Theory of Computer Games: Concluding Remarks

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Abstract

- **Introducing practical issues.**
  - **The open book.**
  - **The graph history interaction (GHI) problem.**
  - **Smart usage of resources.**
    - ▷ *time during searching*
    - ▷ *memory*
    - ▷ *coding efforts*
    - ▷ *debugging efforts*
  - **Opponent models**
- **How to combine what we have learned in class together to get a working game program.**
- **How to test your program?**

The open book (1/2)

- **During the open game, it is frequently the case**
  - **branching factor is huge;**
  - **it is difficult to write a good evaluating function;**
  - **the number of possible distinct positions up to a limited length is small as compared to the number of possible positions encountered during middle game search.**
- **Acquire game logs from**
  - **books;**
  - **games between masters;**
  - **games between computers;**
    - ▷ ***Use off-line computation to find out the value of a position for a given depth that cannot be computed online during a game due to resource constraints.***
  - · · ·

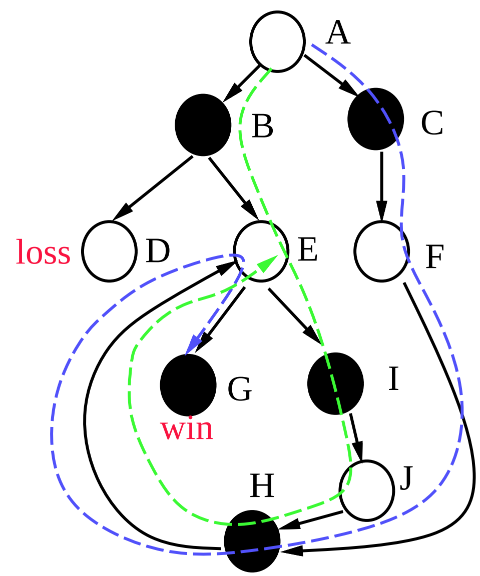
The open book (2/2)

- Assume you have collected r games.
  - For each position in the r games, compute the following 3 values:
    - ▷ win: *the number of games reaching this position and then wins.*
    - ▷ $loss$: *the number of games reaching this position and then loss.*
    - ▷ $draw$: *the number of games reaching this position and then draw.*
- When r is large and the games are trustful, then use the 3 values to compute an estimated goodness for this position.
- Comments:
  - Pure statistically.
  - Can build a static open book.
  - You program may not be able to take over when the open book is over.
  - It is difficult to acquire large amount of "trustful" game logs.
  - Automatically analysis of game logs written by human experts. [Chen et. al. 2006]
  - Using high-level meta-knowledge to guide the way in searching:
    - ▷ *Dark chess: adjacent attack of the opponent's Cannon. [Chen and Hsu 2013]*

Graph history interaction problem

- **The graph history interaction (GHI) problem [Campbell 1985]:**
  - **In a game graph, a position can be visited by more than one paths.**
  - **The value of the position depends on the path visiting it.**
    - *It can be win. loss or draw for Chinese chess.*
    - *It can only be draw for Western chess.*
    - *It can only be loss for Go.*
- **In the transposition table, you record the value of a position, but not the path leading to it.**
  - **Values computed from rules on repetition cannot be used later on.**
  - **It takes a huge amount of storage to store all the paths visiting it.**
- **This is a very difficult problem to be solved in real time [Wu et al. '05].**

GHI problem – example



- $A \to B \to E \to I \to J \to H \to E$ **is loss because of rules of repetition.**
  - ▷ ***Memorized*** H ***as a loss position.***
- $A \to B \to D$ **is a loss.**
- $A \to C \to F \to H$ **is loss because** H **is recorded as loss.**
- A **is loss because both branches lead to loss.**
- **However,** $A \to C \to F \to H \to E \to G$ **is a win.**

Using resources

- **Time [Hyatt 1984] [Šolak and Vučković 2009]**
  - **For human:**
    - *More time is spent in the beginning when the game just starts.*
    - *Stop searching a path further when you think the position is stable.*
  - **Pondering:**
    - *Use the time when your opponent is thinking.*
    - *Guessing and then pondering.*
- **Memory**
  - **Using a large transposition table occupies a large space and thus slows down the program.**
    - *A large number of positions are not visited too often.*
  - **Using no transposition table makes you to search a position more than once.**
- **Other resources.**

