Lexical Analyzer — Scanner

ASU Textbook Chapter 3.1, 3.3, 3.4, 3.6, 3.7, 3.5

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Main tasks

- **Read the input characters and produce as output a sequence of tokens to be used by the parser for syntax analysis.**
  - **tokens: terminal symbols in grammar.**
- **Lexeme: a sequence of characters matched by a given pattern associated with a token.**
  - **Example:**

Lexeme	pi	=	3.1416	;
token	ID	ASSIGN	FLOAT-LIT	SEMI-COL

  - **patterns:**
    - ▷ *identifier (variable names) starts with a letter or "_", and follows by letters, digits or "_";*
    - ▷ *floating point number starts with a string of digits, follows by a dot, and terminates with another string of digits;*

Strings

- **Definitions and operations.**
  - **alphabet** : a finite set of symbols (characters);
  - **string** : a finite sequence of symbols from the alphabet;
  - $|S|$: length of a string S;
  - empty string: ϵ;
  - xy: concatenation of strings x and y
    - ▷ $\epsilon x \equiv x\epsilon \equiv x$;
  - exponention:
    - ▷ $s^0 \equiv \epsilon$;
    - ▷ $s^i \equiv s^{i-1}s$, $i > 0$.

Parts of a string

- **Parts of a string: example string "necessary"**
  - **prefix: deleting zero or more tailing characters; eg: "nece"**
  - **suffix: deleting zero or more leading characters; eg: "ssary"**
  - **substring: deleting prefix and suffix; eg: "ssa"**
  - **subsequence: deleting zero or more not necessarily contiguous symbols; eg: "ncsay"**
  - **Proper prefix, suffix, substring or subsequence: one that cannot equal to the original string;**

Language

- **Language : any set of strings over an alphabet.**
- **Operations on languages:**
  - **union:** $L \cup M = \{s | s \in L$ **or** $s \in M\}$;
  - **concatenation:** $LM = \{st | s \in L$ **and** $t \in M\}$;
  - $L^0 = \{\epsilon\}$;
  - **Kleene closure** : $L^* = \cup_{i=0}^{\infty} L^i$;
  - **Positive closure** : $L^+ = \cup_{i=1}^{\infty} L^i$;
  - $L^* = L^+ \cup \{\epsilon\}$.

Regular expressions

- **A regular expression r denotes a language $L(r)$, also called a regular set.**
- **Operations on regular expressions:**

regular expression	language
$\emptyset$	empty set $\{\}$
ϵ	the set containing the empty string $\{\epsilon\}$
a	$\{a\}$ where a is a legal symbol
$r\|s$	$L(r) \cup L(s)$ — union
rs	$L(r)L(s)$ — concatenation
r^*	$L(r)^*$ — Kleene closure

- **Example:**

$a\|b$	$\{a, b\}$
$(a\|b)(a\|b)$	$\{aa, ab, ba, bb\}$
a^*	$\{\epsilon, a, aa, aaa, \ldots\}$
$a\|a^*b$	$\{a, b, ab, aab, \ldots\}$
C identifier	$(A\|\cdots\|Z\|a\|\cdots\|z)\ ((A\|\cdots\|Z\|a\|\cdots\|z\|\_)\ \|\ (0\|1\|\cdots\|9)\ \|\ \_)^*$

Regular definitions

- **For simplicity, give names to regular expressions.**
  - **format: name $\rightarrow$ regular expression.**
  - **example 1: digit $\rightarrow 0|1|2|\cdots|9$.**
  - **example 2: letter $\rightarrow a|b|c|\cdots|z|A|B|\cdots|Z$.**
- **Notational standards:**

r^*	$r^+\|\epsilon$
r^+	rr^*
$r^?$	$r\|\epsilon$
$[abc]$	$a\|b\|c$
$[a-z]$	$a\|b\|c\|\cdots\|z$

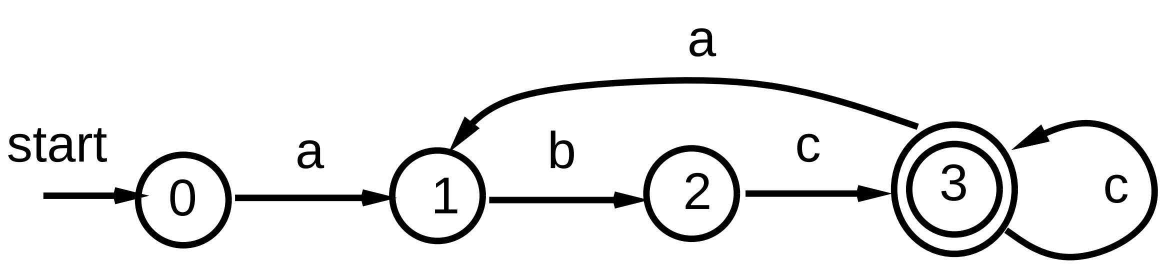
- **Example:**
  - **C variable name:** $[A-Za-z\_][A-Za-z0-9\_]^*$

