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Short Paper

A Study of Disjunctive Information in Fuzzy Relational Databases

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This paper proposes a logical reconstruction of the classical fuzzy relational database model to accommodate fuzzy disjunctive information. In query processing, we use two supplementary measurements, matching information and extra information, to model the concepts of imprecision and uncertainty, respectively. We also use these two supplementary measurements to determine the quality of answers to the query. The answers to the query thus contain sure answers and maybe answers. In addition, we discuss the redundancy problem by presenting a complete set of fuzzy relational algebra with fuzzy disjunctive information. The proposed extended fuzzy relational database model preserves the properties – uniquely-determined and well-defined properties of relational algebra to ensure that the manipulative power of the proposed model is the same as that of the classical fuzzy relational database model.

Keywords: fuzzy relational database, fuzzy disjunctive information, imprecision, uncertainty, quality of answer

1. INTRODUCTION

Ever since the introduction of the relational database model by Codd [6], many works [1-3, 13, 14, 20-25] have proposed additional general database models, called fuzzy relational database models, to handle imprecise information. Table 1 shows an *EMPLOYEE* fuzzy relation, where μ_r denotes the membership value of tuples. In logical interpretation, the disjunctive information can be either inclusive-or or the exclusive-or disjunctive information, but the fuzzy disjunctive information, such as "(John, engineering, 35000, 1) $\vee$ (John, manager, (high,0.9), 0.9)," can not be represented semantically by

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those fuzzy relational database models. In order to accommodate such information, a fuzzy relational database can be considered as a particular kind of first order logical interpretation [9, 16, 26, 29]. This paper will propose an extended fuzzy relational database model to handle fuzzy inclusive-or disjunctive information.

Table 1. The *EMPLOYEE* fuzzy relation.

Name	*Job*	*Salary*	μ_r
John	engineering, manager	35,000, (high,0.9)	0.9
Roy	sales, manager	40,000, high	1
Mutry	manager	high	1

On the other hand, numerous attempts have been made to obtain the better answers to queries from fuzzy relational databases with imprecise information. Most researchers have traditionally defined a membership value as representing the best matching degree of answers to a query in fuzzy relational databases [1-4, 7, 14, 19, 20-25]. However, if we let $\sigma_{\varphi(Salary)=\{\text{high}\}}$ be a query, then in Table 1, the second and the third tuples yield the same best matching degree, "one," in response to the query. Indeed, following the introduction of fuzzy disjunctive information into the fuzzy relational database, the third tuple is a better answer than the second tuple to the query. Therefore, measuring the matching degree between membership values of the tuple and query is not always enough to determine which answers are better [5, 9].

Measure of uncertainty [8, 10, 11, 32] is another method for handling query evaluation with imprecise information. De Luca and Termini [12] demonstrated that the uncertainty measure can be viewed as a measure of fuzziness. Following De Luca and Termini's suggestion, other researchers [10, 28] have suggested that any measure of fuzziness should be a measure of the lack of distinction between the fuzzy set and its complement. Moreover, Buckles and Petry [2] adapted De Luca and Termini's formula to evaluate the uncertainty of answers to a query. They specified a formula for computing the fuzzy entropy of a fuzzy relation, with respect to a query. Therefore, we employ two supplementary measurements, matching information and extra information, to model the concepts of imprecision and uncertainty, respectively. We also use these two supplementary measurements to determine the quality of answers to the query. The answers to the query thus contain sure answers and maybe answers.

2. PRELIMINARIES

2.1 Classical Fuzzy Relational Database Model

In this section, the basic definition and concept related to the classical fuzzy relational model are explained. The technique of employing sets of values to express imprecision in fuzzy relational databases was proposed by Buckles and Petry [1, 2], and investigated in several excellent works [14, 19, 21, 23, 32]. As pointed by Raju and Majumdar [21], a fuzzy relation over a relational schema can be defined as follows.

