A Semantics for Logics of Preference Based on Possibility Theory

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Abstract

In this paper, we study the semantics for the logics of preference based on possibility theory. Possibility distributions representing the preference between worlds are associated with the possible world models for dynamic logics. Then the preference between actions are determined by comparing some measures of their consequences. We will define different logics of preference by considering the comparisons of possibility measures and guaranteed possibility measures. Some properties of the proposed logics will be studied and their relationships with deontic logics will also be considered.
Key words: Possibility theory, deontic logic, logic of preference, dynamic logic.

1 Introduction

Deontic logic is the logic for reasoning about norms. Deontic reasoning has been extensively exploited in ethics and legal philosophy since the ancient times. However, the first modern formal system for deontic logic is not established until the fifties[14]. Though the system is influential on the later work, there are arising many paradoxes when it is applied to practical deontic reasoning, so alternative systems have also been proposed since then for the resolution of paradoxes[2]. Among them, the Meyer's approach is the most interesting one[10]. His system is based on the reduction to dynamic logic[8], so actions and propositions can be both represented and the distinction of "ought-to-do" and "ought-to-be" can be made clearly. This in turn clarifies much confusion caused by the inappropriate translation of practical deontic reasoning to formal systems.

On the other hand, while the different formal systems are mainly the consequence of foundational studies of deontic reasoning, the applications of deontic logics to computer science and artificial intelligence has received more and more attention recently[11]. The applications include automated legal reasoning, electronic commerce, system specification, and so on. Since Meyer's system is strongly based on dynamic logic and the latter is the logic for reasoning about computer program, it is in particular suitable for the potential application.

In addition to the reduction to dynamic logic, another important feature of Meyer's logic is the use of a special propositional atom V, meaning the violation of law (or something like sanction, punishment, etc.). This special atom is originally introduced by Anderson[1] for reducing deontic logic to alethic modal logic. By using the special atom, an action is forbidden if the execution of it will necessarily lead to states in which V holds, and it is permitted if not forbidden. Moreover, an action is obligatory if failing to executing it will result in violation of law.

Though Meyer's logic is successful in reasoning about normative actions, it is inadequate in the representation of action preference. However, the norms are usually relative and conflict with other ones, so we may have to make some decision choices between conflicting actions. For example, the violation of constitution is considered more serious than that of regulation laws, so we will try to obey the former instead of the latter provided that it is impossible to enforce both in the same time. Under the situations, we will need the capability of reasoning about action preference. In this paper, we will show that by extending Meyer's semantics with possibility theory constructs, we can achieve the purpose. Intuitively, the possibility distribution will model the degrees of satisfaction of norms, and an action will be preferred to another one if it satisfies the norms to a higher degree than the other.

The rest of the paper is organized as follows. First, Meyer's deontic logic and possibility theory are reviewed in the next section. Then a logic for deontic degree is proposed and its semantics is presented. In section 4, we consider a logic of action preference. Its relationship with deontic logics will also be studied. Then an alternative logic based on the minimal semantics (neighborhood semantics) of modal logics[4] is considered. Finally, we discuss some related works and further generalization in the concluding section.

2 Review of Deontic Logic and Possibility Theory

2.1 Deontic logic as dynamic logic

The system of Meyer's logic is called PD_eL. The elementary symbols of the PD_eL language consist of

1. A set of propositional letters, $PV = \{p, q, r, \ldots\}$ and a special propositional letter V not in PV, and
2. a set of atomic actions, $A = \{a, b, c, \ldots\}$ and three distinguished action symbols $\emptyset$, $\mathbf{u}$, and $\mathbf{i}$[1].

The set of well-formed formulas(Φ) and the set of action expressions(Π) are defined inductively in the following way.

1. Φ is the smallest set such that
   - $PV \cup \{V\} \subseteq \Phi$, and
   - if $\varphi, \psi \in \Phi$ and $\alpha \in \Pi$, then $\neg\varphi, \varphi \vee \psi, [\alpha]\varphi \in \Phi$.
2. Π is the smallest set such that
   - $A \cup \{\emptyset, \mathbf{u}, \mathbf{i}\} \subseteq \Pi$, and
   - if $\alpha, \beta \in \Pi$ and $\varphi \in \Phi$, then $\alpha;\beta, \alpha \cup \beta, \alpha\&\beta, \overline{\alpha}, \varphi \rightarrow \alpha/\beta \in \Pi$.

