Many-Valued Dynamic Logic for Qualitative Decision Theory

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Abstract. This paper presents an integration of the dynamic logic semantics and rational decision theory. Logics for reasoning about the expected utilities of actions are proposed. The well-formed formulas of the logics are viewed as the possible goals to be achieved by the decision maker and the truth values of the formulas are considered as the utilities of the goals. Thus the logics are many-valued dynamic logics. Based on different interpretations of acts in the logics, we can model different decision theory paradigms, such as possibilistic decision theory and case-based decision theory.

1 Introduction

Rational decision theory is a very important research topic in many academic fields such as economics, politics, and philosophy. Recently, it has also received more and more attention of the AI community due to the development of intelligent agent systems. The basic execution loop of an intelligent agent consists of three phases: perception, deliberation, and action. In the perception phase, the agent senses the status of the environment and receives information from other agents. Then, in the deliberation phase, the agent reasons with the observed and received information and plans its actions for achieving its goals. Finally, in the action phase, the plan is really executed. The capabilities of both reasoning about actions and decision making are crucial to the success of the deliberation phase since it has to know the possible effects of actions and select the appropriate actions for achieving its goals.

A variety of formalisms for reasoning about actions have been developed in AI, theoretical computer science, and philosophical logic. Among them, dynamic logic is originally proposed for reasoning about program behavior[10], and subsequently adopted for reasoning about actions by the AI community. Though the advantages of using dynamic logic for reasoning about actions have been emphasized in [6], the traditional dynamic logic has only limited capability in handling uncertainty.

In dynamic logic, a formula $[\alpha]\varphi$ denotes that φ holds after the execution of (possibly compound) action α, so in principle, if the agent's goal is φ and $[\alpha]\varphi$ can be derived from the description of the initial situation, then α is a feasible plan for achieving the goal. This gives rise to a decision theoretic reading of dynamic logic semantics. Since nondeterministic actions are allowed in dynamic logic, it may capture the agent's ignorance on the possible effects of actions to some extent. However, uncertainty pervades the whole deliberation phase, so further extensions of dynamic logic for handling different forms of uncertainty are needed. In general, there are three forms of uncertainty in the deliberation process.

- The perception of the agent may be imperfect and its received information may be incomplete and faulty, so its knowledge in the status of the environment is uncertain. Sometimes, the probabilistic instead of the exact knowledge is available. However, in other times, even probabilistic knowledge is not available, so more general consideration is also needed.
- In multi-agent systems, our agent may not be the only one which can cause the change of the world, so it has only partial knowledge about the possible effects of the actions. The classical dynamic logic may handle the case when the knowledge is *imprecise* (i.e. the effect of an action is represented as a set of states). However, the knowledge may be also probabilistic or possibilistic (i.e. the effect of an action are represented as a probability or possibility distribution on the set of states). We recall that a possibility distribution on a set X is a mapping $\pi : X \rightarrow [0,1]$ such that $\pi(x)$ measures the extent to which x is likely to be the actual consequence[14]. The dynamic logic should be extended to cover such cases.
- Since dynamic logic is two-valued, the goal for an agent to achieve must be crisp and non-flexible. A goal is either satisfied or non-satisfied. However, sometimes, we may want to describe more flexible goals. A goal may be satisfied to some degree. In decision theory, this is in general described by a real-valued utility function. Recently, more general notions of ordinal preference are considered[1, 13]. To represent the flexible goals in dynamic logic, we will generalize its semantics to a many-valued one.