Non-regular sets

- **Balanced or nested construct**
  - **Example: if $\cdots$ then $\cdots$ else**
  - **Recognized by context free grammar.**
- **Matching strings:**
  - $\{wcw\}$**, where w is a string of a's and b's and c is a legal symbol.**
  - **Cannot be recognized even using context free grammars.**
- **Remark: anything that needs to "memorize" "non-constant" amount of information happened in the past cannot be recognized by regular expressions.**

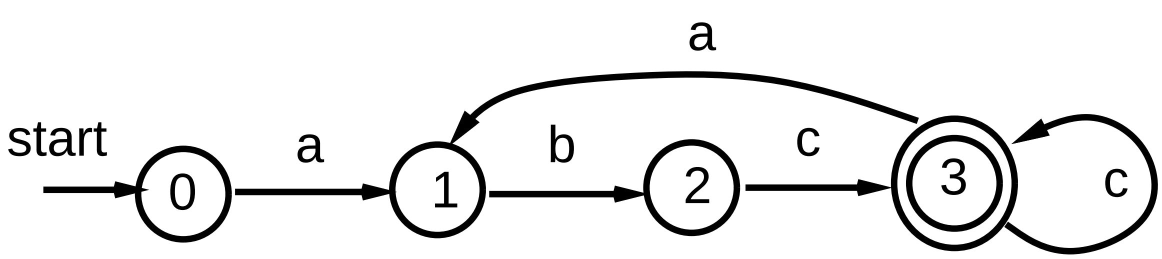
Finite state automata (FA)

- FA is a mechanism used to recognize tokens specified by a regular expression.
- Definition:
  - A finite set of states, i.e., vertices.
  - A set of transitions, labeled by characters, i.e., labeled directed edges.
  - A starting state, i.e., a vertex with an incoming edge marked with "start".
  - A set of final (accepting) states, i.e., vertices of concentric circles.
- Example: transition graph for the regular expression $(abc^+)^+$



Transition graph and table for FA

- **Transition graph:**



- **Transition table:**

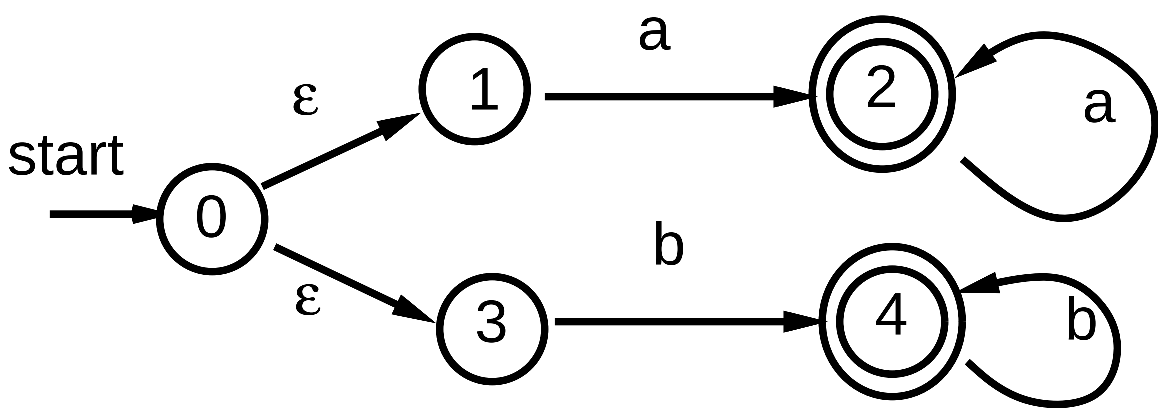
	a	b	c
0	**1**		
1		**2**	
2			**3**
3	**1**		**3**

  - **Rows are input symbols.**
  - **Columns are current states.**
  - **Entries are resulting states.**
  - **Along with the table, a starting state and a set of accepting states are also given.**

This is also called a GOTO table.

Types of FA's

- **Deterministic FA (DFA):**
  - **has a unique next state for a transition**
  - **and does not contain ϵ-transitions, that is, a transition takes ϵ as the input symbol.**
- **Nondeterministic FA (NFA):**
  - **either "could have more than one next state for a transition;"**
  - **or "contains ϵ-transitions."**
  - **Example:** $aa^*|bb^*$.



