Lexical Analyzer — Scanner

ALSU Textbook Chapter 3.1–3.4, 3.6, 3.7, 3.5, 3.8

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Main tasks

- **Read the input characters and produce as output a sequence of tokens to be used by the parser for syntax analysis.**
  - **tokens: terminal symbols in grammar.**
- **Lexeme: a sequence of characters matched by a given pattern associated with a token.**
- **Examples:**
lexemes:	**pi**	**=**	**3.1416**	**;**
tokens:	**ID**	**ASSIGN**	**FLOAT-LIT**	**SEMI-COL**
  - **patterns:**
    - ▷ *identifier (variable name) starts with a letter or "_", and follows by letters, digits or "_";*
    - ▷ *floating point number starts with a string of digits, follows by a dot, and terminates with another string of digits;*

Strings

- **Definitions.**
  - **alphabet**: a finite set of symbols or characters;
  - **string**: a finite sequence of symbols chosen from the alphabet;
  - $|S|$: length of a string S;
  - empty string: ϵ;
- **Operations.**
  - **concatenation** of strings x and y: xy
    - ▷ $\epsilon x \equiv x\epsilon \equiv x$;
  - **exponention**:
    - ▷ $s^0 \equiv \epsilon$;
    - ▷ $s^i \equiv s^{i-1}s$, $i > 0$.

Parts of a string

- **Parts of a string: example string "necessary"**
  - **prefix : deleting zero or more tailing characters;** $\boxed{\text{eg: "nece"}}$
  - **suffix : deleting zero or more leading characters;** $\boxed{\text{eg: "ssary"}}$
  - **substring : deleting prefix and suffix;** $\boxed{\text{eg: "ssa"}}$
  - **subsequence : deleting zero or more not necessarily contiguous symbols;** $\boxed{\text{eg: "ncsay"}}$
  - **proper prefix, suffix, substring or subsequence: one that cannot equal to the original string;**

Language

- **Language**: a set of strings over an alphabet.
- **Operations on languages:**
  - **union:** $L \cup M = \{s | s \in L$ **or** $s \in M\}$;
  - **concatenation:** $LM = \{st | s \in L$ **and** $t \in M\}$;
  - $L^0 = \{\epsilon\}$;
  - $L^1 = L$;
  - $L^i = LL^{i-1}$ **if** $i > 1$;
  - **Kleene closure**: $L^* = \cup_{i=0}^{\infty} L^i$;
  - **Positive closure**: $L^+ = \cup_{i=1}^{\infty} L^i$;
  - $L^* = L^+ \cup \{\epsilon\}$.

Regular expressions

- **A regular expression r denotes a language $L(r)$ which is also called a regular set [Kleene 1956].**
- **Atomic items of regular expressions and operations on them:**

regular expression	language
$\emptyset$	empty set $\{\}$
ϵ	$\{\epsilon\}$ where ϵ is the empty string
a	$\{a\}$ where a is a legal symbol
$r\|s$	$L(r) \cup L(s)$ — union
rs	$L(r)L(s)$ — concatenation
r^*	$L(r)^*$ — Kleene closure

- **Example:**

$$\begin{array}{ll} a|b & \{a, b\} \\ (a|b)(a|b) & \{aa, ab, ba, bb\} \\ a^* & \{\epsilon, a, aa, aaa, \ldots\} \\ a|a^*b & \{a, b, ab, aab, \ldots\} \end{array}$$

Algebraic laws of R.E.

- **Assume r, s and t are arbitrary regular expressions.**

Law	**Description**
$r \mid s = s \mid r$	$\mid$ **(union) is commutative**
$r \mid (s \mid t) = (r \mid s) \mid t$	$\mid$ **is associative**
$r(st) = (rs)t$	**Concatenation is associative**
$r(s \mid t) = rs \mid rt$ $(s \mid t)r = sr \mid tr$	**Concatenation distributes over union**
$\epsilon \mid r = r \mid \epsilon = r$	ϵ **is the identity for union**
$\epsilon r = r\epsilon = r$	ϵ **is the identity for concatenation**
$r^* = (r \mid \epsilon)^*$	ϵ **is guaranteed in a closure**
$r^{**} = r^*$	$*$ **is idempotent**

- **Algebraic structure:**
  - **Without the Kleene closure operation, it is a semi-ring, i.e., a ring without an inverse for union.**
  - **With the Kleene closure operation, it is a Kleene algebra.**

Regular definitions

- **For simplicity, give names to regular expressions and use names later in defining other regular expressions.**
  - **similar to the idea of macros or subroutine calls without parameters**
  - **format:**
    - ▷ ***name*** $\rightarrow$ ***regular expression***
  - **examples:**
    - ▷ ***digit*** $\rightarrow 0 \mid 1 \mid 2 \mid \cdots \mid 9$
    - ▷ ***letter*** $\rightarrow a \mid b \mid c \mid \cdots \mid z \mid A \mid B \mid \cdots \mid Z$
- **Notational standards:**

$\{r\}$	r **is a regular definition**
r^*	$r^+ \mid \epsilon$
r^+	rr^*
$r?$	$r \mid \epsilon$
$[abc]$	$a \mid b \mid c$
$[a-z]$	$a \mid b \mid c \mid \cdots \mid z$

