Games solved: Now and in the future

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Abstract

- **Which game characters are predominant when the solution of a game is the main target?**
  - **It is concluded that decision complexity is more important than state-space complexity.**
  - **There is a trade-off between knowledge-based methods and brute-force methods.**
  - **There is a clear correlation between the first-player's initiative and the necessary effort to solve a game.**

Definitions I

- **Domain: two-person zero-sum games with perfect information.**
  - **Zero-sum means one player's loss is exactly the other player's gain, and vice versa. There is no way for both players to win at the same time.**
- **Game-theoretic value of a game: the outcome, i.e., win, loss or draw, when all participants play optimally.**
- **Classification of games' solutions according to L.V. Allis in 1994.**
  - **Ultra-weakly solved: the game-theoretic value of the initial position has been determined.**
  - **Weakly solved: for the initial position a strategy has been determined to achieve the game-theoretic value against any opponent.**
    - ▷ *The strategy must be efficient and practical in terms of resource usage.*
  - **Strongly solved: a strategy has been determined for all legal positions.**

Definitions II

- **State-space complexity of a game: the number of all legal positions in a game.**
- **Game-tree (or decision) complexity of a game: the number of all leaf nodes in a *solution search tree*.**
  - ▷ *A solution search tree is a tree where the game-theoretic value of the root position can be decided.*
- **A convergence game: the size of the state space decreases as the game progress.**
  - **Start with many pieces on the board and pieces are gradually removed during the course of the game.**
- **A divergence game: the size of the state space increases as the game progress.**
  - **Start with an empty board and pieces are gradually added during the course of the game.**
- **Initiative: the right to move first**
- **A fair game: the game-theoretic value is draw and both players have roughly an equal probability on making a mistake.**

Prediction made in 1990

- **Predictions were made at 1990 for the year 2000 concerning the expected playing strength of computer programs.**

solved	over champion	world champion	grand master	amateur
Connect-four	Checkers (8*8)	Chess	Go (9*9)	Go (19*19)
Qubic	Renju	Draughts (10*10)	Chinese chess	
Nine Men's Morris	Othello		Bridge	
Go-Moku	Scrabble			
Awari	Backgammon			

  - ▷ *Over champion means definitely over the best human player.*
  - ▷ *World champion means equaling to the best human player.*
  - ▷ *Grand master means equaling to most human players.*

Game space

- **A double dichotomy of the game space**

$\uparrow$

$\log\log$**(state-space complexity)**

category 3 **if solvable at all,** **then by knowledge-based methods**	**category 4** **unsolvable by any method**
category 1 **solvable by any method**	**category 2** **if solvable at all,** **then by brute-force methods**

$\log\log$**(game-tree complexity)** $\longrightarrow$

Questions to be researched

- Can perfect knowledge obtained from solved games be translated into rules and strategies which human beings can assimilate?
- Are such rules generic, or do they constitute a multitude of ad hoc recipes?
- Can methods be transferred between games?
  - More specifically, are there generic methods for all category-n games, or is each game in a specific category a law unto itself?

Convergent games

- **Can be solved by the method of endgame databases if we can enumerate and store all possible positions.**
- **Problems solved:**
  - **Nine Men's Morris: in the year 1995, a total of 7,673,759,269 states.**
    - *The game theoretic value is draw.*
  - **Mancala games**
    - *Awari: in the year 2002.*
    - *Kalah: in the year 2000 upto, but not equal, Kalah(6,6)*
  - **Checkers: in the year 1994**
    - *By combining Endgame databases, middle-game databases and verification of opening analysis.*
    - *Solved the so called 100-year position.*
  - **Chess endgames**
  - **Chinese chess endgames**

Divergent games

- **Connection games**
  - **Connect-four (6*7)**
  - **Qubic (4*4*4)**
  - **Go-Moku (15*15)**
  - **Renju**
  - k**-in-a-row games**
  - **Hex (10*10 or 11*11)**
- **Polynmino games**
  - **Pentominoes**
  - **Domineering**
- **Othello**
- **Chess**
- **Chinese chess**
- **Shogi**
- **Go**