On the other hand, in the decision theoretic contexts, far richer notions of uncertainty have been explored. Besides classical decision theory, in which the notions of probability and expected utility are of central importance, some alternatives, such as possibilistic decision theory[1], case-based decision theory[7], and belief function-based decision theory[11] have been proposed and axiomatically justified in different settings recently. The main concern of decision theory is to choose an action which will maximize the expected utility of performing the action given some knowledge on the effects of the action and the desirability of these effects. In the extreme case that the utility function is two-valued and the available knowledge is imprecise, this is just a rephrasing of the decision theoretic interpretation of dynamic logic. The only difference is that in general the set of available acts in decision theory is not algebraically structured as in dynamic logic. Thus, due to the usefulness of dynamic logic in reasoning about actions and the rich notions of uncertainty in decision theories, the combination of decision theory and dynamic logic semantics will have the advantages of cross-fertilization.

In this paper, we suggest a kind of integration between decision theoretic notions and dynamic logic semantics. The dynamic logic semantics is enhanced to a many-valued one, so the truth value of a formula in a state plays a two-fold role. One is the degree of satisfaction of the formula and the other is the utility of the state. Since each formula corresponds to a goal and we can consider more than one formulas in the same time, this means that we can easily describe the multiple objective decision-making in the logic. In classical dynamic logic, each action is interpreted as a binary transition relation on the set of states. Here, depending on the different uncertainty handling formalisms, we can generalize it to a fuzzy relation, a set of probability distributions, or a set of possibility distributions generated from a similarity relation. Thus, we will try to develop several many-valued dynamic logics for reasoning with different uncertainty formalisms.

In what follows, we will first review the basic notions from classical decision theory and some recent proposals of qualitative alternatives. Then the dynamic logics for possibilistic decision theory and case-based decision theory are considered respectively. Finally, conclusion is given and some possible research directions are suggested.

2 Review of Some Decision Theories

Classical quantitative decision theory considers expected utility maximization (EUM) as the criteria of rational choice. In the theory, a decision framework is a 4-tuple (D, X, μ, u), where D is a set of available decision acts, X is a set of possible outcomes, $\mu : D \times X \rightarrow [0, 1]$ assigns to each decision act $d \in D$ a probability distribution $\mu(d, \cdot)$ on X, and $u : X \rightarrow \Re$ is the utility function. Then the expected utility of a decision d is defined as

$$U(d) = \Sigma_{x \in X} \mu(d, x) \cdot u(x),$$

and the decision maker will choose d_0 such that $U(d_0) = \max_{d \in D} U(d)$.

While the computation of $E(d)$ relies on the arithmetic operations (mainly $+$ and $\cdot$) on real numbers, qualitative decision theory concentrates more on the decision maker's ordinal preference and uncertainty about the possible outcomes. Recently, a qualitative decision theory(PODT) based on possibilistic logic is proposed[1]. In the theory, a possibilistic decision framework is a 4-tuple (D, X, π, u), where D and X are defined as above, $\pi : D \times X \rightarrow T_1$ assigns to each decision act $d \in D$ a possibility distribution $\pi(d, \cdot) : X \rightarrow T_1$, and $u : X \rightarrow T_2$ is the utility assignment function. Here, T_1 and T_2 are linearly ordered scales, and under the commensurability assumption, we can assume $T_1 = T_2 = T$ without loss of generality. Typical examples of T are [0,1] or a subset of [0,1]. Let $n : T \rightarrow T$ be an order-reversing map on T, then two qualitative expected utilities for a decision d can be defined. For the risk-averse decision maker, the pessimistic expected utility is

$$U_*(d) = \min_{x \in X} \max(n(\pi(d, x)), u(x)),$$

and for a risk-prone decision maker, the optimistic expected utility is

$$U^*(d) = \max_{x \in X} \min(\pi(d, x), u(x)).$$

The PODT is particularly suitable for complicate situations in which complete probabilistic information is rarely available.