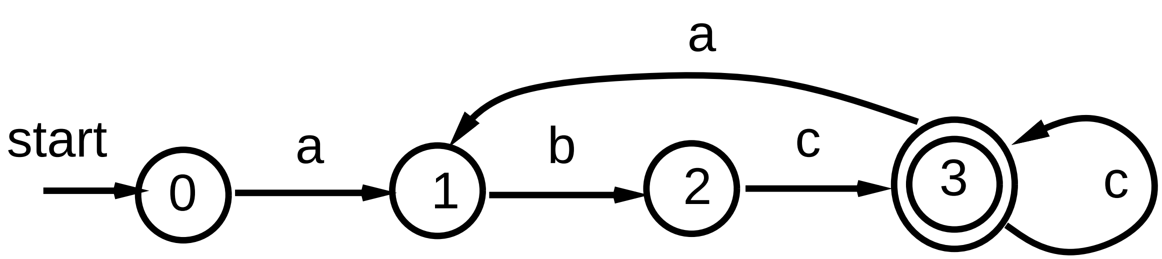
- **Example: C variable name**
  - $[A-Za-z\_][A-Za-z0-9\_]^*$
  - $[$**{letter}**$\_][$**{letter}{digit}**$\_]^*$

Non-regular sets

- **Balanced or nested construct**
  - **Example:**
    if $cond_1$ then if $cond_2$ then $\cdots$ else $\cdots$ else $\cdots$
  - **Can be recognized by context free grammars.**
- **Matching strings:**
  - $\{wcw\}$, **where w is a string of a's and b's and c is a legal symbol.**
  - **Cannot be recognized even using context free grammars.**
- **Remark: anything that needs to "memorize" "non-constant" amount of information happened in the past cannot be recognized by regular expressions.**

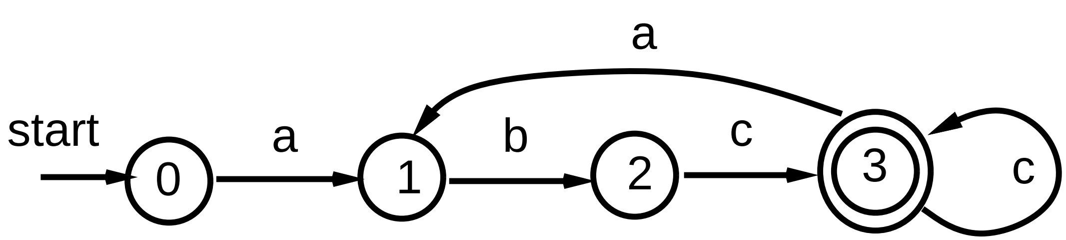
Finite state automata (FA)

- **FA is a mechanism used to recognize tokens specified by a regular expression.**
- **Definition:**
  - **A finite set of states, i.e., vertices.**
  - **A set of transitions, labeled by characters, i.e., labeled directed edges.**
  - **A starting state, i.e., a vertex with an incoming edge marked with "start".**
  - **A set of final (accepting) states, i.e., vertices of concentric circles.**
- **Example: transition graph for the regular expression** $(abc^+)^+$



Transition graph and table for FA

- **Transition graph:**



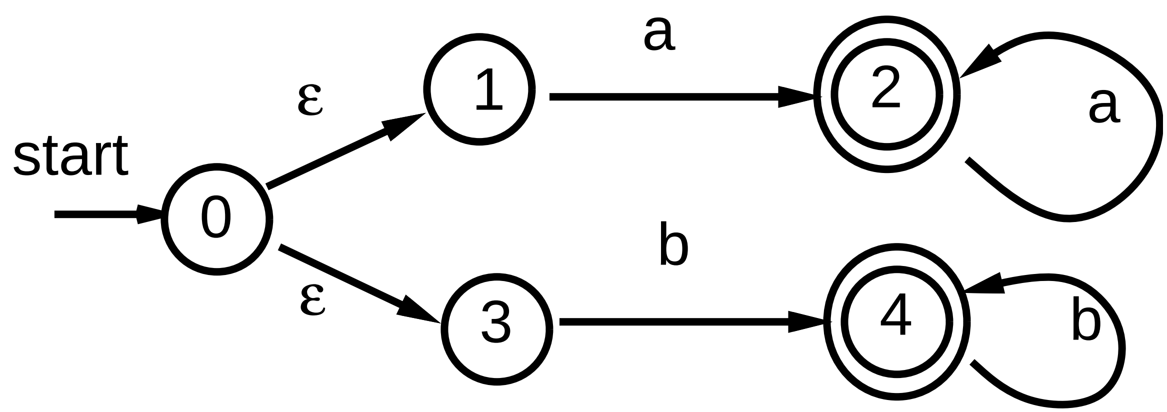
- **Transition table:**

	a	b	c
0	$\{1\}$	$\emptyset$	$\emptyset$
1	$\emptyset$	$\{2\}$	$\emptyset$
2	$\emptyset$	$\emptyset$	$\{3\}$
3	$\{1\}$	$\emptyset$	$\{3\}$

  - Rows are input symbols.
  - Columns are current states.
  - Entries are resulting states.
  - Along with the table, a starting state and a set of accepting states are also given.
- **Transition table is also called a GOTO table.**

Types of FA's

- **Deterministic FA (DFA):**
  - **has a unique next state for a transition**
  - **and does not contain ϵ-transitions, that is, a transition takes ϵ as the input symbol.**
- **Nondeterministic FA (NFA):**
  - **either "could have more than one next state for a transition;"**
  - **or "contains ϵ-transitions."**
  - **Note: can have both of the above two.**
  - **Example: regular expression: $\boxed{aa^*|bb^*}$.**