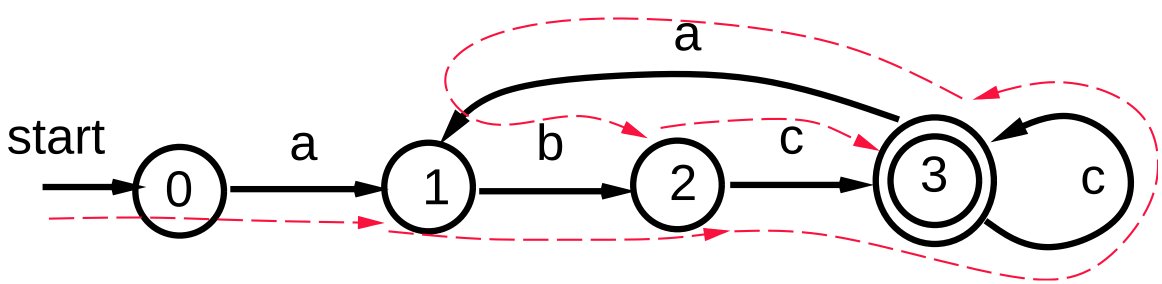
How to execute a DFA

- **Algorithm:**

> $s \leftarrow$ **starting state;**
>
> **while there are inputs and** s **is a legal state do**
>
> $s \leftarrow$ **Table**$[s, input]$
>
> **end while**
>
> **if** $s \in$ **accepting states then ACCEPT else REJECT**

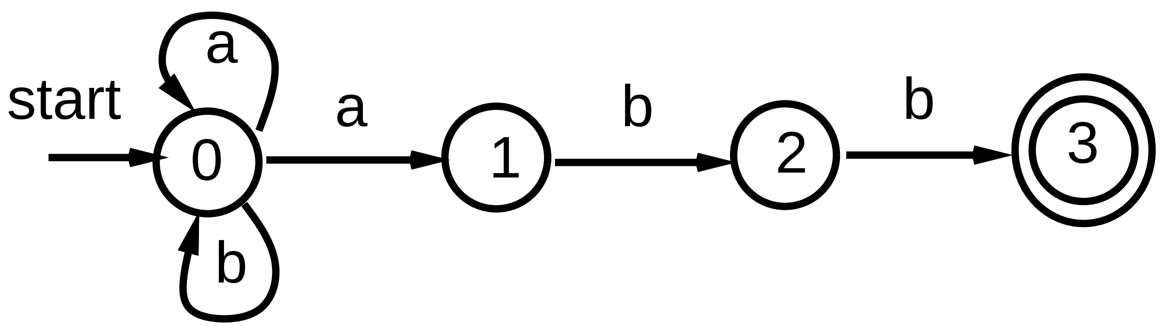
- **Example: input "abccabc". The accepting path:**

$$0 \xrightarrow{a} 1 \xrightarrow{b} 2 \xrightarrow{c} 3 \xrightarrow{c} 3 \xrightarrow{a} 1 \xrightarrow{b} 2 \xrightarrow{c} 3$$



How to execute an NFA (informally)

- **An NFA accepts an input string x if and only if there is some path in the transition graph initiating from the starting state to some accepting state such that the edge labels along the path spell out x.**
- **Could have more than one path. (Note DFA has at most one.)**
- **Example: regular expression: $(a|b)^*abb$; input $aabb$**



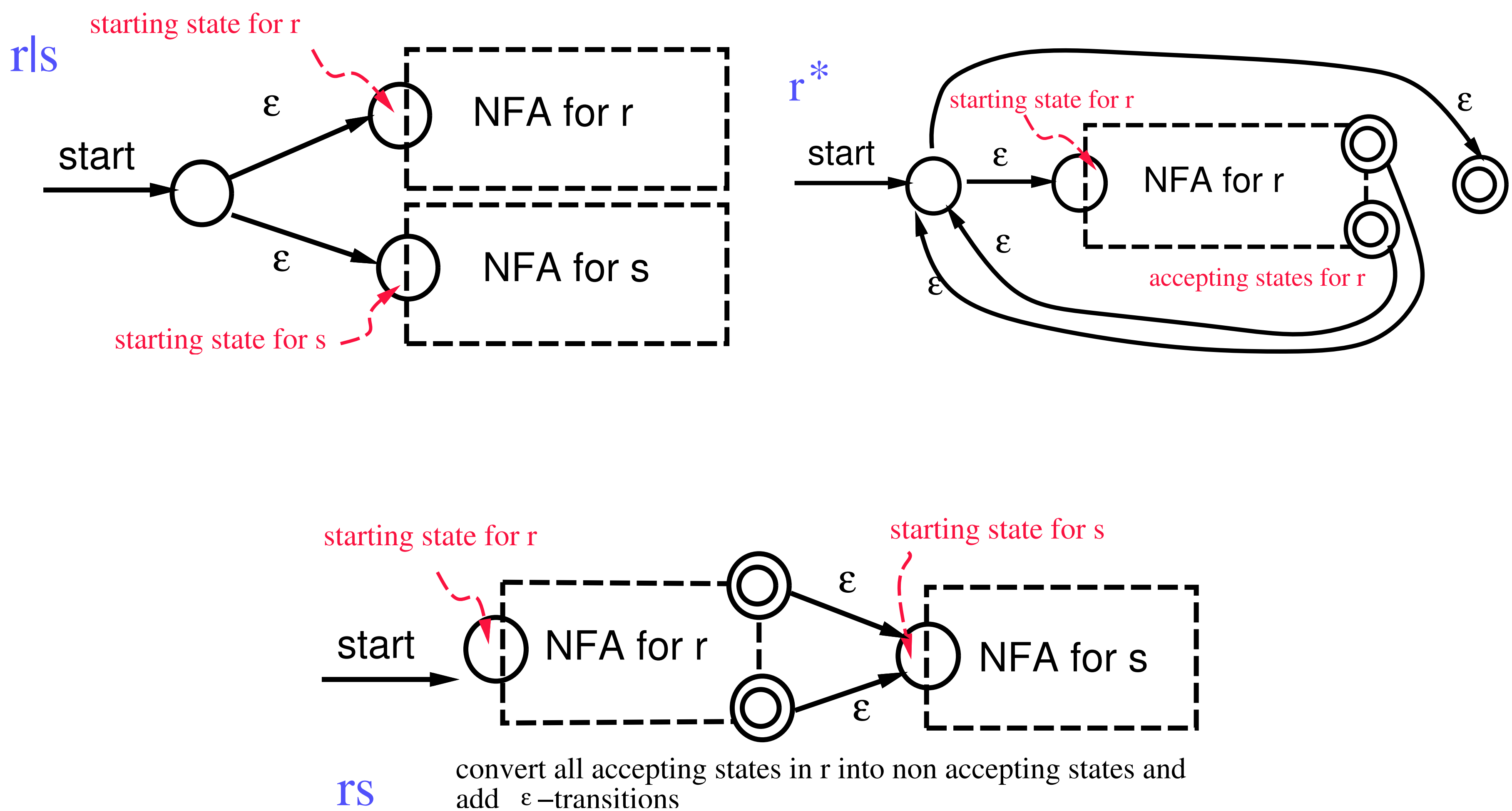
	a	b
0	**{0,1}**	**{0}**
1		**{2}**
2		**{3}**