Another recent proposed alternative decision theory is the case-based one. According to [7], the purpose of case-based decision theory(CBDT) is to model decision making under uncertainty by formalizing reasoning by analogies. It suggests that decision makers tend to choose actions which performed well in similar past cases. Each case is viewed as a triple of a situation (i.e. a decision problem), the action chosen in it, and the consequence of performing the action. Thus, a case-based decision framework is a 6-tuple (P, D, X, C, s, u), where D, X, and u are defined as above, P is a set of situations, $C \subseteq P \times D \times X$ is a finite set of cases, called the memory of the decision maker, and $s : P \times P \to [0, 1]$ is a similarity function which measures the similarity between situations. Then, for a given situation p, the expected utility for a decision d is defined as

$$U_c(d) = \sum_{(q,d,x) \in C} s(p, q)u(x).$$

However, it is also pointed out that U_c is cumulative in nature, so the number of times a certain act was chosen in the past will affect perceived desirability. Thus, an act that was chosen repeatedly producing bad results may be considered superior to an act that was chosen only once but producing good result. To overcome the difficulty, the average utility is considered in [7], namely, in the above equation, the similarity function s is replaced by s' which is defined as

$$s'(p, q) = \begin{cases} s(p, q)/\sum_{(q',d,x) \in C} s(p, q') & \text{if well–defined} \\ 0 & \text{otherwise} \end{cases}$$

In [2], a more qualitative version of expected utility is considered which can also eliminate the cumulation assume the utility values are normalized to the range $[0, 1]$. The definition is analogous to the pessimistic and optimistic expected utility in the PODT.

$$U_{c*}(d) = \min_{(q,d,x) \in C} \max(n(s(p, q)), u(x)),$$

$$U_c^*(d) = \max_{(q,d,x) \in C} \min(s(p, q), u(x)).$$

While in PODT, it is observed that the criterion of maximizing $U^*(d)$ is sometimes over-optimistic[5], it seems that for CBDT, the pessimistic utility has some counterintuitive results. For example, if an act a was only adopted in the past for the cases that are completely different with the present situation, i.e., for all $(q, a, x) \in C$, $s(p.q) = 0$, then we will have $U_{c*}(a) = 1$ which is the maximum value. This phenomenon is due to the fact that the case memory is only a partial description of the world, so we may encounter a novel situation in which no past experience can be followed. In CBDT, we tend to find the most similar past case and apply its solution to the new situation, so the optimistic criterion seems more suitable.

3 Possibilistic Dynamic Logic

The possibilistic dynamic logic ($PoDL$) provides the integrated treatment of possibilistic decision theory and dynamic logic. The syntax and semantics of $PoDL$ is an extension of that for dynamic logic[10] and fuzzy modal logic $S\L MV$ in [8], which in turn bases on rational Pavelka logic proposed in [9].

3.1 Syntax

The alphabet of $PoDL$ consists of

1. A set of propositional letters, $PV = \{p, q, \ldots\}$,
2. a set of atomic actions, $A = \{a, b, c, \ldots\}$,
3. the set of truth constants r for each rational $r \in [0, 1]$, and
4. the logical symbols $\sim$, $\rightarrow$, $\wedge$, $\vee$, [,], ;, *, $\cup$, and ?.

The set of well-formed formulas(Φ) and the set of action expressions(Ξ) are defined inductively in the following way.

1. Φ is the smallest set such that
   - $PV \subseteq \Phi$ and $\mathsf{r} \in \Phi$ for all rational $r \in [0, 1]$, and
   - if $\varphi, \psi \in \Phi$ and $\alpha \in \Xi$, then $\sim \varphi$, $\varphi \rightarrow \psi, \varphi \wedge \psi, \varphi \vee \psi, [\alpha]\varphi \in \Phi$.
2. Ξ is the smallest set such that
   - $A \subseteq \Xi$, and
   - if $\alpha, \beta \in \Xi$ and $\varphi \in \Phi$, then $\alpha; \beta, \alpha \cup \beta, \alpha^*, \varphi? \in \Pi$.