Opponent models

- **In a normal alpha-beta search, it is assumed that you and the opponent use the same strategy.**
  - **What is good to you is bad to the opponent and vice versa!**
  - **Hence we can reduce a minimax search to a NegaMax search.**
  - **This is normally true when the game ends, but may not be true in the middle of the game.**
- **What will happen when there are two strategies or evaluating functions f_1 and f_2 so that**
  - **for some positions p, $f_1(p)$ is better than $f_2(p)$**
    - ▷ ***"better" means closer to the real value $f(p)$***
  - **for some positions q, $f_2(q)$ is better than $f_1(q)$**
- **If you are using f_1 and you know your opponent is using f_2, what can be done to take advantage of this information.**
  - **This is called OM (opponent model) search [Carmel and Markovitch 1996].**
    - ▷ ***In a MAX node, use f_1.***
    - ▷ ***In a MIN node, use f_2***

Opponent models – comments

- **Comments:**
  - **Need to know your opponent model precisely.**
  - **How to learn the opponent on-line or off-line?**
  - **When there are more than 2 possible opponent strategies, use a probability model (PrOM search) to form a strategy.**

Search with chance nodes

- **Chinese dark chess**
  - **Two player, zero sum, complete information**
  - **Perfect information**
  - **Stochastic**
  - **There is a chance node during searching [Ballard 1983].**
    - ▷ ***The value of a node is a distribution, not a fixed value.***
- **Previous work**
  - **Alpha-beta based [Ballard 1983]**
  - **Monte-Carlo based [Lancoto et al 2013]**

Basic ideas for searching chance nodes

- **Assume a chance node x has a score probability distribution function $Pr(*)$ with the range of possible outcomes from 1 to N where N is a positive integer.**
  - **For each possible outcome i, there is a $score(i)$ to be computed.**
  - **The expected value $E = \sum_{i=1}^{N} score(i) * Pr(x = i)$.**
  - **The minimum value is $m = \min_{i=1}^{N}\{score(i) \mid Pr(x = i) > 0\}$.**
  - **The maximum value is $M = \max_{i=1}^{N}\{score(i) \mid Pr(x = i) > 0\}$.**
- **Example: in Chinese dark chess.**
  - **For the first ply, $N = 14 * 32$.**
    - ▷ ***Using symmetry, we can reduce it to 7*8.***
  - **We now consider the chance node of flipping the piece at the cell a1.**
    - ▷ $N = 14$.
    - ▷ ***Assume $x = 1$ means a black King is revealed and $x = 8$ means a red King is revealed.***
    - ▷ ***Then*** $score(1) = score(8)$.
    - ▷ $Pr(x = 1) = Pr(x = 8) = 1/14$.

Bounds in a chance node

- **Assume the various possibilities of a chance node is evaluated one by one in the order that at the end of phase i, $i = N$ is evaluated.**
  - **Assume $v_{min} \leq score(i) \leq v_{max}$.**
- **How do the lower and upper bounds, namely m_i and M_i, of the chance node change at the end of phase i?**
  - $i = 0$.
    - ▷ $m_0 = v_{min}$
    - ▷ $M_0 = v_{max}$
  - $i = 1$, **we first compute $score(1)$, and then know**
    - ▷ $m_1 \geq score(1) * Pr(x = 1) + v_{min} * (1 - Pr(x = 1))$, *and*
    - ▷ $M_1 \leq score(1) * Pr(x = 1) + v_{max} * (1 - Pr(x = 1))$.
  - $\cdots$
  - $i = i^*$, **we have computed $score(1), \ldots, score(i^*)$, and then know**
    - ▷ $m_{i^*} \geq \sum_{i=1}^{i^*} score(i) * Pr(x = i) + v_{min} * (1 - \sum_{i=1}^{i^*} Pr(x = i))$, *and*
    - ▷ $M_{i^*} \leq \sum_{i=1}^{i^*} score(i) * Pr(x = i) + v_{max} * (1 - \sum_{i=1}^{i^*} Pr(x = i))$.

Example: Chinese dark chess

- **Assumption:**
  - **The range of the scores of Chinese dark chess is [-10,10] inclusive.**
  - $N = 7$.
  - $Pr(x = i) = 1/N = 1/7$.
- **Calculation:**
  - $i = 0$,
    - ▷ $m_0 = -10$.
    - ▷ $M_0 = 10$.
  - $i = 1$ **and if** $score(1) = -2$, **then**
    - ▷ $m_1 = -2 * 1/7 + -10 * 6/7 = -62/7 \simeq -8.86$.
    - ▷ $M_1 = -2 * 1/7 + 10 * 6/7 = 58/7 \simeq 8.26$.
  - $i = 1$ **and if** $score(1) = 3$, **then**
    - ▷ $m_1 = 3 * 1/7 + -10 * 6/7 = -57/7 \simeq -8.14$.
    - ▷ $M_1 = 3 * 1/7 + 10 * 6/7 = 63/7 = 9$.