$0 \xrightarrow{a} 0 \xrightarrow{a} 1 \xrightarrow{b} 2 \xrightarrow{b} 3$ **Accept!**

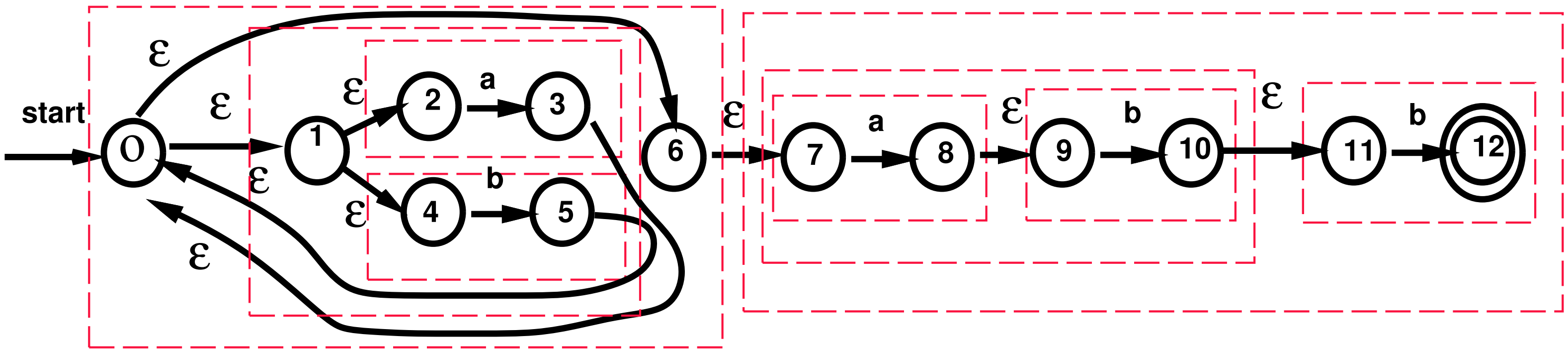
$0 \xrightarrow{a} 0 \xrightarrow{a} 0 \xrightarrow{b} 0 \xrightarrow{b} 0$ **Reject!**

From regular expressions to NFA's

- **Structural decomposition:**
  - **atomic items: $\emptyset$, ϵ and a legal symbol.**