Some abbreviations of $PoDL$ include $\neg\varphi = \varphi \rightarrow \mathsf{0}$, $\varphi \otimes \psi = \neg(\varphi \rightarrow \neg\psi)$, $\varphi \oplus \psi = \neg\varphi \rightarrow \psi$, and $\langle\alpha\rangle\varphi = \neg[\alpha]\neg\varphi$.

3.2 Semantics

The semantics of $PoDL$ is defined relative to a given Kripke structure $M = (W, \tau, [\![\cdot]\!], w_0)$, where W is a set of possible worlds, $\tau : W \times PV \rightarrow [0, 1]$ is the truth valuation function, $[\![\cdot]\!] : A \rightarrow (W \times W \rightarrow [0, 1])$ is the action denotation function, and $w_0 \in W$ is a designated world. The mappings τ and $[\![\cdot]\!]$ is extended to Φ and Ξ as follows.

1. $\tau(w, \mathsf{r}) = r$,
2. $\tau(w, \sim \varphi) = \begin{cases} 0 \text{ if } \ \tau(w, \varphi) = 1 \\ 1 \text{ otherwise} \end{cases}$
3. $\tau(w, \varphi \rightarrow \psi) = I(\tau(w, \varphi), \tau(w, \psi))$,
4. $\tau(w, \varphi \wedge \psi) = \min(\tau(w, \varphi), \tau(w, \psi))$,

5. $\tau(w, \varphi \vee \psi) = \max(\tau(w, \varphi), \tau(w, \psi))$,
6. $\tau(w, [\alpha]\varphi) = \inf_{x \in W} \max(1 - \|\alpha\|(w, x), \tau(x, \varphi))$,
7. $\|\alpha; \beta\| = \|\alpha\| \circ \|\beta\|$, i.e,, composition of two fuzzy relations $\|\alpha\|$ and $\|\beta\|$,
8. $\|\alpha \cup \beta\| = \|\alpha\| \cup \|\beta\|$,
9. $\|\alpha^*\| = \|\alpha\|^*$, i.e., the reflexive and transitive closure of $\|\alpha\|$,
10. $\|\varphi?\|(w, x) = \begin{cases} \tau(w, \varphi) & \text{if } w = x \\ 0 & \text{otherwise} \end{cases}$

where $I : [0,1] \times [0,1] \to [0,1]$ is an implication function. Typical implication functions include material implication $I(x, y) = \max(1 - x, y)$, Łukasiewicz's implication $I(x, y) = \min(1, 1 - x + y)$, and Gödel implication $I(x, y) = 1$ if $x \leq y$ and $= y$ if $x > y$. Here we will let I denote the Łukasiewicz's implication.

Let $w \models_M \varphi$ denote $\tau(w, \varphi) = 1$, then φ is true in M, written as $M \models \varphi$ if $w_0 \models_M \varphi$ and for a set Σ of wffs, we write $M \models \Sigma$ if $M \models \varphi$ for all $\varphi \in \Sigma$. Furthermore, φ is said to be an (external) logical consequence of Σ, denoted by $\Sigma \models \varphi$, if for any model M, $M \models \Sigma$ implies $M \models \varphi$. When Σ is the empty set, this is abbreviated as $\models \varphi$ and φ is said to be valid. Moreover, φ is satisfiable if $\sim \varphi$ is not valid and weakly satisfiable if $\neg\varphi$ is not valid.

3.3 Discussion

In the semantics above, if we consider Ξ as the set of available acts, and Φ the set of goals, then a Kripke structure is a generalization of possibilistic decision framework. It can be seen that $\tau(w, [\alpha]\varphi)$ and $\tau(w, \langle\alpha\rangle\varphi)$ are respectively the pessimistic and optimistic utility of doing α under state w with respect to the decision objective φ.

