Belief Function Semantics for Comparative Confirmation

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1 Introduction

The logic of confirmation is first proposed by Hempel to study the confirmatory power of a piece of evidence on a hypothesis[3]. However, a paradox is soon found because according to the logic, an observation of white shoes confirms the hypothesis " All ravens are black". Though Hempel argued that this is not really a paradox by saying " The impression of a paradoxical situation is not objectively founded; it is a psychological illusion."([3], p. 18), not many philosophers agree with him on this point. Therefore, some alternative formalisms for confirmation logics have been proposed to resolve the paradox. Some of the most well-known resolutions is the selective confirmation theory proposed by Goodman and refined by Scheffler and its relativized version by Grandy[1, 6, 2, 4]. In selective confirmation theory, a piece of evidence confirms a hypothesis and disconfirms its contrary in the same time. On the other hand, Popper[5] considers it impossible to confirm a universal scientific hypothesis. According to his viewpoint, only falsification of such statements is possible.

Although these viewpoints are radically different, they all concern the relationship between evidence and hypotheses. Therefore, it would be interesting to explore the uniform semantic basis of different theories of confirmation. In this paper, we will present a general theory of comparative confirmation based on the belief function semantics. The belief function theory is proposed by Dempster and developed systematically by Shafer later, so also known as Dempster-Shafer theory[7]. A belief function structure(BFS) consists of two finite sets, called evidence frame and hypothesis frame respectively, and a binary compatibility relation between them. When a probability distribution on the evidence frame is given, we can determine the belief and plausibility measures of the hypotheses in the hypothesis frame. We will show that different definitions of hypothesis frame and compatibility relation lead to different confirmation theories. So the general setting will provide a ground to understand the difference among several confirmation theories.

2 Belief Function Structures and Comparative Confirmation

Let $\mathcal{L}$ denote a logic language and $\mathcal{L}_h$ and $\mathcal{L}_e$ be sublanguages of $\mathcal{L}$ for describing hypotheses and observation reports respectively. Then a *comparative confirmation theory* (CCT) is a triplet

$(\Theta, \Omega, \sharp)$, where Θ and Ω are finite subsets of $\mathcal{L}_e$ and $\mathcal{L}_h$ respectively, and $\sharp \subseteq \Theta \times \Omega$ is a compatibility relation between Θ and Ω. We further require that the sentences in Ω (resp. Θ) are pairwisely inconsistent under the logic $\mathcal{L}$. For each observation report $e \in \Theta$, define $C(e) = \{h : e\sharp h\}$ as the compatible hypotheses of e. We assume the closed world assumption[8] that requires $C(e) \neq \emptyset$ for all $e \in \Theta$.

Let $\mu : \Theta \rightarrow [0, 1]$ be a probability distribution on Θ such that $\sum_{e\in\Theta} \mu(e) = 1$ and $\prod_{e\in\Theta} \mu(e) > 0$. We call μ a *positive probability distribution* (ppd). Then the corresponding belief and plausibility measures on 2^{Ω} can be defined as

$$Bel(H) = \sum\{\mu(e) : C(e) \subseteq H\}$$

and

$$Pl(H) = 1 - Bel(\overline{H}).$$

Given two ppds μ_1 and μ_2 and an observation report e, we say that μ_2 is an *e-increment* of μ_1 if $\mu_2(e) > \mu_1(e)$ and $\mu_1(e') > \mu_2(e')$ for all $e' \in \Theta \setminus \{e\}$. Then we have the following main definition.

Definition 1 *Let $(\Theta, \Omega, \sharp)$ be a CCT, e be an observation report in Θ, and H be a subset of Ω. Then*

1. *e confirms H iff for all ppds μ_1 and its e-increment μ_2 $Bel_2(H) > Bel_1(H)$ where Bel_i is the corresponding belief measure of μ_i $(i = 1, 2)$,*
2. *e disconfirms H iff for all ppds μ_1 and its e-increment μ_2 $Pl_2(H) < Pl_1(H)$ where Pl_i is the corresponding plausibility measure of μ_i $(i = 1, 2)$,*
3. *e is neutral to H otherwise.*

Note in the definition above, we do not need to specify the ppds really. What we need to know is just the direction of changes of belief measures w.r.t. e-increments of arbitrary ppds, so the definition is essentially qualitative though it seems resorting to quantitative probability and belief functions.

According to the definition, a valid sentence can not be confirmed. This seems to violate our intuition at a first glance. However, it is reasonable since a tautology is not empirically significant.

3 Three Examples

In this section, let $\mathcal{L}_h$ be a first order language and $\mathcal{L}_e$ be a set of ground sentences generated from the predicate symbols of $\mathcal{L}_h$ and a finite set of new individual constants. We consider the Hempel's black-raven example. Let us use the following notations.

$$\begin{array}{ll}
h_0 = \forall x(Rx \supset Bx) & e_1 = Ra \wedge Ba \\
h_1 = \exists x Rx \wedge \forall x(Rx \supset Bx) & e_2 = Ra \wedge \neg Ba \\
h_2 = \exists x Rx \wedge \forall x(Rx \supset \neg Bx) & e_3 = \neg Ra \\
h_3 = \forall x \neg Rx, &
\end{array}$$

where a is a constant fresh to $\mathcal{L}_h$.

Example 1 (Hempel's logic) Define the CCT with $\Theta = \{e_1, e_2, e_3\}$, $\Omega = \{h_0, \neg h_0\}$, and $e \sharp h$ iff $e \models h\{a\}$, where $h\{a\}$ is the development of h for $\{a\}$([3], p.36). Then it can be easily verified that for all ppds μ, $Bel(h) = Pl(h) = 1 - \mu(e_2)$, so it is obviously e_2 disconfirms h whereas e_1 and e_3 confirm h. This is essentially the result of Hempel's logic

Example 2 (Popperian view) The Θ and Ω are defined as above, but the compatibility relation is now given by the logical consistency notion. That is, $e \sharp h$ iff e and h is consistent under $\mathcal{L}$. Then for all ppds μ, $Bel(h) = 0$ and $Pl(h) = 1 - \mu(e_2)$, so e_2 disconfirms (or falsifies, in the terminology of Popper[5]) h, and e_1 and e_3 remain neutral to h. This is a naive Popperian viewpoint. It says that universal statement can only be falsified but not be confirmed and existential statement can only be confirmed but not be falsified.

Example 3 (Selective confirmation) Let Θ and $\sharp$ be defined as in Example 2, but $\Omega = \{h_1, h_2, h_3\}$. Then for all ppds μ, $Bel(h_1) = \mu(e_1)$, $Pl(h_1) = 1 - \mu(e_2)$, $Bel(h_2) = \mu(e_2)$, $Pl(h_2) = 1 - \mu(e_1)$, $Bel(h_3) = 0$ and $Pl(h_3) = \mu(e_3)$, so e_1 confirms h_1 but disconfirms h_2 and h_3, etc.

According to the belief function semantics, we can find that the difference between Popperian and Hempelian viewpoint is due to the adoption of different compatibility relations, while the one between Popperian viewpoint and selective confirmation is due to the available hypothesis set.

4 Conclusion

The belief function semantics for confirmation logic shows that confirmation is not an isolated work. It depends on at least three factors, i.e., the possible observation reports, the available competitive hypotheses, and the definition of compatibility between evidence and hypothesis frames. The different viewpoints of confirmation logicians can be traced back to the difference of these factors. The details and implications of the proposed semantics will be discussed in the full paper.

References

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