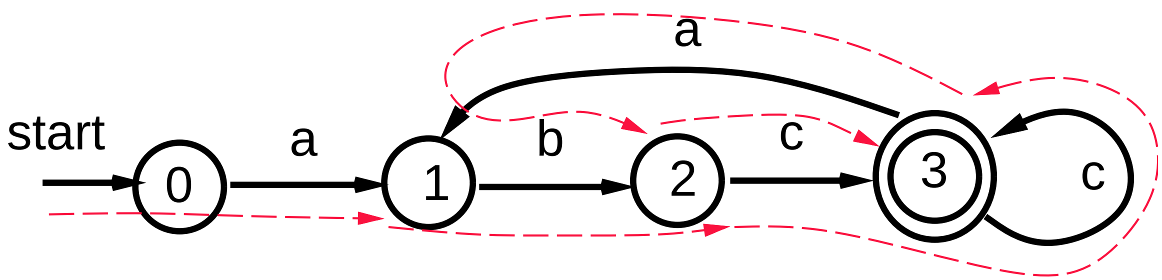
How to execute a DFA

- **Algorithm:**

> $s \leftarrow$ **starting state;**
>
> **while there are inputs and** s **is a legal state do**
>
> $s \leftarrow$ **Table**$[s, input]$
>
> **end while**
>
> **if** $s \in$ **accepting states then ACCEPT else REJECT**

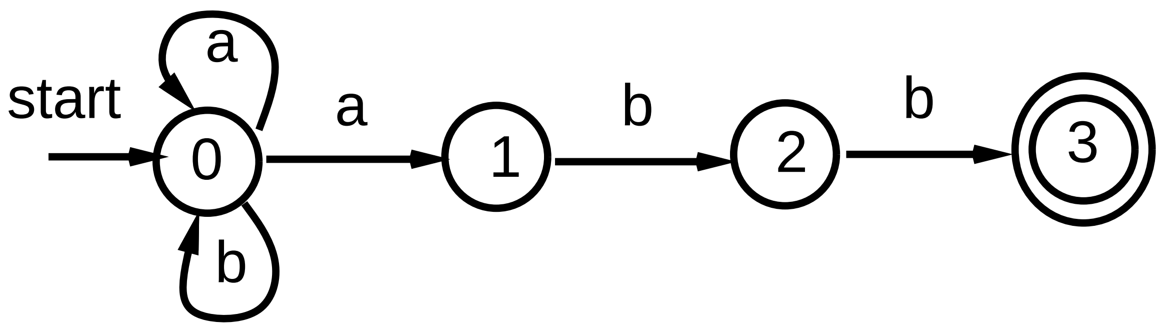
- **Example: input:** $\boxed{\textbf{abccabc}}$**. The accepting path:**

$$0 \xrightarrow{a} 1 \xrightarrow{b} 2 \xrightarrow{c} 3 \xrightarrow{c} 3 \xrightarrow{a} 1 \xrightarrow{b} 2 \xrightarrow{c} 3$$



How to execute an NFA (informally) (1/2)

- **An NFA accepts an input string x if and only if there is some path in the transition graph initiating from the starting state to some accepting state such that the edge labels along the path spell out x.**
- **Could have more than one path. (Note DFA has at most one.)**
- **Example: regular expression: $\boxed{(a|b)^*abb}$; input: $\boxed{aabb}$.**



How to execute an NFA (informally) (2/2)

- **Goto table:**

	a	b
0	**{0,1}**	**{0}**
1	$\emptyset$	**{2}**
2	$\emptyset$	**{3}**

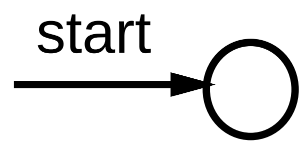
- **Two possible traces.**

$$0 \xrightarrow{a} 0 \xrightarrow{a} 1 \xrightarrow{b} 2 \xrightarrow{b} 3 \textbf{ Accept!}$$

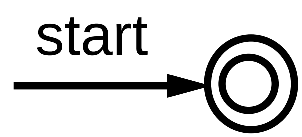
$$0 \xrightarrow{a} 0 \xrightarrow{a} 0 \xrightarrow{b} 0 \xrightarrow{b} 0 \textbf{ Reject!}$$

From regular expressions to NFA's (1/3)