To see how the $PoDL$ model generalize a possibilistic decision framework, let us consider the formal correspondence between them. Let $M = (W, \tau, \|\cdot\|, w_0)$ be a $PoDL$ model and φ be a fixed wff, then $\mathcal{D}(M, \varphi) = (D, X, \pi, u)$ is a possibilistic decision framework, where

- $D = \Xi$,
- $X = W$,
- $\pi : D \times X \to [0, 1]$, $\pi(\alpha, w) = \|\alpha\|(w_0, w)$ for all $\alpha \in D$ and $w \in X$,
- $u : X \to [0, 1]$, $u(w) = \tau(w, \varphi)$ for all $w \in X$.

The difference is made explicit from the formal correspondence. First, in a possibilistic decision framework, the set of available decision acts is taken as primitive, so it is only needed to specify the possible effects of each act from the initial situation w_0, which is implicitly assumed. On the other hand, in $PoDL$, the set of actions is composed from some atomic ones, so to know the effects of an action under the initial situation, we have to know also effects of its constitutive actions under different situations. In other words, the decision maker choose plans instead of a single action in $PoDL$ model. Second, in a possibilistic decision framework, a utility function is given, which is implicitly assumed to correspond to a goal of the decision maker, whereas in $PoDL$, we have a bundle of utility functions, each corresponding to a wff of the language. Thus, we can know the utilities of not only some primitive goals p and q, but also the conjunctive goal $p \wedge q$, the aggregated goal $p \otimes q$, the negated goal $\neg p$, and so on. This is in particular suitable for multiple objectives decision making. Moreover, because of the use of rational Pavelka logic, the prioritized or thresholded goals mentioned in [3] can also be expressed in $PoDL$. For example, we can write goals like $\varphi \vee (1 - \mathsf{r})$ or $\mathsf{r} \rightarrow \varphi$.

When we can completely specify a possibilistic decision framework or a $PoDL$ model M, then the decision making process amount to the model checking problem in M. For example, if $M \models \mathsf{r} \rightarrow [\alpha]\varphi$, then we know that for goal φ, the plan α has pessimistic expected utility at least r. If $M \models \sim ([\alpha]\varphi \rightarrow [\beta]\varphi)$, then α is a better plan than β for satisfying φ according to the criterion of maximizing U_*. Sometimes, it is not easy to have a complete specification of the decision framework. Instead, we may have only some partial description of the status of the environment and the preconditions and effects of the primitive actions. Some typical sentences for the description are non-modal wffs of $PoDL$, or formulas of the form $\varphi \rightarrow (\mathsf{r} \rightarrow [a]\psi)$ where a is atomic and φ and ψ are non-modal. In this case, assume $\Sigma \subseteq \Phi$ is the set of descriptions, then the decision making process amount to deduction problem in the logic. We must try to derive the formulas like $\mathsf{r} \rightarrow [\alpha]\varphi$ or $\sim ([\alpha]\varphi \rightarrow [\beta]\varphi)$ from Σ by proof methods of the logic. Though the development of proof methods for the logic is beyond the purely semantic concern of the paper, it is indeed a very interesting direction for further research.

4 Case-based Dynamic Logic

Analogous to $PoDL$, in this section, we develop a case-based dynamic logic($CbDL$) for reasoning about actions and decisions according to CBDT. Though the syntax of $CbDL$ is similar to that of $PoDL$, we will add a similarity-based modal operator which is of independent interest to fuzzy reasoning[4, 8]. Furthermore, a more classical dynamic operator $\{\cdot\}$ is used for describing primitive actions in case bases.

Thus, the alphabet of $CbDL$ consists of those for $PoDL$ and three additional logical symbols $\{, \}, \nabla$ and the following formation rules are added to those for $PoDL$,

- if $\varphi \in \Phi$ and $a \in A$, then $\{a\}\varphi$ and $\nabla\varphi \in \Phi$.

To define the semantics for $CbDL$, we first recall the definition of fuzzy similarity relation. A fuzzy relation $S : X \times X \rightarrow [0, 1]$ is a similarity one if it satisfies the following three properties, for all $x, y \in X$,

1. reflexivity: $S(x, x) = 1$,
2. symmetry: $S(x, y) = S(y, x)$, and
3. transitivity: $S(x, y) \geq \sup_{z \in X} \min(S(x, z), S(z, y))$.