The wff $\neg[\alpha]\neg\varphi$ is abbreviated as $\langle\alpha\rangle\varphi$ and the other classical connectives($\top, \bot, \wedge, \supset, \equiv$) are defined as usual. The wff $[\alpha]\varphi$ means that if action α is done, φ will hold. The action expressions $\alpha;\beta, \alpha \cup \beta, \alpha\&\beta$ denote the sequential composition, nondeterministic choice, and simultaneous execution of α and β respectively, whereas $\overline{\alpha}$ means the non-execution of α and $\varphi \rightarrow \alpha/\beta$ denotes that if φ holds then execute α else execute β.

Note that the language of PD_eL is not the traditional one for dynamic logic. First, the Kleene star α^* is not in Π. This is because in the deontic reasoning domain, the repetition of some actions is not so usual as in computer program. On the other hand, the simultaneous execution and non-execution of actions are nonstandard in dynamic logic. In fact, the two constructs significantly complicate the formal semantics of PD_eL. Since the formal semantics is rather involved, we will only present the informal one based on the ordinary state transition semantics of dynamic logics.

A Kripke model for PD_eL is a quadruple $\langle W, \models, [\![\cdot]\!], opt\rangle$, where W is a set of possible worlds and *opt* is a nonempty subset of W, meaning the best elements of W, whereas $\models \subseteq W \times \Phi$ and $[\![\cdot]\!] : \Pi \rightarrow \mathcal{P}(W \times W)$ define the truth relation and the action denotation function respectively. It is required that $\models$ and $[\![\cdot]\!]$ must satisfy the following constraints.

- For all $w \in W$, $\varphi, \psi \in \Phi$, and $[\alpha] \in \Pi$,

  ($\models$0) $w \models V \Leftrightarrow w \notin opt$,

  ($\models$1) $w \models \varphi \vee \psi \Leftrightarrow w \models \varphi$ or $w \models \psi$,

  ($\models$2) $w \models \neg\varphi \Leftrightarrow w \not\models \varphi$,

  ($\models$3) $w \models [\alpha]\varphi \Leftrightarrow \forall u \in [\![\alpha]\!](w), u \models \varphi$, where $[\![\alpha]\!](w)$ is defined as $\{u \mid (w, u) \in [\![\alpha]\!]\}$.

- For all $w \in W$, $\alpha, \beta \in \Pi$, and $\varphi \in \Phi$,

  ($[\![\cdot]\!]$1): $[\![\emptyset]\!] = \emptyset$, $[\![\mathbf{u}]\!] = W \times W$, $[\![\mathbf{i}]\!] = \{(w, w) \mid w \in W\}$.

  ($[\![\cdot]\!]$2): $[\![\alpha \cup \beta]\!] = [\![\alpha]\!] \cup [\![\beta]\!]$.

  ($[\![\cdot]\!]$3): $[\![\alpha;\beta]\!] = [\![\alpha]\!] \circ [\![\beta]\!]$.[2]

[1] The last symbol is added by me for the latter use.
[2] i.e. the relational composition of $[\![\alpha]\!]$ and $[\![\beta]\!]$.

($[\![\cdot]\!]4$): $[\![\alpha\&\beta]\!] \subseteq [\![\alpha]\!] \cap [\![\beta]\!]$.

($[\![\cdot]\!]5$): (negated actions)

1. $[\![\overline{\overline{\alpha}}]\!] = [\![\alpha]\!]$
2. $[\![\alpha \cup \overline{\alpha}]\!] = [\![\mathbf{u}]\!]$, $[\![\alpha\&\overline{\alpha}]\!] = [\![\emptyset]\!]$.
3. $[\![\overline{\alpha \cup \beta}]\!] = [\![\overline{\alpha}\&\overline{\beta}]\!]$
4. $[\![\overline{\alpha\&\beta}]\!] = [\![\overline{\alpha} \cup \overline{\beta}]\!]$
5. $[\![\overline{\alpha;\beta}]\!] = [\![\overline{\alpha} \cup (\alpha;\overline{\beta})]\!]$
6. $[\![\overline{\varphi \to \alpha/\beta}]\!] = [\![\varphi \to \overline{\alpha}/\overline{\beta}]\!]$