- **Structural decomposition:**
  - **atomic items:**
    - ▷ $\emptyset$

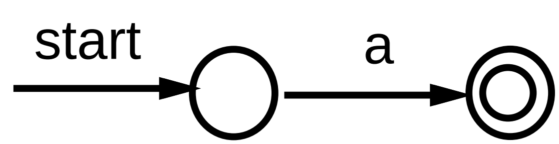


    - ▷ ϵ



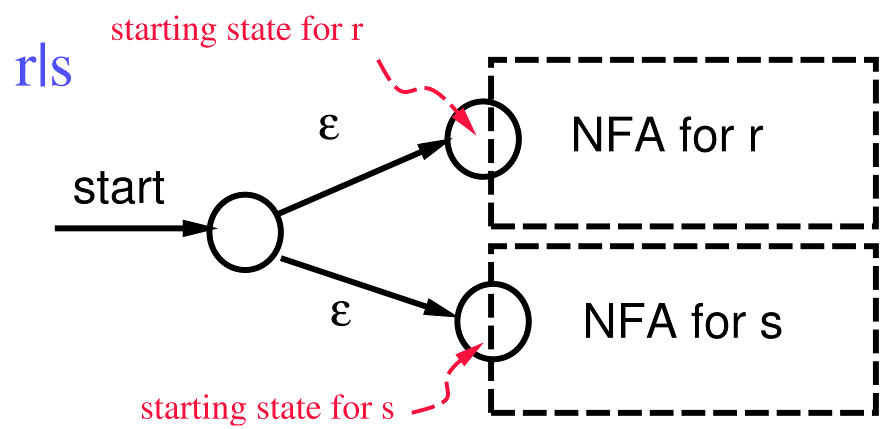
    - ▷ ***a legal symbol***

a

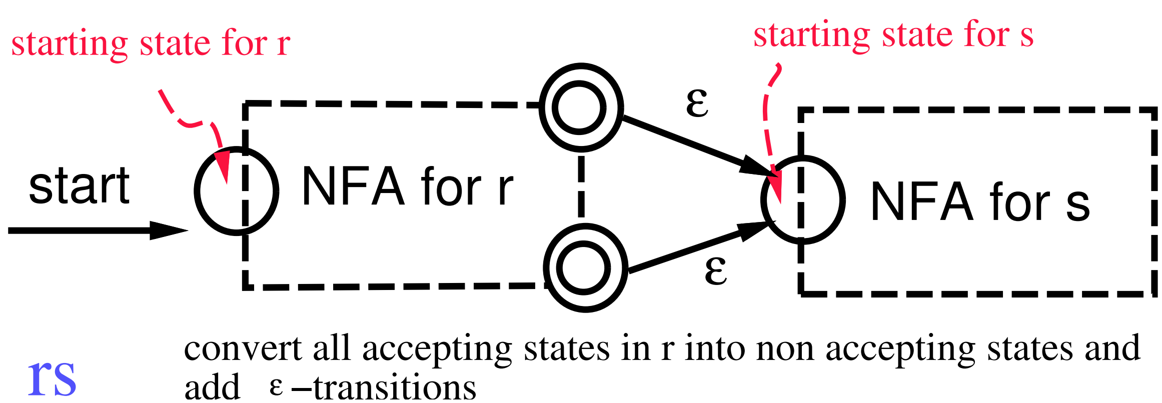


From regular expressions to NFA's (2/3)

- **union**

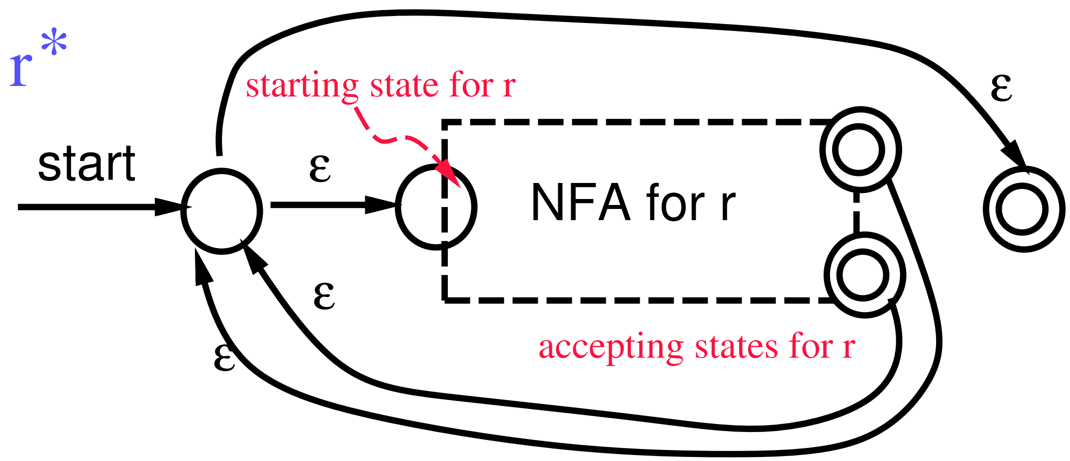


- **concentration**

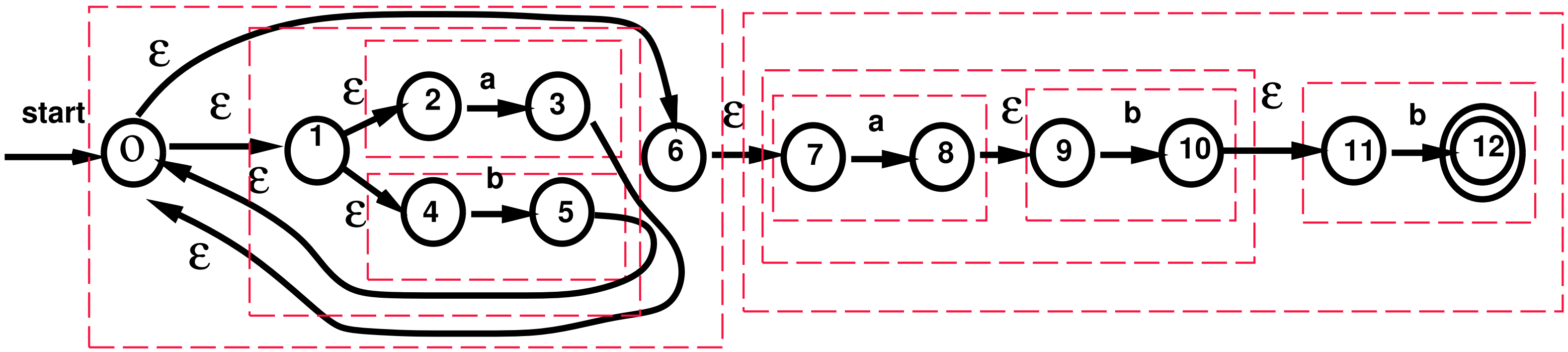


From regular expressions to NFA's (3/3)