How to use these bounds

- **The lower and upper bounds of the expected score can be used to do alpha-beta pruning.**
  - **Nicely fit into the alpha-beta search algorithm.**
- **Can do better by not searching the DFS order.**
  - **It is not necessary to finish search completely for the subtree of $x = 1$, and then start to look at the subtree of $x = 2$.**
  - **Assume it is a MAX chance node, e.g., the opponent takes a flip.**
    - ▷ ***Knowing some value v'_1 of a subtree for $x = 1$ gives an upper bound, i.e., $score(1) \geq v'_1$.***
    - ▷ ***Knowing some value v'_2 of a subtree for $x = 2$ gives an upper bound, i.e., $score(2) \geq v'_2$.***
    - ▷ ***These bounds can be used to make the search window further narrower.***
- **For Monte-Carlo based algorithm, we need to use a sparse sampling algorithm to efficiently estimate the expected value of a chance node [Kearn et al 2002].**

Putting everything together

- **Game playing system**
  - **Use some sorts of open book.**
  - **Middle-game searching: usage of a search engine.**
    - *Main search algorithm*
    - *Enhancements*
    - *Evaluating function: knowledge*
  - **Use some sorts of endgame databases.**
- **Debugging and testing**

Testing

- You have two versions P_1 and P_2.
- You make the 2 programs play against each other using the same resource constraints.
- To make it fair, during a round of testing, the numbers of a program plays first and second are equal.
- After a few rounds of testing, how do you know P_1 is better or worse than P_2?

How to know you are successful

- **Assume during a self-play experiment, two copies of the same program are playing against each other.**
  - **Since two copies of the same program are playing against each other, the outcome of each game is an independent random trial and can be modeled as a trinomial random variable.**
  - **Assume for a copy playing first,**

$$Pr(game_{first}) = \begin{cases} p & \textbf{if win} \\ q & \textbf{if draw} \\ 1-p-q & \textbf{if lose} \end{cases}$$

  - **Hence for a copy playing second,**

$$Pr(game_{last}) = \begin{cases} 1-p-q & \textbf{if win} \\ q & \textbf{if draw} \\ p & \textbf{if lose} \end{cases}$$

Outcome of self-play games

- **Assume $2n$ games, $g_1, g_2, \ldots, g_{2n}$ are played.**
  - **In order to offset the initiative, namely first player's advantage, each copy plays first for n games.**
  - **We also assume each copy alternatives in playing first.**
  - **Let g_{2i-1} and g_{2i} be the ith pair of games.**
- **Let the outcome of the ith pair of games be a random variable X_i from the prospective of the copy who plays g_{2i-1}.**
  - **Assume we assign a score of x for a game won, a score of 0 for a game drawn and a score of $-x$ for a game lost.**
- **The outcome of X_i and its occurrence probability is thus**

$$Pr(X_i) = \begin{cases} p(1-p-q) & \textbf{if } X_i = 2x \\ pq + (1-p-q)q & \textbf{if } X_i = x \\ p^2 + (1-p-q)^2 + q^2 & \textbf{if } X_i = 0 \\ pq + (1-p-q)q & \textbf{if } X_i = -x \\ (1-p-q)p & \textbf{if } X_i = -2x \end{cases}$$

How good we are against the baseline?

- **Properties of X_i.**
  - **The mean $E(X_i) = 0$.**
  - **The standard deviation of X_i is**

$$\sqrt{E(X_i^2)} = x\sqrt{2pq + (2q + 8p)(1 - p - q)},$$

  and it is a multi-nominally distributed random variable.
- **When you have played n pairs of games, what is the probability of getting a score of s, $s > 0$?**
  - **Let $X[n] = \sum_{i=1}^{n} X_i$.**
    - ▷ ***The mean of $X[n]$, $E(X[n])$, is 0.***
    - ▷ ***The standard deviation of $X[n]$, σ_n, is $x\sqrt{n}\sqrt{2pq + (2q + 8p)(1 - p - q)}$,***
  - **If $s > 0$, we can calculate the probability of $Pr(|X[n]| \leq s)$ using well known techniques from calculating multi-nominal distributions.**