($[\![\cdot]\!]6$): (conditional actions)

$$[\![\varphi \to \alpha/\beta]\!](w) = \begin{cases} [\![\alpha]\!](w), & \text{if } w \models \varphi, \\ [\![\beta]\!](w), & \text{if } w \not\models \varphi. \end{cases}$$

As usual, the denotation of an action is just a state transition relation. However, unlike standard compound actions in traditional dynamic logics, the denotations of $\overline{\alpha}$ and $\alpha\&\beta$ are not functionally determined by those of their component actions. This makes it impossible to define the denotation function only for primitive actions and then extend it to all actions.

Then the deontic wffs are defined as abbreviations,

$$F_m\alpha = [\alpha]V,$$

$$P_m\alpha = \neg F_m\alpha = \langle\alpha\rangle\neg V,$$

$$O_m\alpha = F_m\overline{\alpha} = [\overline{\alpha}]V,$$

for all $\alpha \in \Pi$. Here we use subscript m to indicate that the deontic operators are according to Meyer's definitions.

2.2 Possibility theory

Possibility theory is developed by Zadeh from fuzzy set theory[15]. Given a universe W, a *possibility distribution* on W is a function $\pi : W \to [0,1]$. In general, we require the normalization condition is satisfied. That is, $\sup_{w\in W} \pi(w) = 1$ must hold. Obviously, π is a characteristic function of a fuzzy subset of W. Two measures on W can be derived from π. They are called possibility and necessity measures and denoted by Π and N respectively. Formally, $\Pi, N : 2^W \to [0,1]$ are defined as

$$\Pi(A) = \sup_{w\in A} \pi(w),$$

$$N(A) = 1 - \Pi(\overline{A}),$$

where $\overline{A}$ is the complement of A with respect to W. Another measure, called guaranteed possibility and denoted by Δ, is defined as

$$\Delta(A) = \inf_{w\in A} \pi(w),$$

in [7]. Possibility and guaranteed possibility measures will be used in this paper. For convenience, we assume $\sup\emptyset = \inf\emptyset = 0$.

3 A Graded Deontic Logic

While deontic logics are relevant to reasoning with norms, we may be also interested in the more general logic of value concepts. For example, in the decision-making context, we may select some actions to do according to our preference. Most research on the logics of preference is based on the analysis of probability and utility theory. However, since possibility theory models ordering relation in a natural way, it should be suitable to use the theory in the analysis of preference relations. In this section, we will first develop a kind of graded deontic logic, called GD_eL, with semantics based on possibility theory. This logic, though does not involve with the comparison of action preference directly, may represent different degrees of permission, obligation and prohibition and will serve as a basis of prohairetic logics.

The alphabet of GD_eL is that of PD_eL without the symbol V and with the addition of six classes of graded deontic operators O_g^c, P_g^c, F_g^c, $O_g^{>c}$, $P_g^{>c}$, and $F_g^{>c}$ for all $c \in [0,1]$. The formation rule of wffs will include

If $\alpha \in \Pi$, then $\Box\alpha \in \Phi$ for any deontic operator $\Box$ in the six classes.

The intuitive meaning of these graded deontic formulas is to represent the degree of obligation, permission and prohibition respectively. For example, $P_g^c\alpha$ means that doing α is permitted at least to the degree c.

A model of GD_eL will be a quadruple $\langle W, \models, [\![\cdot]\!], \pi\rangle$, where W and $[\![\cdot]\!]$ are defined as above, π is a normalized possibility distribution on W such that $\pi(W)$ is finite, and $\models$ is as above except that the constraint ($\models$0) is dropped and the following are added:

($\models$4) $w \models P_g^c\alpha$ (resp. $P_g^{>c}\alpha$) iff $\Pi([\![\alpha]\!](w)) \geq c$ (resp. $> c$).