- **Kleene closure**



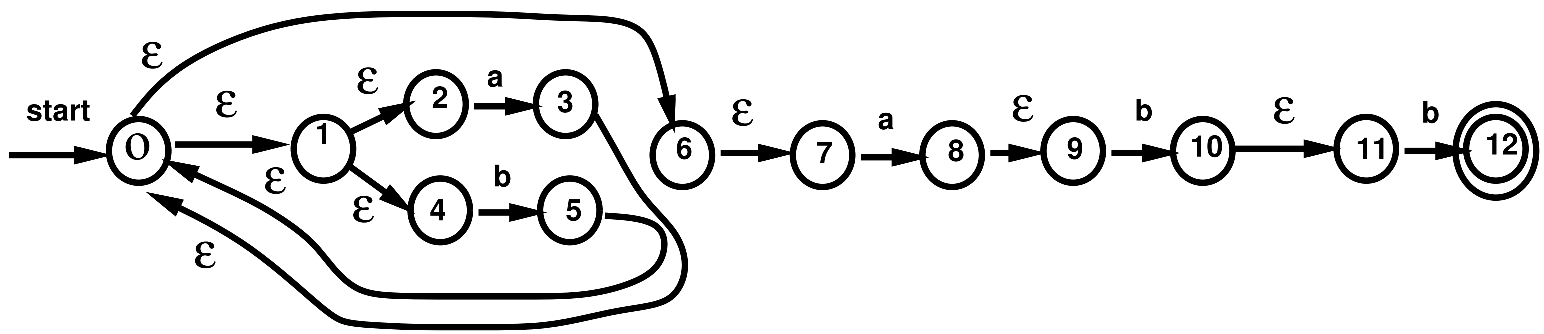
Example: $(a|b)^*((ab)b)$



- **This construction produces only ϵ-transitions, and never produce multiple transitions for an input symbol.**
- **It is possible to remove all ϵ-transitions from an NFA and replace them with multiple transitions for an input symbol, and vice versa.**
- **Theorem [Thompson 1969]:**
  - **Any regular expression can be expressed by an NFA.**

Converting an NFA to a DFA

- **Definitions: let T be a set of states and a be an input symbol.**
  - **ϵ-closure(T): the set of NFA states reachable from some state $s \in T$ using ϵ-transitions.**
  - **$move(T, a)$: the set of NFA states to which there is a transition on the input symbol a from state $s \in T$.**
  - **Both can be computed using standard graph algorithms.**
  - **ϵ-closure$(move(T, a))$: the set of states reachable from a state in T for the input a.**
- **Example: NFA for $(a|b)^*((ab)b)$**



  - **ϵ-closure$(\{0\}) = \{0, 1, 2, 4, 6, 7\}$, that is, the set of all possible starting states**
  - $move(\{2, 7\}, a) = \{3, 8\}$

Subset construction algorithm

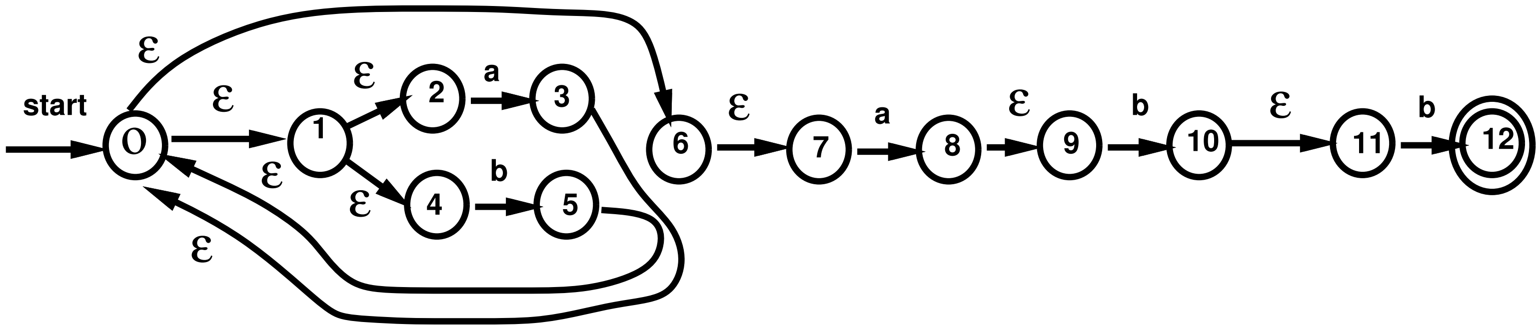
- **In the converted DFA, each state represents a subset of NFA states.**
  - $T \xrightarrow{a} \epsilon$-**closure**$(move(T, a))$
- **Subset construction algorithm: [Rabin & Scott 1959]**

initially, we have an unmarked state labeled with ϵ-closure$(\{s_0\})$, where s_0 is the starting state.

> **while there is an unmarked state with the label T do**
>
> ▷ *mark the state with the label T*
>
> ▷ *for each input symbol a do*
>
> ▷ $\quad$ $U \leftarrow \epsilon$-*closure*$(move(T, a))$
>
> ▷ $\quad$ *if U is a subset of states that is never seen before*
>
> ▷ $\quad$ *then add an unmarked state with the label U*
>
> ▷ *end for*
>
> **end while**

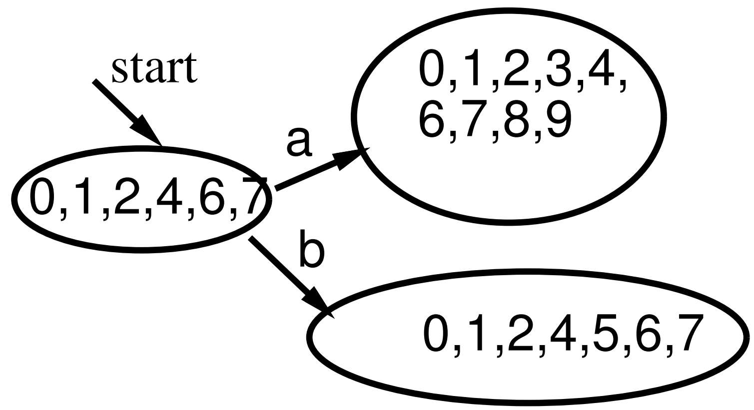
- **New accepting states: those contain an original accepting state.**

Example (1/2)