Definition 1 A fuzzy relation r on a relational schema $R(A_1, ..., A_n)$ is a fuzzy subset of the Cartesian product of universes, $Dom(A_1) \times \cdots \times Dom(A_n)$. ❑

Definition 2 Let $R(A_1, ..., A_n, \mu_r)$ be a fuzzy relational schema. An n-ary fuzzy relation r over R is a fuzzy subset or a set of fuzzy subsets of $Dom(A_1) \times \cdots \times Dom(A_n)$, which is characterized by the following membership function :

$$\mu_r(t_i) : Dom(A_1) \times \ldots \times Dom(A_n) \rightarrow [0, 1].$$ ❑

The type-1 fuzzy relation r can be expressed as

$$r = \sum_{\forall t_i, t_i \in r} \mu(t_i) / t_i \quad \text{or} \quad r = \sum_{\forall t_i, t_i \in r} \mu(d_{i1}, ..., d_{in}) / (d_{i1}, ..., d_{in}),$$

where for each $d_{ij} = \mu(a_{ij}) / a_{ij}$ and $\mu(d_{i1}, ..., d_{in}) = min(\mu(a_{i1}), ..., \mu(a_{in}))$, $a_{ij} \in Dom(A_j)$ and $j = 1, ..., n$. Consider a tuple t_i in the type-2 fuzzy relation; each component d_{ij} of t_i allows a set of index terms to be a fuzzy set, that is,

$$d_{ij} = \{\mu(a_{ij1}) / a_{ij1}, ..., \mu(a_{ijk_j}) / a_{ijk_j}\},$$

where $a_{ijl} \in Dom(A_j)$, $l = 1, ..., k_j$ and $\mu(a_{ijl})$ is the membership degree of a_{ijl}. The attribute value d_{ij} in a type-2 fuzzy relation can be a set of scalar terms, a set of range data, discrete scalars, or discrete numbers. Thus, the membership value $\mu_r(t_i)$ must satisfy the following equation:

$$\mu_r(t_i) = \min_{\forall d_{ij}}(\mu(d_{i1}), ..., \mu(d_{in})),$$

where $\mu(d_{ij}) = max(\mu(a_{ijl}), ..., \mu(a_{ijk_j}))$ for all $a_{ijl} \in d_{ij}$ and $l = 1, ..., k_j$.

As in type-1 fuzzy relations, $\mu_r(t_i)$ can be explained either as a possibility measure of the association between attribute values or as a fuzzy truth value of a fuzzy predicate associated with the fuzzy relation r. Zadeh [31] extended the concepts of select, projection, and natural join operations of classical relational algebra to fuzzy relations.

2.2 Extended Fuzzy Relational Database Model

Reiter [22] proposed a generalized relational theory to specify the logical semantics of disjunctive information in the context of first order logic and to accommodate disjunctive information. Liu and Sunderraman [17, 18] proposed a generalization of the relational database model to represent indefinite and maybe information. Pons and Vila [16, 26] logically defined fuzzy relational databases. Chiang *et al*. [5] defined an extended fuzzy relational model using first order logic. As stated in Section 1, fuzzy disjunctive information can not be represented semantically by the classical fuzzy relational database models. The classical fuzzy relational database model must be modified to represent sure tuples and maybe tuples with fuzzy disjunctive information. The proposed extended fuzzy relational language is based on Reiter's [22] relational language. Based on the extended fuzzy relational language and the suggestion made by Lipski [11], the extended fuzzy relation can be defined as follows.

Definition 3 Let $R(A_1, ..., A_n, \mu_r)$ be an extended fuzzy relational schema, where A_i is an attribute and $Dom(A_i)$ denotes the attribute domain of A_i. Then, an extended fuzzy relation r over R consists of two components, r_{sure} and r_{maybe}, which can be defined as follows:

$$
\begin{aligned}
r_{sure} = \{(\boldsymbol{t}, \mu_r(\boldsymbol{t})) \mid\ & (\boldsymbol{t}, \mu_r(\boldsymbol{t})) = \{(t_1, \mu_r(t_1)), ..., (t_k, \mu_r(t_k))\} \wedge k \geq 1 \wedge \\
& t_i \in Dom(A_1) \times \ldots \times Dom(A_n) \wedge \mu_r(t_i) > 0 \wedge \\
& \mu_r(t_i) = min(\mu(t_i[A_1]), ..., \mu(t_i[A_n])) \wedge i = 1, ..., k\}, \text{ and} \\
r_{maybe} = \{(t, \mu_r(t)) \mid\ & t \in Dom(A_1) \times \ldots \times Dom(A_n) \wedge \\
& \mu_r(t) > 0 \wedge |t| = 1 \wedge \mu_r(t) = min(\mu(t[A_1]), ..., \mu(t[A_n]))\},
\end{aligned}
$$

where for each $(t_i, \mu_r(t_i))$, $i = 1,..., k$ is a sub-tuple (interpretation) of the sure tuple $(\boldsymbol{t}, \mu_r(\boldsymbol{t}))$ and $|t|$ denotes the length of a tuple t. ❑

In logical representation, the disjunctive form of $(\boldsymbol{t}, \mu_r(\boldsymbol{t}))$ is $(t_1, \mu_r(t_1)) \vee ... \vee (t_k, \mu_r(t_k))$. For a sure tuple $(\boldsymbol{t}, \mu_r(\boldsymbol{t}))$, if $(\boldsymbol{t}, \mu_r(\boldsymbol{t}))$ contains only one sub-tuple, then it is a *definite sure tuple*; otherwise, $(\boldsymbol{t}, \mu_r(\boldsymbol{t}))$ is an *indefinite sure tuple*. Furthermore, $(t, \mu_r(t))$ denotes a maybe tuple, and for each $(t, \mu_r(t))$, it contains exactly one sub-tuple. The semantics of tuples in the extended fuzzy relation can be explained as follows.

Definition 4 Let r be an extended fuzzy relation with respect to an extended fuzzy relational database, DB. The semantics of fuzzy tuples can be described as follows:

(1) When $(\boldsymbol{t}, \mu_r(\boldsymbol{t})) = (t_1, \mu_r(t_1)) \vee ... \vee (t_k, \mu_r(t_k))$ and $(\boldsymbol{t}, \mu_r(\boldsymbol{t})) \in r_{sure}$, that is, $DB \vdash r_{sure}((\boldsymbol{t}, \mu_r(\boldsymbol{t})))$, this means that $DB \overset{p}{\vdash} r((t_j, \mu_r(t_j)))$, where $(t_j, \mu_r(t_j)) \in (\boldsymbol{t}, \mu_r(\boldsymbol{t}))$, $j=1,\ldots,k$, and there is at least one sub-tuple $(t_i, \mu_r(t_i)) \in (\boldsymbol{t}, \mu_r(\boldsymbol{t}))$ such that $DB \vdash r((t_i, \mu_r(t_i)))$, $1 \leq i \leq k$.

(2) When $(t, \mu_r(t)) \in r_{maybe}$, that is, $DB \vdash r_{maybe}((t, \mu_r(t)))$, this means that $DB \overset{p}{\vdash} r((t, \mu_r(t)))$. ❑

In Definition 4, $\vdash$ stands for a tuple (or a sub-tuple) that can be derived from DB, and $\overset{p}{\vdash}$ stands for a tuple (or a sub-tuple) that can possibly be derived from DB in logical interpretation.

3. MEASURING THE QUALITIES OF ANSWERS

3.1 The Problem to Be Solved

Ever since the relational database theory was introduced by Codd [6], studies have investigates query answering in relational databases with incomplete information. As suggested by Lipski [11], answers to a query for an incomplete database should be divided into two categories: sure answers and possible (maybe) answers. On the other hand, to deal with imprecise information, most recent research has focused on using Zadeh's fuzzy set theory and fuzzy logic [30, 31]. This theory provides a mathematical framework for dealing with imprecise information in fuzzy relational databases. Measuring the matching degree of the membership value of a query has been extensively studied [3, 15, 20, 25]. For measuring uncertainty, Vila *et al.* [27] provided a method that can used to measure imprecision and uncertainty of fuzzy information simultaneously. Yang [29] proposed an implicit predicate for handling uncertainty in fuzzy databases. However, the problem of measuring the quality of answers and finding more likely answers to a query has not yet been thoroughly investigated.