($\models$5) $w \models F_g^c\alpha$ (resp. $F_g^{>c}\alpha$) iff $w \models \neg P_g^{>(1-c)}\alpha$ (resp. $w \models \neg P_g^{1-c}\alpha$).

($\models$6) $w \models O_g^c\alpha$ (resp. $O_g^{>c}\alpha$) iff $w \models F_g^c\,\overline{\alpha}$ (resp. $F_g^{>c}\,\overline{\alpha}$).

For any world w, the possibility value $\pi(w)$ denotes the degree of "goodness" of w, so the graded deontic formulas reflect the degree of goodness (or badness) of the states to which some action will lead.

A GD_eL model can be viewed as a generalization of a PD_eL one since the characteristic function of the subset *opt* in the latter can be seen as a (crisp) possibility distribution. Hence though the special atom V is not in GD_eL, we can expect that it is more expressive than PD_eL in the representation of deontic formulas[3]. The following result verifies the expectation. For the sake of comparison, we restrict the atom V to being used only in the definitions of deontic wffs and still called the logic PD_eL from now on.

Let us fixed any $c > 0$. Define a translation mapping $\tau_1 : PD_eL \rightarrow GD_eL$ such that $\tau_1(\Box_m\alpha) = \Box_g^c\alpha$ for any $\alpha \in \Pi$ and $\Box = O, P$, or F, and τ_1 is an identity function on PV and is homomorphic with respect to other connectives.

Theorem 1 *If S is a subset of wffs of PD_eL and φ is a wff of PD_eL, then $S \models \varphi$ iff $\tau_1(S) \models \tau_1(\varphi)$, where $\models$ is the semantic consequence relation in the respective systems defined in the usual way.*

4 Logics of Action Preference

Though GD_eL can represent the degree of permission, obligation, and prohibition of actions directly, the comparison of preference between actions is not made explicit in the language. To exploit the full generality of our semantics, we can develop logics that include the comparative constructs directly.

To compare the preference between two actions, we will compare the resultant subsets of worlds after doing them. Now, given a possibility distribution on W and two subsets of W, A and B, there are at least four ways to compare them. Namely, we can define

$$A >_1 B \Leftrightarrow \Pi(A) > \Pi(B)$$

$$A >_2 B \Leftrightarrow \Delta(A) > \Pi(B)$$

$$A >_3 B \Leftrightarrow \Delta(A) > \Delta(B)$$

$$A >_4 B \Leftrightarrow \Pi(A) > \Delta(B),$$

and we have $A >_2 B$ implies $A >_1 B$ and $A >_3 B$ which in turn imply the weakest $A >_4 B$. In this section, we will explore a logic of action preference with binary connectives based on the orderings $>_1$ and $>_2$. The resultant logic will be called LAP. The orderings $>_3$ and $>_4$ will be left for the future research.

The basic symbols of LAP are those of PD_eL without the atom V, but two binary connectives $\succ_1$ and $\succ_2$ with the following formation rules are added.

If α and $\beta \in \Pi$, then $\alpha \succ_1 \beta, \alpha \succ_2 \beta \in \Phi$.

The wff $\alpha \succ_i \beta$ is also written as $\beta \prec_i \alpha$ for $i = 1, 2$.

The semantic models are those for GD_eL with the following constraints.

($\models$7) $w \models \alpha \succ_1 \beta$ iff $[\![\alpha]\!](w) >_1 [\![\beta]\!](w)$

[3] The atom V can of course be added back to GD_eL if necessary, however we feel that the symbol is redundant in the representation of deontic formulas.

($\models$8) $w \models \alpha \succ_2 \beta$ iff $[\![\alpha]\!](w) >_2 [\![\beta]\!](w)$

The basic properties of LAP are as follows.