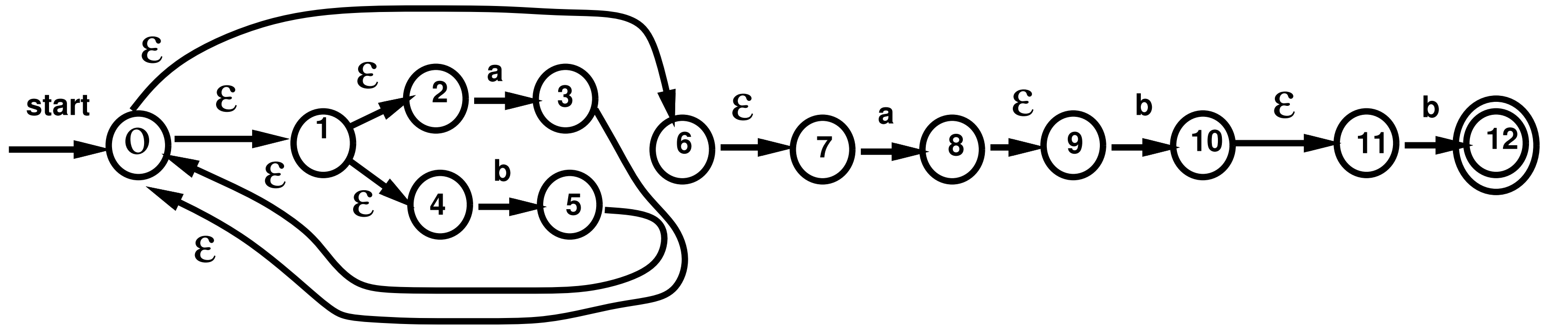


First step:

- ϵ-**closure({0}) = {0,1,2,4,6,7}**
- $move(\{0,1,2,4,6,7\},a)$ = **{3,8}**
- ϵ-**closure({3,8}) = {0,1,2,3,4,6,7,8,9}**
- $move(\{0,1,2,4,6,7\},b)$ = **{5}**
- ϵ-**closure({5}) = {0,1,2,4,5,6,7}**



Example (2/2)

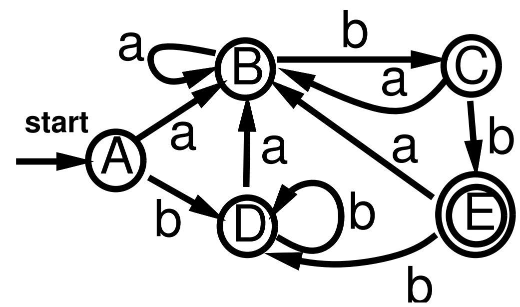


transition table:

states:

- $A = \{0, 1, 2, 4, 6, 7\}$
- $B = \{0, 1, 2, 3, 4, 6, 7, 8, 9\}$
- $C = \{0, 1, 2, 4, 5, 6, 7, 10, 11\}$
- $D = \{0, 1, 2, 4, 5, 6, 7\}$
- $E = \{0, 1, 2, 4, 5, 6, 7, 12\}$

	a	b
A	B	D
B	B	C
C	B	E
D	B	D
E	B	D



Construction theorems (I)

- **Facts:**
  - **Lemma [Thompson 1968]:**
    - ▷ *Any regular expression can be expressed by an NFA.*
  - **Lemma [Rabin & Scott 1959]**
    - ▷ *Any NFA can be converted into a DFA.*
    - ▷ *By using the Subset Construction Algorithm.*
- **Conclusion:**
  - **Theorem: Any regular expression can be expressed by a DFA.**
- **Note: It is possible to convert a regular expression directly into a DFA [McNaughton & Yamada 1960].**

Construction theorems (II)

- **Facts:**
  - **Theorem [previous slide]: Any regular expression can be expressed by a DFA.**
  - **Lemma [Brzozowski & McCluskey 1963]: Every DFA can be expressed as a regular expression.**
    - ▷ *Define extended FA that has labels of regular expressions on the edges.*
    - ▷ *Repeatly merge states.*
- **Conclusion:**
- **Theorem: DFA and regular expression have the same expressive power.**
- **Q: How about the power of DFA and NFA?**

Algorithm for executing an NFA

- **Algorithm:** s_0 **is the starting state,** F **is the set of accepting states.**

$S \leftarrow \epsilon$-**closure**$(\{s_0\})$

while next input a **is not EOF do**

▷ $S \leftarrow \epsilon$-***closure***$(move(S, a))$

end while

if $S \cap F \neq \emptyset$ **then ACCEPT else REJECT**

- **Execution time is** $O(r^2 \cdot s)$**, where**
  - ▷ *r is the number of NFA states, and s is the length of the input.*
  - ▷ *Need $O(r^2)$ time in running ϵ-closure(T) assuming using an adjacency matrix representation and a constant-time hashing routine with linear-time preprocessing to remove duplicated states.*
- **Space complexity is** $O(r^2 \cdot c)$ **using a standard adjacency matrix representation for graphs, where** c **is the cardinality of the alphabet.**

- **Have better algorithms by using compact data structures and techniques.**

Trade-off in executing NFA's

- **Can also convert an NFA to a DFA and then execute the equivalent DFA.**
  - **Running time: linear in the input size.**
  - **Space requirement: linear in the size of the DFA.**
- **Catch:**
  - **May get $O(2^r)$ DFA states by converting an r-state NFA.**
  - **The converting algorithm may also take $O(2^r \cdot c)$ time in the worst case.**

  ▷ ***For typical cases, the execution time is $O(r^3)$.***

- **Time-space tradeoff:**

	space	time
NFA	$O(r^2 \cdot c)$	$O(r^2 \cdot s)$
DFA	$O(2^r \cdot c)$	$O(s)$

  - **If memory is cheap or programs will be used many times, then use the DFA approach;**
  - **otherwise, use the NFA approach.**