Example: $(a|b)^*abb$



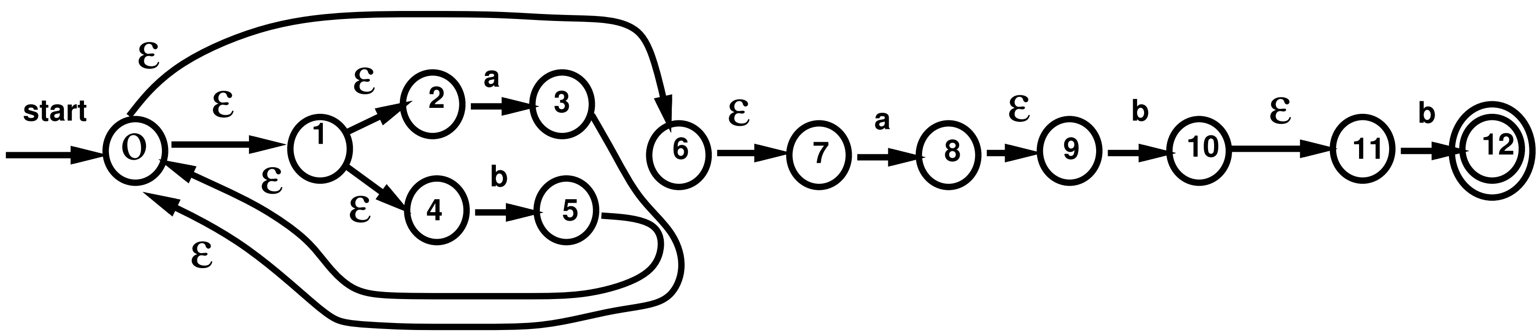
- **This construction produces only ϵ-transitions, and never produces multiple transitions for an input symbol.**
- **It is possible to remove all ϵ-transitions from an NFA and replace them with multiple transitions for an input symbol, and vice versa.**

Construction theorems

- **Theorem #1:**
  - **Any regular expression can be expressed by an NFA.**
  - **Any NFA can be converted into a DFA.**
- **That is, any regular expression can be expressed by a DFA.**
- **How to convert an NFA to a DFA:**
  - **Find out what is the set of possible states that can be reached from an NFA state using ϵ-transitions.**
  - **Find out what is the set of possible states that can be reached from an NFA state on an input symbol.**
- **Theorem #2:**
  - **Every DFA can be expressed as a regular expression.**
  - **Every regular expression can be expressed as a DFA.**
  - **DFA and regular expressions have the same expressive power.**
- **How about the power of DFA and NFA?**

Converting an NFA to a DFA

- **Definitions: let T be a set of states and a be an input symbol.**
  - **ϵ-closure(T): the set of NFA states reachable from some state $s \in T$ using ϵ-transitions.**
  - $move(T, a)$**: the set of NFA states to which there is a transition on the input symbol a from state $s \in T$.**
  - **Both can be computed using standard graph algorithms.**
  - **ϵ-closure**$(move(T, a))$**: the set of states reachable from a state in T for the input a.**
- **Example: NFA for $(a|b)^*abb$**



  - **ϵ-closure**$(\{0\}) = \{0, 1, 2, 4, 6, 7\}$**, that is, the set of all possible starting states**
  - $move(\{2, 7\}, a) = \{3, 8\}$

Subset construction algorithm

- **In the converted DFA, each state represents a subset of NFA states.**
  - $T \xrightarrow{a} \epsilon$-**closure**$(move(T, a))$
- **Subset construction algorithm :**

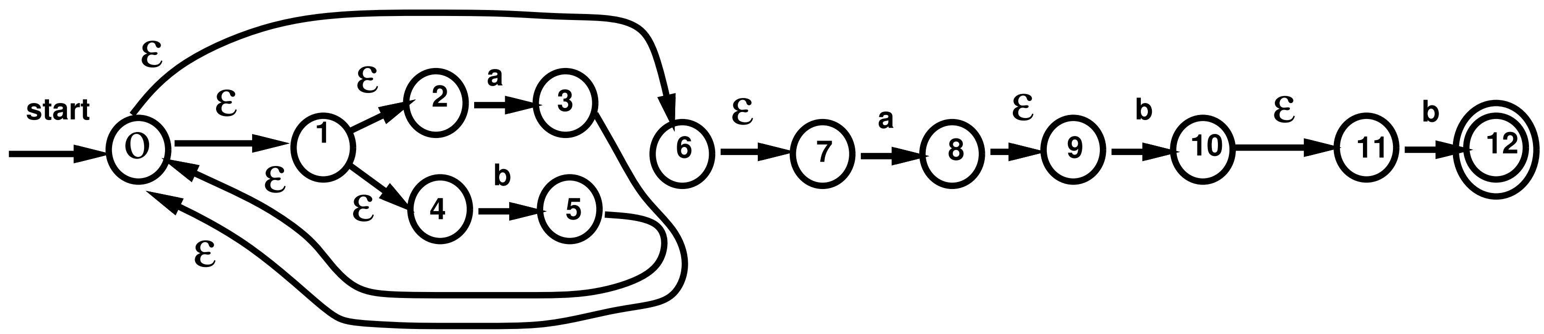
initially, we have an unmarked state labeled with ϵ-closure$(\{s_0\})$, where s_0 is the starting state.