Proposition 1 *1.* $\models (\alpha_1 \cup \alpha_2 \succ_1 \beta) \equiv (\alpha_1 \succ_1 \beta) \vee (\alpha_2 \succ_1 \beta)$

2. $\models (\alpha_1 \cup \alpha_2 \prec_1 \beta) \equiv (\alpha_1 \prec_1 \beta) \wedge (\alpha_2 \prec_1 \beta)$

3. $\models (\alpha_1 \& \alpha_2 \succ_1 \beta) \supset (\alpha_1 \succ_1 \beta) \wedge (\alpha_2 \succ_1 \beta)$

4. $\models \neg(\alpha \& \beta \succ_1 \beta)$

5. $\models \neg(\alpha \succ_1 \alpha \cup \beta)$

6. $\models \alpha \succ_1 \beta \supset \neg(\overline{\alpha} \succ_1 \overline{\beta})$

7. $\models (\alpha; \beta_1 \succ_1 \alpha; \beta_2) \supset \langle\alpha\rangle(\beta_1 \succ_1 \beta_2)$

8. $\models (\varphi \rightarrow \alpha_1/\alpha_2 \succ_1 \beta) \equiv (\varphi \supset \alpha_1 \succ_1 \beta) \wedge (\neg\varphi \supset \alpha_2 \succ_1 \beta)$

Proposition 2 *1.* $\models (\alpha_1 \cup \alpha_2 \succ_2 \beta) \equiv (\alpha_1 \succ_2 \beta) \wedge (\alpha_2 \succ_2 \beta)$

2. $\models (\alpha_1 \cup \alpha_2 \prec_2 \beta) \equiv (\alpha_1 \prec_2 \beta) \wedge (\alpha_2 \prec_2 \beta)$

3. $\models (\alpha_1 \succ_2 \beta) \supset (\alpha_1 \& \alpha_2 \succ_2 \beta)$

4. $\models (\alpha \succ_2 \beta_1) \supset (\alpha \succ_2 \beta_1 \& \beta_2)$

5. $\models \alpha \succ_2 \beta \supset \neg(\overline{\alpha} \succ_2 \overline{\beta})$

6. $\models (\alpha; \beta_1 \succ_2 \alpha; \beta_2) \supset [\alpha](\beta_1 \succ_2 \beta_2)$

7. $\models (\varphi \rightarrow \alpha_1/\alpha_2 \succ_2 \beta) \equiv (\varphi \supset \alpha_1 \succ_2 \beta) \wedge (\neg\varphi \supset \alpha_2 \succ_2 \beta)$

The action preference wffs can be combined with the conditional action wffs to denote a decision choice action. More specifically, define

$$\alpha \oplus \beta = (\alpha \succ_1 \beta) \rightarrow \alpha/(\alpha \prec_1 \beta \rightarrow \beta/(\alpha \cup \beta)).$$

Then doing $\alpha \oplus \beta$ will mean doing α or β selectively according to the weak preference relation $\succ_1$. The same definition can be carried out for $\succ_2$.

We can also embody the unary deontic operators of PD_eL by using the special actions $\mathbf{u}$ and $\emptyset$ or the comparison between an action and its negation. Define the following translation mappings, $\tau_2, \tau_3, \tau_4 : PD_eL \rightarrow LAP$ such that

$$\tau_2 : P_m\alpha \mapsto \neg(\mathbf{u} \succ_1 \alpha), F_m\alpha \mapsto \mathbf{u} \succ_1 \alpha, O_m\alpha \mapsto \mathbf{u} \succ_1 \overline{\alpha}$$

$$\tau_3 : P_m\alpha \mapsto \alpha \succ_1 \emptyset, F_m\alpha \mapsto \neg(\alpha \succ_1 \emptyset), O_m\alpha \mapsto \neg(\overline{\alpha} \succ_1 \emptyset)$$

and

$$\tau_4 : P_m\alpha \mapsto \neg(\overline{\alpha} \succ_1 \alpha), F_m\alpha \mapsto \overline{\alpha} \succ_1 \alpha, P_m\alpha \mapsto \alpha \succ_1 \overline{\alpha},$$

then we have the following result.