LEX

- **An UNIX utility [Lesk 1975].**
  - **It has been ported to lots of OS's and platforms.**
    - ▷ ***Flex (GNU version), and JFlex and JLex (Java versions).***
- **An easy way to use regular expressions to specify "patterns".**
- **Convert your LEX program into an equivalent C program.**
- **Depending on implementation, may use NFA or DFA algorithms.**
- **file.l $\longrightarrow$ $\boxed{\text{lex file.l}}$ $\longrightarrow$ lex.yy.c**
- **lex.yy.c $\longrightarrow$ $\boxed{\text{cc -ll lex.yy.c}}$ $\longrightarrow$ a.out**
  - **May produce .o file if there is no main().**
- **input $\longrightarrow$ $\boxed{\text{a.out}}$ $\longrightarrow$ output a sequence of tokens**
- **May have slightly different implementations and libraries.**

LEX formats (1/2)

- **Source format:**
  - Declarations —- a set of regular definitions, i.e., names and their regular expressions.
  - %%
  - Translation rules — actions to be taken when patterns are encountered.
  - %%
  - Auxiliary procedures
- **Built-in global variables:**
  - $yytext$: current matched string
  - $yyleng$: length of the current matched string
  - ...
- **Built-in service routines:**
  - $yylex()$: the scanner routine
    - ▷ *returns the value 0 when EOF is encountered*
  - $yywrap()$: called when EOF is encountered
  - $yyerror()$: called when there is an error
  - ...

LEX formats (2/2)

- **Declarations:**
  - **C language code between %{ and %}.**
    - ▷ ***variables;***
    - ▷ ***manifest constants, i.e., identifiers declared to represent constants.***
  - **Regular expressions.**
- **Translation rules:**

  P_1 **{action$_1$}**

  if regular expression P_1 is encountered, then action$_1$ is performed.
- **LEX internals:**
  - **regular expressions $\longrightarrow$ NFA $\xrightarrow{\text{if needed}}$ DFA**
  - **regular expressions $\xrightarrow{\text{directly}}$ DFA**

test.l — Declarations

```
%{
    /* some initial C programs */
#define START_OF_SYMBOLS 1
// 0 is reserved for EOF
#define BEGINSYM 1
#define INTEGER  2
#define IDNAME   3
#define REAL     4
#define STRING   5
#define SEMICOLONSYM 6
#define ASSIGNSYM 7
#define END_OF_SYMBOLS 7
%}
Digit        [0-9]
Letter       [a-zA-Z]
IntLit       {Digit}+
Id           {Letter}({Letter}|{Digit}|_)*
```

test.l — Rules

```
%%
[ \t\n] {/* skip white spaces */}
[Bb][Ee][Gg][Ii][Nn]                  {return(BEGINSYM);}
{IntLit}                              {return(INTEGER);}
{Id}                {
                     printf("var has %d characters, ",yyleng);
                     return(IDNAME);
                    }
({IntLit}[.]{IntLit})([Ee][+-]?{IntLit})? {return(REAL);}
\"[^\"\n]*\"   {stripquotes(); return(STRING);}
";"                                   {return(SEMICOLONSYM);}
":="                                  {return(ASSIGNSYM);}
.          {printf("error --- %s\n",yytext);}
```

test.l — Procedures

```
%%
/* some final C programs */
stripquotes()
{
  /* handling string within a quoted string */
  int frompos, topos=0, numquotes = 2;
  for(frompos=1; frompos<yyleng; frompos++){
    yytext[topos++] = yytext[frompos];
  }
  yyleng -= numquotes;
  yytext[yyleng] = '\0';
}
void main(){
  int i;
  i =  yylex();
  while(i>=START_OF_SYMBOLS && i <= END_OF_SYMBOLS){
    printf("<%s> is %d\n",yytext,i);
    i =  yylex();     }   }
```

Sample run

```
austin% lex test.l
austin% cc lex.yy.c -ll
austin% cat data
Begin
123.3  321.4E21
x := 365;
"this is a string"
austin% a.out < data
<Begin> is 1
<123.3> is 4
<321.4E21> is 4
var has 1 characters, <x> is 3
<:=> is 7
<365> is 2
<;> is 6
<this is a string> is 5
%austin
```

More LEX formats

- **Special format requirement:**

P_1

```
  { action_1
  ...
  }
```

Note: { and } must indent.