Connection games I

- **Connect-four (6*7)**
  - **Solved by J. Allen in 1989 using a brute-force depth first search with alpha-beta pruning, a transposition table, and killer-move heuristic.**
  - **Also solved by L.V. Allis in 1988 using a knowledge-based approach by combining 9 strategic rules that identified potential threats of the opponent.**
    - ▷ ***Threats are something like forced moved or moves you have little choices.***
    - ▷ ***Moves with predictable counter-moves.***
  - **It is a first-player win.**
  - **Weakly solved on a SUN 4 workstation using 300+ hours.**
- **Qubic (4*4*4)**
  - **A three-dimensional version of Tic-Tac-Toe.**
  - **Solve in 1980 by O. Patashnik by combining the usual depth-first search with expert knowledge for ordering the moves.**

Connection games II

- **Go-Moku (15*15)**
  - **First-player win**
  - **Weakly solved by L.V. Allis in 1995 using a combination of threat-space search and database construction.**
- **Renju**
  - **Does not allow the first player to play certain moves.**
  - **An *asymmetric* game.**
  - **Weakly solved by Wágner and Viráag in 2000 by combining search and knowledge.**
    - ▷ *It is still first-player win.*
  - **Took advantage of an iterative-deepening search based on threat sequences up to 17 plies.**
- **k-in-a-row games**
  - **mnk-Game: a game playing on a board of m rows and n columns with the goal of obtaining a straight line of length k.**
  - **Variations: first ply picks only one stone, the rest picks two stones in a ply.**
    - ▷ *Try to balance the advantage of the initiative!*

Hex (10*10 or 11*11)

- **Properties:**
  - **It is a finite game.**
  - **It is not possible for both players to win at the same.**
  - **Exactly one of the players can win.**
    - ▷ ***A 2-D induction and topological argument.***
    - ▷ ***True for Hex 2*2***
    - ▷ ***Assume it is true for any Hex*** $i * j$***, where*** $i \le n$ ***or*** $j \le m$***.***
    - ▷ ***Induction step: try to prove this is true on Hex*** $n * m$
    - ▷ ***Delete the first row or the last row give you two white chains*** w_1 ***and*** w_2***, respectively.***
    - ▷ ***Delete the first column or the last column give you two black chains*** b_1 ***and*** b_2***, respectively.***
    - ▷ ***Delete first row, last row, first column and last column gives you either a white chain*** w_3 ***or a black chain*** b_3***.***
    - ▷ ***Either*** w_3 ***intersects with*** b_1 ***or*** b_2***, or*** b_3 ***intersects with*** w_1 ***or*** w_2***; both are contradicting statements.***
    - ▷ ***Other arguments in graph theory exist.***

Strategy-stealing argument

- The *unrestricted form is a first-player win.*
  - Using the "strategy-stealing" argument made by John Nash in 1949.
    - ▷ *If there is a winning strategy for the second player, the first player can still win by making an arbitrary first move and using the second-player strategy from then on.*
    - ▷ *If using the second-player strategy requires playing the chosen first move or any move played before, then make another arbitrary move.*
    - ▷ *An arbitrary extra move can never be a disadvantage in Hex.*
    - ▷ *This is not true for every games.*
    - ▷ *Not a constructive proof.*
  - This argument works for any symmetry games when an arbitrary extra move can never be a disadvantage.
  - The one-move-equalization rule: one player plays an opening move and the other player then has to decide which color to play for the reminder of the game.
  - The revised version is a second-player win (ultra-weakly).

Solutions to HEX

- **Hex exhibits considerable mathematical structure.**
- **Hex has been proved to be PSPACE-complete by Even and Tarjan in 1976 by converting it to a Shannon switching game.**
- **The state-space and decision complexities are comparable to those of Go on equally-sized boards.**
- **(Weakly or strongly) solved on 6*6 boards in 1994.**
- **Maybe possible to solve the 7*7 case.**
- **Not likely to solve the 8*8 version without fundamental breakthroughs.**

More divergent games

- Polynmino games: placing 2-D pieces that forms a connected subset of a square grid on a grid.
  - Pentominoes
  - Domineering
  - Games on smaller boards are solved.
- Othello
  - M. Buro's LOGISTELLO beat the resigning World Champion by 6-0 in 1997.
  - Weakly solved on 6*6 boards by J. Feinstein in 1993.
- Chess
  - DEEP BLUE beat the human World Champion in 1997.
- Chinese chess
  - Still in progress,
  - Professional 7-dan in 2007.
- Shogi
  - Still in progress,
  - Professional 2-dan in 2007.
- Go
  - Still in progress,
  - Amateur 4 kyu.