Practical setup

- **Parameters that are usually used.**
  - $x = 1$.
  - **For Chinese chess,** q **is about** 0.3161**,** $p = 0.3918$ **and** $1 - p - q$ **is** 0.2920**.**
    - ▷ ***Data source: 63,548 games played among masters recorded at www.dpxq.com.***
    - ▷ ***This means the first player has a better chance of winning.***
  - **The mean of** $X[n]$**,** $E(X[n])$**, is** 0**.**
  - **The standard deviation of** $X[n]$**,** σ_n**, is**

$$x\sqrt{n}\sqrt{2pq + (2q + 8p)(1 - p - q)} = \sqrt{1.16n}.$$

Results (1/3)

$Pr(\|X[n]\| \leq s)$	$s = 0$	$s = 1$	$s = 2$	$s = 3$	$s = 4$	$s = 5$	$s = 6$
$n = 10,\ \sigma_{10} = 3.67$	0.108	0.315	0.502	0.658	0.779	0.866	0.924
$n = 20,\ \sigma_{20} = 5.19$	0.076	0.227	0.369	0.499	0.613	0.710	0.789
$n = 30,\ \sigma_{30} = 6.36$	0.063	0.186	0.305	0.417	0.520	0.612	0.693
$n = 40,\ \sigma_{40} = 7.34$	0.054	0.162	0.266	0.366	0.460	0.546	0.624
$n = 50,\ \sigma_{50} = 8.21$	0.049	0.145	0.239	0.330	0.416	0.497	0.571

Results (2/3)

$Pr(\|X[n]\| \leq s)$	$s = 7$	$s = 8$	$s = 9$	$s = 10$	$s = 11$	$s = 12$	$s = 13$
$n = 10,\ \sigma_{10} = 3.67$	0.960	0.981	0.991	0.997	0.999	1.000	1.000
$n = 20,\ \sigma_{20} = 5.19$	0.851	0.899	0.933	0.958	0.974	0.985	0.991
$n = 30,\ \sigma_{30} = 6.36$	0.761	0.819	0.865	0.902	0.930	0.951	0.967
$n = 40,\ \sigma_{40} = 7.34$	0.693	0.753	0.804	0.847	0.883	0.912	0.934
$n = 50,\ \sigma_{50} = 8.21$	0.639	0.699	0.753	0.799	0.839	0.872	0.900

Results (3/3)

$Pr(\|X[n]\| \leq s)$	$s = 14$	$s = 15$	$s = 16$	$s = 17$	$s = 18$	$s = 19$	$s = 20$
$n = 10,\ \sigma_{10} = 3.67$	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$n = 20,\ \sigma_{20} = 5.19$	0.995	0.997	0.999	0.999	1.000	1.000	1.000
$n = 30,\ \sigma_{30} = 6.36$	0.978	0.986	0.991	0.994	0.997	0.998	0.999
$n = 40,\ \sigma_{40} = 7.34$	0.952	0.966	0.976	0.983	0.989	0.992	0.995
$n = 50,\ \sigma_{50} = 8.21$	0.923	0.941	0.956	0.967	0.976	0.983	0.988

Statistical behaviors

- **Hence assume you have two programs that are playing against each other and have obtained a score of $s+1$, $s>0$, after trying n pairs of games.**
  - **Assume $Pr(|X[n]| \leq s)$ is say 0.95.**
    - ▷ ***Then this result is meaningful, that is a program is better than the other, because it only happens with a low probability of 0.05.***
  - **Assume $Pr(|X[n]| \leq s)$ is say 0.05.**
    - ▷ ***Then this result is not very meaningful, because it happens with a high probability of 0.95.***
- **In general, it is a very rare case, e.g., less than 5% of chance that it will happen, that your score is more than $2\sigma_n$.**
  - **For our setting, if you perform n pairs of games, and your net score is more than $2 * \sqrt{1.16} * \sqrt{n} \simeq 2.154\sqrt{n}$, then it means something statistically.**
- **You can also decide your "definition" of "a rare case".**

Concluding remarks

- **Consider your purpose of studying a game:**
  - **It is good to solve a game completely.**
    - ▷ *You can only solve a game once!*
  - **It is better to acquire the knowledge about why the game wins, draws or loses.**
    - ▷ *You can learn lots of knowledge.*
  - **It is even better to discover knowledge in the game and then use it to make the world a better place.**
    - ▷ *Fun!*
- **Try to use the techniques learned from this course in other areas!**

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