Then a $CbDL$ model is a 5-tuple $M = (W, \tau, [\![\cdot]\!]_0, S, w_0)$, where W, τ, and w_0 are as above, $[\![\cdot]\!]_0 : A \rightarrow 2^{W \times W}$, and $S : W \times W \rightarrow [0, 1]$ is a similarity relation.

The mapping $\llbracket \cdot \rrbracket : A \to (W \times W \to [0,1])$ is defined by

$$\llbracket a \rrbracket(x,y) = \sup_{z \in W, (z,y) \in \llbracket a \rrbracket_0} S(x,z).$$

That is, $\llbracket a \rrbracket = S \circ \llbracket a \rrbracket_0$. Then the mappings τ and $\llbracket \cdot \rrbracket$ is extended to Φ and Ξ as in *PoDL* with the extra rules for wffs of the forms $\nabla\varphi$ and $\{a\}\varphi$,

10. $\tau(w, \nabla\varphi) = \inf_{x \in W} \max(1 - S(w,x), \tau(x,\varphi))$,
11. $\tau(w, \{a\}\varphi) = \inf_{x \in W, (w,x) \in \llbracket a \rrbracket_0} \tau(x,\varphi)$.

The definitions of logical consequence, validity, and satisfiability, etc. are analogous to those of *PoDL*. Sometimes, to distinguish the logical consequence relations between *PoDL* and *CbDL*, we will add the subscripts to them.

The mapping $\llbracket \cdot \rrbracket_0$ is to model the case memory, The definition is such that a case (x,a,y) is in the memory iff $(x,y) \in \llbracket a \rrbracket_0$. This restricts that the actions appearing in the memory must be atomic. This restriction is not so restrictive as it seems at the first glance. Imagine that the agent has a detail trace of the execution of the actions in the past cases. Then for a compound action like $\alpha;\beta$, if we know the intermediate state after the execution of α, then we can decompose a case $(x, \alpha;\beta, y)$ into two cases (x,α,z) and (z,β,y), and store the latter two on the memory. According to the original restriction of case memory in [7], there do not exist two cases (p,a,x) and (p,a',x') in the memory such that $a \neq a'$ or $x \neq x'$, so we can also require that $\llbracket a \rrbracket_0$ is a partial function in the *CbDL* model. However, for generality, we do not impose the restriction on the models.

The definition of $\llbracket \cdot \rrbracket$ from $\llbracket \cdot \rrbracket_0$ and S make the following a valid axiom schema in *CbDL*. That is,

$$\models [a]\varphi \equiv \nabla\{a\}\varphi,$$

for $a \in A$ and $\varphi \in \Phi$.

Apparently, the *CbDL* and *PoDL* models have some correspondence. In the semantics for *CbDL*, we have constructed $\llbracket \cdot \rrbracket$ from $\llbracket \cdot \rrbracket_0$ and S, if we then ignore the latter two components, a *PoDL* model is obtained. Since the language of *PoDL* is a sublanguage of *CbDL*, this means that *CbDL* is an extension of *PoDL*. Namely, if Σ is a set of wffs of *PoDL* and φ is a wff of *PoDL*, then $\Sigma \models_{PoDL} \varphi$ implies $\Sigma \models_{CbDL} \varphi$.