while there is an unmarked state with the label T do
▷ *mark the state with the label T*
▷ *for each input symbol a do*
▷ $U \leftarrow \epsilon$-**closure**$(move(T, a))$
▷ *if U is a subset of states that is never seen before*
▷ *then add an unmarked state with the label U*
▷ *end for*

end while

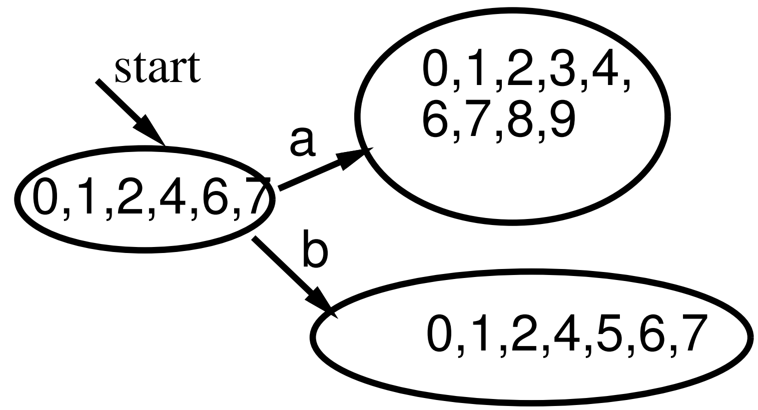
- **New accepting states: those contain an original accepting state.**

Example

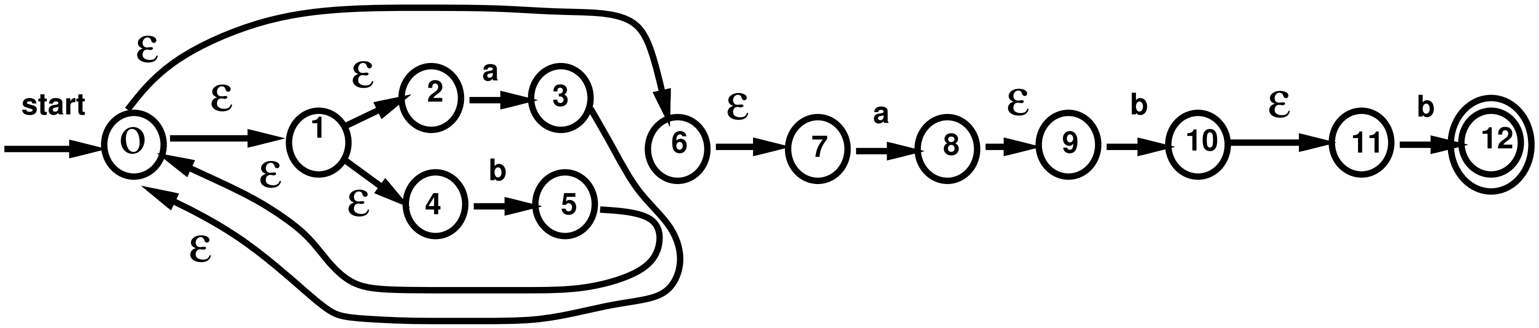


First step:

- ϵ-**closure({0}) = {0,1,2,4,6,7}**
- $move(\{0,1,2,4,6,7\},a)$ = **{3,8}**
- ϵ-**closure({3,8})** = **{0,1,2,3,4,6,7,8,9}**
- $move(\{0,1,2,4,6,7\},b)$ = **{5}**
- ϵ-**closure({5}) = {0,1,2,4,5,6,7}**



Example — cont.

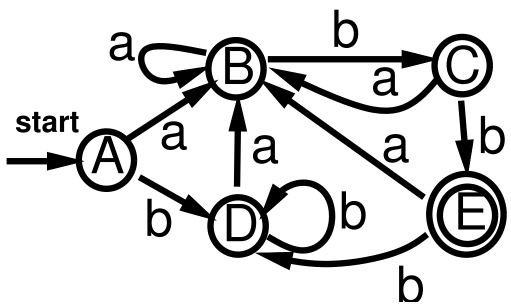


transition table:

states:

- $A = \{0, 1, 2, 4, 6, 7\}$
- $B = \{0, 1, 2, 3, 4, 6, 7, 8, 9\}$
- $C = \{0, 1, 2, 4, 5, 6, 7, 10, 11\}$
- $D = \{0, 1, 2, 4, 5, 6, 7\}$
- $E = \{0, 1, 2, 4, 5, , 6, 7, 12\}$

	a	b
A	B	D
B	B	C
C	B	E
D	B	D
E	B	D



Algorithm for executing an NFA

- **Algorithm:** s_0 **is the starting state,** F **is the set of accepting states.**

  > $S \leftarrow \epsilon$-**closure**$(\{s_0\})$
  >
  > **while next input** a **is not EOF do**
  >
  > $\triangleright$ $S \leftarrow \epsilon$-***closure***$(move(S, a))$
  >
  > **end while**
  >
  > **if** $S \cap F \neq \emptyset$ **then ACCEPT else REJECT**

- **Execution time is** $O(r^2 \cdot s)$**, where**
  - r **is the number of NFA states, and** s **is the length of the input.**
  - **Need** $O(r^2)$ **time in running** ϵ**-closure(**T**) assuming using an adjacency matrix representation and a linear-time hashing routine to remove duplicated states.**
- **Space complexity is** $O(r^2 \cdot c)$ **using a standard adjacency matrix representation for graphs, where** c **is the cardinality of the alphabets.**
- **May have slightly better algorithms.**

