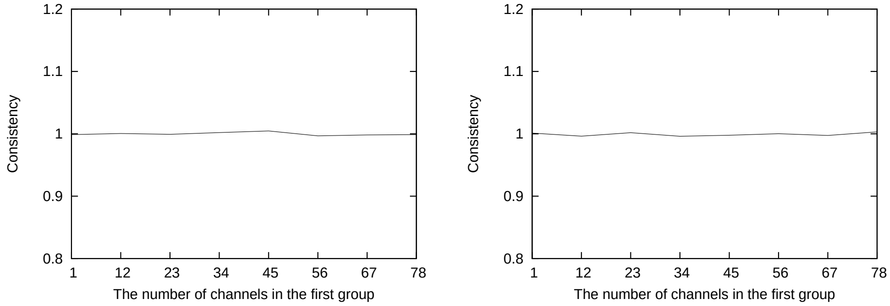
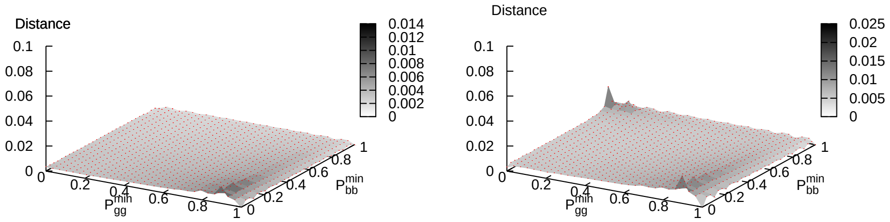
[3] Using Eq. 2, we know that, when $P_{gg} = 0.8$, $P_g(i) = 0.7143$ if $P_{bb}(i) = 0.5$, and $P_g(i) = 0.3333$ if $P_{bb}(i) = 0.9$.



(a) Ordinary Bluetooth Hopping Kernel

(b) (b) AFH Hopping Kernel

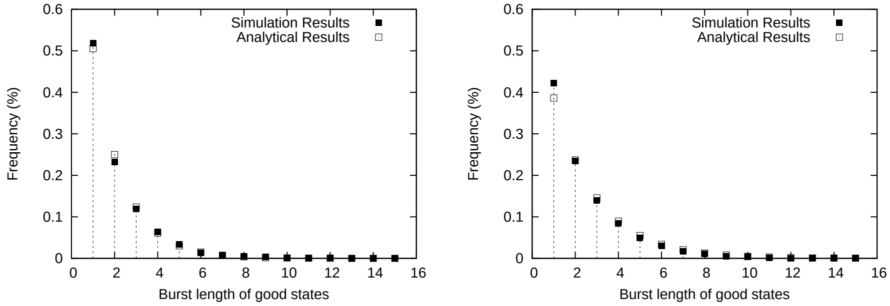
Fig. 7 The consistency results using different numbers of channels in the first group of channels in semi-homogeneous scenarios.



(a) Ordinary Bluetooth Hopping Kernel

(b) (b) AFH Hopping Kernel

Fig. 8 The K-L distance between the simulation results and the analytical results.



(a) Ordinary Bluetooth Hopping Kernel

(b) (b) AFH Hopping Kernel

Fig. 9 Comparison of the frequency distribution of various burst good state lengths using ordinary hopping kernels and AFH kernels.

range $[P_{gg}^{min}, 1]$ and $[P_{bb}^{min}, 1]$ respectively (of course, $0 < P_{gg}^{min}, P_{bb}^{min} < 1$). Similar to the previous evaluation, the state sequence of 20,000 consecutively hopped channels was recorded in each simulation run, and the results were derived by averaging the results of 200 simulations. We measured the *Kullback-Leibler Distance* (KL-distance), which is the relative entropy of two probability mass functions [9], between the simulation results and our analytical results. Figure 8 shows the K-L distance for different P_{gg}^{min} and P_{bb}^{min} values. The results show that the K-L distance is very small in almost all test cases. Note that, in Figure 8(b), the K-L distance is slightly larger when P_{bb}^{min} is approximately 1 and P_{gg}^{min} is approximately 0 (i.e., the Bluetooth network is exceptionally error-prone). This is because the more error-prone the network is, the greater the likelihood that the MCMC simulations and the proposed analytical model will enter different AFH modes. As a result, the accumulated entropy (K-L distance) is larger than that in other settings.

We then observed the distribution of the burst length of *good* states with the focus on bursty channels. We set the values of both P_{gg}^{min} and P_{bb}^{min} to 0.8. The burst length distribution is shown in Figure 9. The results confirm our intuition that the frequency value decreases as the burst length increases. Moreover, the results indicate that when the AFH scheme is used, it is more likely to have a burst of *good* hopped states than the ordinary Bluetooth hopping kernel. Finally, the results again verify the accuracy of the proposed analytical model, since the frequency distribution of the analytical results is very close to that of the simulation results in all test cases.

5 Conclusion

We have investigated the fundamental properties of the FHSS mechanism used in Bluetooth, and proposed two wireless error models to represent, respectively, the channel behavior of Bluetooth networks when the ordinary hopping kernel and the AFH kernel is implemented. Using Markov Chain Monte Carlo (MCMC) simulations, we compare the simulation results and our analytical results in various network scenarios. The results demonstrate that the proposed models can represent the channel behavior of Bluetooth networks accurately in all test cases. The precision and simplicity of the proposed models make them ideal for providing representative wireless error models of Bluetooth networks in future research.

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