However, unlike *PoDL*, we can not find a direct correspondence between *CbDL* models and CBDT framework. This is due to the fact that in CBDT, the set of situation P and the set of consequence X are not necessarily the same, whereas in *CbDL* models, we model the past cases by a set of binary relations on W, so W plays both the roles of P and X. To transform a CBDT framework (P,D,X,C,s,u) into a *CbDL* model, we can let $W = P \cup X$, and then extend the similarity function s to $W \times W$ and the utility function u to W. However, is is likely that s is not well-defined outside $P \times P$ and u is not definable in P. In this case, a simple approach is just let $u(p) = 0$ for all $p \in P$ and $s(x,y) = 0$ for $x \in X$ or $y \in X$. Then the extended s is just the similarity fuzzy relation S and for the extended u, $u(w)$ is the truth value $\tau(w,\varphi)$ for some fixed goal

φ. The mapping $[\![\cdot]\!]_0$ is derived from the case memory C, i.e., $(p, x) \in [\![a]\!]_0$ if $(p, a, x) \in C$. If the process of extending s and u does not distort the original CBDT framework, then we have a complete specification of $CbDL$ model. This reduces the process of reasoning about actions and decision in the original CBDT framework to the model checking problem in $CbDL$ as in the case of $PoDL$.

On the other hand, from a more practical viewpoint, we may have only a partial description of the whole CBDT framework from the beginning. In this case, we assume there is a subset of crisp propositional symbols $PV_0 \subseteq PV$ for describing the case memory. Let Φ_0 is the set of sentences resulting from Boolean combinations of symbols in PV_0. Then in general, we have four sets of proper axioms for the description of the framework. The first one Σ_s is to describe the similarity function, so each sentence in it is of the form $\varphi \to (\mathsf{r} \to \nabla\psi)$ for some $\varphi, \psi \in \Phi_0$, the second is Σ_c for the case memory, so its sentences are in the form of $\varphi \to \{a\}\psi$ for some $\varphi, \psi \in \Phi_0$ and $a \in A$, the third is Σ_u which specifies the utility functions, so each sentence is of the form $\varphi \to (\mathsf{r} \to \psi)$ for some $\varphi \in \Phi_0$ and $\psi \in \Phi$, and the last is $\Sigma_0 = \{p \vee \neg p \mid p \in PV_0\}$ to enforce that each $p \in PV_0$ is two-valued. These four sets are proper axioms instead of premises because we require that they are true not only in w_0 but also in all possible worlds of a model. Let $\Omega = \Sigma_s \cup \Sigma_c \cup \Sigma_u \cup \Sigma_0$ and suppose the agent faces a new problem described by a set of (possibly just propositional) wffs Σ, then our problem is a theorem-proving one. For example, if we have $\Sigma \models_\Omega \sim (\langle\alpha\rangle\varphi \to \langle\beta\rangle\varphi)$, then α will be a better plan for the goal φ with respect to the criterion of maximizing U_c^*, where $\models_\Omega$ means the logical consequence relation in a $CbDL$ system with Ω as the set of proper axioms.

5 Future Works and Conclusion

We have outlined two logical languages for reasoning about actions and decisions based on the dynamic logic semantics and decision theory framework. One is based on possibilistic decision theory and the other on case-based decision theory. The language for $PoDL$ is a subset of $CbDL$ and it is shown that $CbDL$ is a conservative extension of $PoDL$. Both logics can be used in two ways. When we have a complete specification of the decision frameworks, the logic can be used in a model checking way and if we have only a partial description of the problem, then the logic should be used in a theorem proving way.

As mentioned above, since theorem proving is in general more difficult than model checking, the development of theorem proving methods for both logics is the first demanding problem for further research.

Second, while the logics developed here are mainly based on qualitative decision theories, we would also like to develop similar logics for quantitative decision theories. In particular, probabilistic dynamic logics [12] should be a good starting point. However, since these logics are aimed at reasoning about the behavior of probabilistic algorithm, they are still two-valued, so the generalization to many-valued ones are needed. If this is successful, we can model the classical decision framework as well as the original quantitative criterion of maximizing U_c in CBDT.

In conclusion, the results reported in this paper is just at the early stage of a long-term goal to integrate logical reasoning and decision theories. We expect the cross-fertilization of both fields can result from the research.

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