Trade-off in executing NFA's

- **Can also convert an NFA to a DFA and then execute the equivalent DFA.**
  - **Running time: linear in the input size.**
  - **Space requirement: linear in the size of the DFA.**
- **Catch:**
  - **May get $O(2^r \cdot c)$ DFA states by converting an r-state NFA.**
  - **The converting algorithm may also takes $O(2^r)$ time.**
- **Time-space tradeoff:**

	space	time
NFA	$O(r^2 \cdot c)$	$O(r^2 \cdot s)$
DFA	$O(2^r \cdot c)$	$O(s)$

  - **If memory is cheap or programs will be used many times, then use the DFA approach;**
  - **otherwise, use the NFA approach.**

LEX

- **An UNIX utility.**
- **An easy way to use regular expressions to do lexical analysis.**
- **Convert your LEX program into an equivalent C program.**
- **Depending on implementation, may use NFA or DFA algorithms.**
- **file.l $\longrightarrow$ [lex file.l] $\longrightarrow$ lex.yy.c**
- **lex.yy.c $\longrightarrow$ [cc -ll lex.yy.c] $\longrightarrow$ a.out**
  - **May produce .o file if there is no main().**
- **input $\longrightarrow$ [a.out] $\longrightarrow$ output sequence of tokens**
- **May have slightly different implementations and libraries.**

LEX formats

- **Source format:**
  - **Declarations —- a set of regular definitions, i.e., names and their regular expressions.**
  - **%%**
  - **Translation rules — actions to be taken when patterns are encountered.**
  - **%%**
  - **Auxiliary procedures**
- **Global variables:**
  - $yyleng$**: length of current string**
  - $yytext$**: current string**
  - $yylex()$**: the scanner routine**
  - **...**

LEX formats – cont.

- **Declarations:**
  - **C language code between %{ and %}.**
    - ▷ *variables;*
    - ▷ *manifest constants, i.e., identifiers declared to represent constants.*
  - **Regular expressions.**
- **Translation rules:**

  P_1 **{action$_1$}**

  if regular expression P_1 is encountered, then action$_1$ is performed.
- **LEX internals: regular expressions $\longrightarrow$ NFA $\xrightarrow{\text{if needed}}$ DFA**

test.l — Declarations

```
%{
    /* some initial C programs */
#define BEGINSYM 1
#define INTEGER  2
#define IDNAME   3
#define REAL     4
#define STRING   5
#define SEMICOLONSYM 6
#define ASSIGNSYM 7
%}
Digit        [0-9]
Letter       [a-zA-Z]
IntLit       {Digit}+
Id           {Letter}({Letter}|{Digit}|_)*
```

test.l — Rules

```
%%
[ \t\n] {/* skip white spaces */}
[Bb][Ee][Gg][Ii][Nn]                   {return(BEGINSYM);}
{IntLit}                               {return(INTEGER);}
{Id}                {
                     printf("var has %d characters, ",yyleng);
                     return(IDNAME);
                    }
({IntLit}[.]{IntLit})([Ee][+-]?{IntLit})? {return(REAL);}
\"[^\"\n]*\"   {stripquotes(); return(STRING);}
";"                                    {return(SEMICOLONSYM);}
":="                                   {return(ASSIGNSYM);}
.         {printf("error --- %s\n",yytext);}
```

test.l — Procedures

```
%%
/* some final C programs */
stripquotes()
{
  /* handling string within a quoted string */
  int frompos, topos=0, numquotes = 2;
  for(frompos=1; frompos<yyleng; frompos++){
    yytext[topos++] = yytext[frompos];
  }
  yyleng -= numquotes;
  yytext[yyleng] = '\0';
}
void main(){
  int i;
  i =  yylex();
  while(i>0 && i < 8){
    printf("<%s> is %d\n",yytext,i);
    i =  yylex();     }   }
```

Sample run

```
austin% lex test.l
austin% cc lex.yy.c -ll
austin% cat data
Begin
123.3  321.4E21
x := 365;
"this is a string"
austin% a.out < data
<Begin> is 1
<123.3> is 4
<321.4E21> is 4
var has 1 characters, <x> is 3
<:=> is 7
<365> is 2
<;> is 6
<this is a string> is 5
%austin
```

More LEX formats

- **Special format requirement:**

P_1
```
  { action1
  ...
  }
```

Note: { and } must indent.