State-space versus game-tree size

- **In 1994, the boundary of solvability by complete enumeration was set at 10^{11}.**
  - **The current estimation is about 10^{13}.**

Table of complexity

Game	$\log_{10}$(state-space)	$\log_{10}$(game-tree size)
Nine Men's Morris	10	50
Pentominos	12	18
Awari	12	32
Kalak(6,4)	13	18
Connect-four	14	21
Domineering (8*8)	15	27
Dakon-6	15	33
Checkers	21	31
Othello	28	58
Qubic	30	34
Draughts	30	54
Chess	46	123
Chinese chess	48	150
Hex (11*11)	57	98
Shogi	71	226
Renju (15*15)	105	70
Go-Moku (15*15)	105	70
Go (19*19)	172	360

Brute-force versus knowledge-based methods

- **Games with both a relative low state-space complexity and a low game-tree complexity have been solved by both methods.**
  - Category 1
  - Connect-four and Qubic
- **Games with a relative low state-space complexity have mainly been solved with brute-force methods**
  - Category 2
  - Namely by constructing endgame database
  - Nine Men's Morris
- **Games with a relative low game-tree-complexities have mainly been solved with knowledge-based methods.**
  - Category 3
  - Namely, by intelligent (heuristic) searching
  - Sometimes, with the helps of endgame databases
  - Go-Moku, Renju, and k-in-a-row games

The advantage of the initiative

- **Theorem (or arguments) made by Singmaster in 1981: The first-player has advantages.**
  - **Two kinds of positions**
    - *P-positions: the previous player can force a win.*
    - *N-positions: the next player can force a win.*
  - **Arguments**
    - *For the first player to have a forced win, just one of the moves must lead to a P-position.*
    - *For the second player to have a forced win, all of the moves must lead to N-positions.*
    - *It is easier to the first player to have a forced win assuming all positions are randomly distributed.*
    - *Can be easily extended to games with draws.*
  - **Remarks:**
    - *One small boards, the second player is able to draw or even to win for certain games.*
    - *Cannot be applied to the infinite board.*

The advantage of the initiative in practice

- **A connection $mn1$-game.**
- **Mentioned in S.K Chang's famous 1978 novel: Chi Wang.**
- **Need to offset the initiative**
  - **Enforce rules so that the first player can win by selective patterns.**
  - **Swapping: a player makes the first move, the second player decides the color to play thereafter.**
  - **The first move plays one stone, the rest plays two stones.**
  - **The first player uses less time.**
  - **Must be simple and try to be as fair as possible.**

How to make use of initiative

- **A potential universal strategy for winning a game:**
  - **Try to obtain a small initiative**
    - *The opponent must react adequately on the moves played by the player.*
  - **To reinforce the initiative the player searches for threats, and even a sequence of threats using an evaluation function E.**
- **Threat-space search**
  - **Search for threats only!**

Methods developed for solving games

- **Brute-force methods**
  - **Retrograde analysis**
  - **Enhanced transposition-table methods**
- **Knowledge-based methods**
  - **Threat-space search and λ-search**
  - **Proof-number search**
  - **Depth-first proof-number search**
  - **Pattern search**
    - ▷ ***To search for threat patterns, which are collections of cells in a position P.***
    - ▷ ***A threat pattern can be thought of as representing the relevant area on the board, an area that human players commonly identify when analyzing a position.***

Conclusions

- **The knowledge-based methods mostly inform us on the structure of the game, while exhaustive enumeration rarely does.**
- **Many ad-hoc recipes are produced currently.**
  - **The database can be used as a corrector of strategies formulated by human experts.**
- **It may be hopeful to use data mining techniques to obtain cross-game methods.**
  - **Currently not very successful.**

New predictions

- **Predictions were made for the year 2010 concerning the expected playing strength of computer programs.**

solved	over champion	world champion	grand master	amateur
Awari	Chess	Go (9*9)	Bridge	Go (19*19)
Othello	Draughts (10*10)	Chinese chess	Shogi	
Checkers (8*8)	Scrabble	Hex		
	Backgammon	Amazons		
	Lines of Action			

References and further readings

* H. J. van den Herik, J. W. H. M. Uiterwijk, and J. van Rijswijck. Games solved: Now and in the future. *Artificial Intelligence*, 134:277–311, 2002.
- Jonathan Schaeffer. The games computers (and people) play. *Advances in Computers*, 52:190–268, 2000.
- L. V. Allis, M. van der Meulen, and H. J. van den Herik. Proof-number search. *Artificial Intelligence*, 66(1):91–124, 1994.