Theorem 2 *If S is a subset of wffs of PD_eL and φ is a wff of PD_eL, then $S \models \varphi$ iff $\tau_i(S) \models \tau_i(\varphi)$ for $i = 2, 3, 4$, where $\models$ is the semantic consequence relation in the systems PD_eL and LAP respectively.*

This result shows that LAP is indeed more expressive than PD_eL in deontic reasoning. Since the weak action preference connective $\succ_1$ is used in the definition of Meyer's deontic operators, it is expected that the same thing can be done for $\succ_2$. However, since $\succ_2$ is stronger than $\succ_1$, the resultant derived deontic operators are also stronger (or weaker in the case of permission operator) than the Meyer's. Let us define

$$O_s\alpha = \alpha \succ_2 \overline{\alpha},$$

$$F_s\alpha = \overline{\alpha} \succ_2 \alpha,$$

$$P_w\alpha = \neg(\overline{\alpha} \succ_2 \alpha),$$

then in LAP, $\models \Box_s\alpha \supset \tau_i(\Box_m\alpha)$, for $\Box = O, F$ and $\models \tau_i(P_m\alpha) \supset P_w\alpha$, for $i = 2, 3, 4$.

Furthermore, we can also defined some derived modal operators by using the special action $\mathbf{i}$. Let

$$I_w\alpha = \alpha \succ_1 \mathbf{i}, D_w\alpha = \overline{\alpha} \succ_1 \mathbf{i},$$

$$I_s\alpha = \alpha \succ_2 \mathbf{i}, D_s\alpha = \overline{\alpha} \succ_2 \mathbf{i},$$

then $I_s\alpha$ (resp. $I_w\alpha$) means it is strongly (resp. weakly) intended to do α, whereas $D_s\alpha$ (resp. $D_w\alpha$) means it is strongly (resp. weakly) declined to do α. The properties of these derived operators, as well as those of strong deontic ones, will be further investigated in the full paper.

5 A Logic for Action under Conflicting Obligations

Recently, a logic for conflicting obligation has been proposed by Brown[3], in which the preference ordering is set between *obligations* instead of worlds, where an obligation is represented by a subset of possible worlds. Since the intersection of two sets of possible worlds may be empty, the conflicting obligations can be represented in the framework naturally. Then four unary deontic operators and a dyadic comparative operator are defined. Their semantics based on the combination of quantifiers on the set of obligation and traditional deontic operators O and P. For example, one of them means for all obligations φ is obligatory. The formal semantic models of Brown's logic are the so-called minimal models in modal logic[4].

In this section, we will propose an alternative logic for action under conflicting obligations ($LACO$) by assimilating Brown's idea into Meyer's framework.

The language is just that of PD_eL (without the symbol V) with six unary deontic operators, $[O]$, $\langle O\rangle$, $[P]$, $\langle P\rangle$, $[F]$, and $\langle F\rangle$, and a binary one, $\succ$. The formation rule for these operators is by applying them to actions.

A model of $LACO$ is then a quadruple $\langle W, \models, [\![\cdot]\!], \pi\rangle$, where W and $[\![\cdot]\!]$ are defined as above. However, π is now a normalized possibility distribution on $\mathcal{P}(W)$ such that $\pi(\emptyset) = 0$, and $\models$ satisfies the constraints ($\models 1$)–($\models 3$) and the following:

($\models 9$): $w \models [P]\alpha$ iff for all $OB \subseteq W$ if $\pi(OB) > 0$, then $OB \cap [\![\alpha]\!](w) \neq \emptyset$.

($\models 10$): $w \models \langle P\rangle\alpha$ iff there exists $OB \subseteq W$ such that $\pi(OB) > 0$ and $OB \cap [\![\alpha]\!](w) \neq \emptyset$.

($\models 11$): $w \models [O]\alpha$ iff for all $OB \subseteq W$ if $\pi(OB) > 0$, then $OB \cap [\![\overline{\alpha}]\!](w) = \emptyset$.

($\models 12$): $w \models \langle O\rangle\alpha$ iff there exists $OB \subseteq W$ such that $\pi(OB) > 0$ and $OB \cap [\![\overline{\alpha}]\!](w) = \emptyset$.

($\models 13$): $w \models [F]\alpha$ iff for all $OB \subseteq W$ if $\pi(OB) > 0$, then $OB \cap [\![\alpha]\!](w) = \emptyset$.

($\models 14$): $w \models \langle F\rangle\alpha$ iff there exists $OB \subseteq W$ such that $\pi(OB) > 0$ and $OB \cap [\![\alpha]\!](w) = \emptyset$.