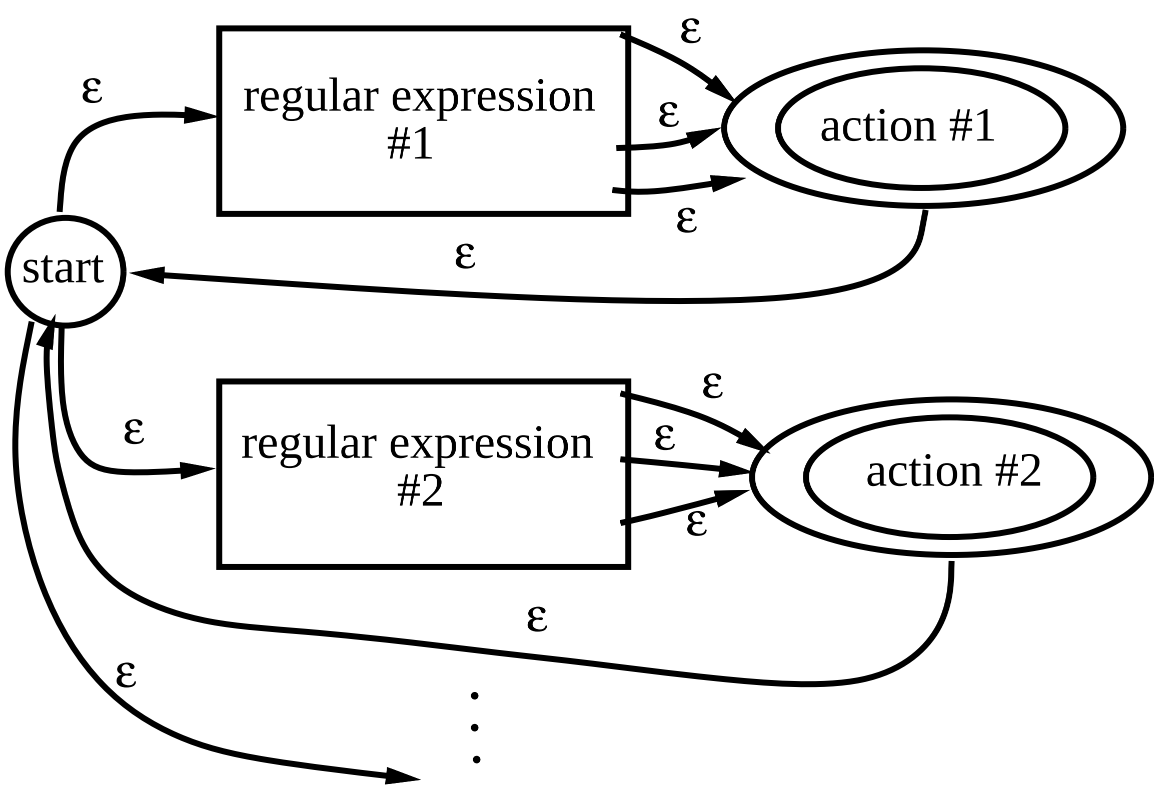
- **LEX special characters (operators):**

```
‘‘ \ [ ] ^ - ? . * + | ( ) $ { } % < >
```

- **When there is any ambiguity in matching, prefer**
  - **longest possible match;**
  - **earlier expression if all matches are of equal length.**

LEX internals

- **LEX code:**
  - **regular expression #1 {action #1}**
  - **regular expression #2 {action #2}**
  - · · ·



LEX internals – cont.

- **How to find a longest possible match if there are many legal matches?**
  - **If an accepting state is encountered, do not immediately accept.**
  - **Push this accepting state and the current input position into a stack and keep on going until no more matches is possible.**
  - **Pop from the stack and execute the actions for the popped accepting state.**
  - **Resume the scanning from the popped current input position.**
- **How to find the earliest match if all matches are of equal length?**
  - **Assign numbers $1, 2, \cdots$ to the accepting states using the order they appear (from top to bottom) in the expressions.**
  - **If you are in multiple accepting states, execute the action associated with the least indexed accepting state.**

Practical considerations

- **key word** v.s. **reserved word**
  - key word:
    - ▷ *def: word has a well-defined meaning in a certain context.*
    - ▷ *example: FORTRAN, PL/1, . . .*

if	*if*	*then*	*else*	=	*then*	;
	id		*id*		*id*	

    - ▷ *Makes compiler to work harder!*
  - reserved word:
    - ▷ *def: regardless of context, word cannot be used for other purposes.*
    - ▷ *example: COBOL, ALGOL, PASCAL, C, ADA, . . .*
    - ▷ *task of compiler is simpler*
    - ▷ *reserved words cannot be used as identifiers*
    - ▷ *listing of reserved words is tedious for the scanner, also makes scanner large*
    - ▷ *solution: treat them as identifiers, and use a table to check whether it is a reserved word.*

Practical considerations – cont.

- **Multi-character lookahead: how many more characters ahead do you have to look in order to decide which pattern to match?**
- **FORTRAN: lookahead until difference is seen without counting blanks.**
  - **DO 10 I = 1, 15 $\equiv$ a loop statement.**
  - **DO 10 I = 1.15 $\equiv$ an assignment statement for the variable DO10I.**
- **PASCAL: lookahead 2 characters with 2 or more blanks treating as one blank.**
  - **10..100: needs to look 2 characters ahead to decide this is not part of a real number.**
- **LEX lookahead operator "/": r_1/r_2: match r_1 only if it is followed by r_2; note that r_2 is not part of the match.**
  - **This operator can be used to cope with multi-character lookahead.**
  - **How is this implemented in LEX?**