- **LEX special characters (operators):**

`‘‘ \ [ ] ^ - ? . * + | ( ) $ { } % < >`

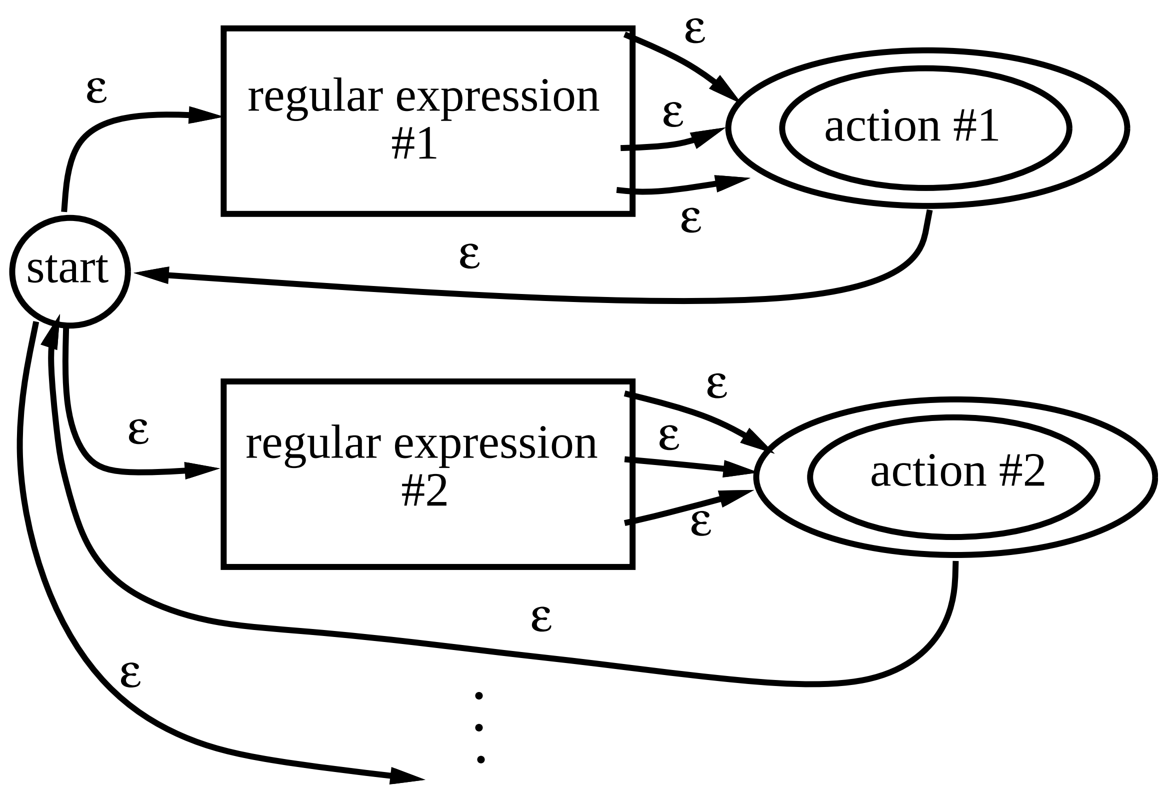
  - **watch out for precedence and associative rules of these operators.**

LEX and regular expressions

- **LEX assumes input is a stream of strings, not just one string.**
  - **How to know it is the end of a lexeme?**
- **LEX allows the specification of multiple regular expressions.**
  - **Assume you have regular expressions R_1 and R_2.**
  - **Assume $L(R_i)$ is the language, i.e., set of strings, defined by R_i.**
  - **Potential problems or ambiguities:**
    - ▷ $L(R_1) \cap L(R_2) \neq \emptyset$.
    - ▷ $\exists s_1 \in L(R_1)$ ***such that*** s_1 ***is a proper prefix of a string*** s_2 ***and*** $s_2 \in L(R_2)$.
- **LEX allows “conditional matches”.**
  - **Lookahead symbols.**
  - **Accept a string only if it is followed by another string.**

LEX internals

- **LEX code:**
  - **regular expression #1 {action #1}**
  - **regular expression #2 {action #2}**
  - $\cdots$



Ambiguity in matching (1/2)

- **Definitions:**
  - **for a given prefix of the input output "accept" for more than one pattern;**
    - ▷ *that is, the languages defined by two patterns have some intersection.*
  - **output "accept" for two different prefixes.**
    - ▷ *An element in a language is a proper prefix of another element in a different language.*
- **When there is any ambiguity in matching, prefer**
  - **longest possible match;**
  - **earlier expression if more than one longest match.**
- **White space is needed only when there is a chance of ambiguity.**

Ambiguity in matching (2/2)

- How to find a longest possible match if there are many legal matches?
  - If an accepting state is encountered, do not immediately accept.
  - Push this accepting state and the current input position into a stack and keep on going until no more matches is possible.
  - Pop from the stack and execute the actions for the popped accepting state.
  - Resume the scanning from the popped current input position.
- How to find the earliest match if there are more than one longest match?
  - Assign numbers $1, 2, \ldots$ to the accepting states using the order they appear (from top to bottom) in the expressions.
  - If you are in multiple accepting states, execute the action associated with the least indexed accepting state.
- What does $yylex()$ do?
  - Find the longest possible prefix from the current input stream that can be accepted by "the regular expression" defined.
  - Extract this matched prefix from the input stream and assign its token meaning according to rules discussed.

Lookahead symbols

- Multi-character lookahead: how many more characters ahead do you have to look in order to decide which pattern to match?
  - Extensions to regular expression when there are ambiguity in matching.
- FORTRAN: lookahead until difference is seen without counting blanks.
  - DO 10 I = 1, 15 $\equiv$ a loop statement.
  - DO 10 I = 1.15 $\equiv$ an assignment statement for the variable DO10I.
- PASCAL: lookahead 2 characters with 2 or more blanks treating as one blank.
  - 10..100: needs to look 2 characters ahead to decide this is not part of a real number.
- LEX lookahead operator "/": r_1/r_2: match r_1 only if it is followed by r_2; note that r_2 is not part of the match.
  - This operator can be used to cope with multi-character lookahead.
  - How is it implemented in LEX?

Practical consideration

- **key word** v.s. **reserved word**
  - **key word:**
    - ▷ *def: word has a well-defined meaning in a certain context.*
    - ▷ *example: FORTRAN, PL/1, . . .*

if	*if*	*then*	*else*	=	*then*	;
	id		*id*		*id*	

    - ▷ *Makes compiler to work harder!*
  - **reserved word:**
    - ▷ *def: regardless of context, word cannot be used for other purposes.*
    - ▷ *example: COBOL, ALGOL, PASCAL, C, ADA, . . .*
    - ▷ *task of compiler is simpler*
    - ▷ *reserved words cannot be used as identifiers*
    - ▷ *listing of reserved words is tedious for the scanner, also makes the scanner larger*
    - ▷ *solution: treat them as identifiers, and use a table to check whether it is a reserved word.*