($\models 15$): $w \models \alpha \succ \beta$ iff $\{OB \mid OB \cap [\![\overline{\alpha}]\!](w) = \emptyset\} >_1 \{OB \mid OB \cap [\![\overline{\beta}]\!](w) = \emptyset\}$

6 Concluding Remarks

In this paper, the possibility theory and Meyer's semantics for deontic logics are combined to provide a semantics for action preference. We develop three logics based on the semantics. The logic GD_eL can reason about graded normative behavior. In the logic LAP, we can compare the preference between actions and do decision choice according to the comparison. The third logic, LACO, is based on the semantics of conflicting obligations. In what follows, we consider some related work and possible further research.

6.1 Related Works

The deontic operators and preference connectives considered in this paper are all applied to actions. However, there have been volumes of works on the deontic and preference logics of propositions[2, 12]. This kind of logics can also be considered in the present framework.

First, for GD_eL, wffs of the form $\Box_g^c\varphi$ and $\Box_g^{>c}\varphi$ can be added for $\Box = O, F, P$ and $\varphi \in \Phi$. In the semantics, replace $[\![\alpha]\!](w)$ by $|\varphi| =_{def} \{w \mid w \models \varphi\}$ and $\overline{\alpha}$ by $\neg\varphi$. Then when φ is a non-modal formula, $P_g^c\varphi$ and $O_g^c\alpha$ are just the wffs $(\varphi\ \Pi\ c)$ and $(\varphi\ N\ c)$ in possibilistic logic[6]. This also gives possibilistic logic a deontic interpretation.

Second, for LAP, we can add wffs of the form $\varphi \succ_1 \psi$ and interpret it as $|\varphi| >_1 |\psi|$ for any wffs φ and ψ. Again, this type of wffs are just equivalent to those of another well-known logics, called qualitative possibility logic (QPL), developed by Farinas del Cerro and Herzig[5]. It has been known that QPL has close relationship with conditional logic and nonmonotonic consequence relation[9], so the extension of the present framework to covering conditional and defeasible obligation will deserve further consideration.

6.2 Further generalization

In the above semantics, a fixed possibility distribution is associated with a model, so LAP is a logic of static preference. However, there is no essential difficulty in associating different possibility distributions with every possible world. Then we can get a variable preference relation. We believe that this step is necessary in developing dyadic deontic logics.

Second, in the present framework, the denotation of each action is a binary transition relation on the set of possible worlds. That is, we do not consider the action uncertainty. Though $[\![\alpha]\!](w)$ denote the all possibilities of doing α in w, it is not known which world is more possible than others. In the further research, we can interpret $[\![\alpha]\!]$ as a fuzzy binary relation so that $[\![\alpha]\!](w)$ is a possibility distribution or assign $[\![\alpha]\!](w)$ as a probability measure on W directly, then the expected payoff of doing α in w can be estimated as

$$\bigoplus_{u \in W} [\![\alpha]\!](w)(u) \otimes \pi(u),$$

where $\otimes$ and $\oplus$ are a kind of t-norms and co-t-norms respectively. Then the preference between two action can be compared via their expected payoff. This will also provide a kind of decision-theoretic semantics for our logic.

Third, the use of possibility theory forces our preference relation on possible worlds (or sets of possible worlds in the case of LACO) to be a connected ordering. This results in that we have only "don't care" incomparability (i.e. $\neg(w > u) \wedge \neg(u > w)$) but do not have "don't know" incomparability. This is indeed a limitation of our semantics, however, it can be easily overcome by allowing partial ordering between worlds. But the semantics of $\succ_1$, $\succ_2$, and $\succ$ have to be written in qualitative terms. For example, $w \models \alpha \succ_1 \beta$ iff there exists $u \in [\![\alpha]\!](w)$ such that for all $v \in [\![\beta]\!](w)$, $u > v$, where $>$ is a partial ordering between worlds.

Finally, our system is only for single agent. To model multiagent environment, we must associate a preference relation for each agent in the model, then some actions, such as "request" and "commit", that will change the agents' preference can be considered in the framework and it can serve as a semantic basis of agent-oriented programming[13]. Moreover, the interference of different agents will change the choice of decision makers just like the situations analyzed by game theoreticians. These problems will all be considered in our long-term goal to the integration of different mental attitudes in a logical framework.

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