When imprecise information is introduced into the fuzzy relational database, we need to consider how to measure uncertainty during query evaluation. It is well known that methods for measuring uncertainty can also be used to measure information [10]. An important concept is that of the minimum information [15] required by the database system to be sure that a tuple satisfies a query. Self-information is a measure used to estimate the probability that an event corresponds to the occurrence of a particular attribute value. Self-information is defined as $-\ ln\ p(x)$, where $p(x)$ is the probability of event x occurring. Entropy information is defined as the average of self-information. The most prominent uncertainty measure is associated with Shannon entropy [7], which is expressed as the following function:

$$H(p(x) \mid x \in X) = -\sum_{x \in X} p(x)\ ln\ p(x)\,,$$

where $(p(x) \mid x \in X)$ is a possibility distribution on a finite set X.

Probability theory is the classical means of handling uncertainty, and it is useful in many applications. However, the problem with using probability theory is that the a priori probability of an object is difficult or impossible to estimate [15]. Influenced by Shannon entropy, De Luca and Termini [12] defined a measure of fuzziness of a fuzzy set A:

$$f(A) = -\sum_{x \in X} (\mu_A(x)\ ln\ \mu_A(x) + (1-\mu_A(x))\ ln\ (1-\mu_A(x))).$$

Moreover, Buckles and Petry [2] pointed out that a tuple's membership value may not be a static one, but is a measure of the appropriateness of a tuple as an answer to a query. Since the membership value of a tuple represents the best matching degree with a query, the membership value $\mu_\sigma(t)$ of tuple t to query σ can be defined as

$$\mu_\sigma(t) = \max_{I \text{ of } t} (\mu_\sigma(t)),$$

where I is an interpretation of tuple t. After producing each tuple's membership value for a query, we can define the fuzzy entropy of a fuzzy relation r with n tuples as

$$H_f(r|\sigma) = -\sum_{i=1}^{n} (\mu_\sigma(t_i)\ ln\ \mu_\sigma(t_i) + (1-\mu_\sigma(t_i))\ ln\ (1-\mu_\sigma(t_i)))\,,$$

where $\mu_\sigma(t_i)$ is the membership value of a tuple t_i, imposed by query σ, and $(\mu_\sigma(t_i)\ ln\ \mu_\sigma(t_i) + (1-\mu_\sigma(t_i))\ ln\ (1-\mu_\sigma(t_i))) = 0$ if and only if $\mu_\sigma(t_i) = 0$ or $\mu_\sigma(t_i) = 1$ for all *i*. Since $\mu_\sigma(t) = \max_{I \text{ of } t}(\mu_\sigma(t))$, two different tuples may have the same membership value with respect to a query. Accordingly, the quality of answers is determined by the matching degree and the uncertainty degree. We will propose a new method to determine the *matching strength* of answers. ❑

3.2 Answers to a Query

For each sub-tuple, $(t_i, \mu_r(t_i)) \in (\boldsymbol{t}, \mu_r(\boldsymbol{t}))$, $(t_i, \mu_r(t_i))$ may or may not satisfy the select condition of a query. Therefore, a tuple, $(\boldsymbol{t}, \mu_r(\boldsymbol{t}))$, can be belong to one of two components of a query ̃ a satisfied part or an unsatisfied part. When $(t_i, \mu_r(t_i))$ satisfies the query, it belongs to the satisfied part; otherwise, it belongs to the unsatisfied part.

Definition 5 Let $(\boldsymbol{t}, \mu_r(\boldsymbol{t})) = (t_1, \mu_r(t_1)) \vee ... \vee (t_k, \mu_r(t_k))$ be a tuple in the extended fuzzy relation *r*, and let σ_φ be a query. Then,

$Sat(\boldsymbol{t}) = \{(t_i, \mu_r(t_i)) \mid (\forall t_i)((t_i, \mu_r(t_i)) \in (\boldsymbol{t}, \mu_r(\boldsymbol{t})) \wedge (t_i, \mu_r(t_i))$ satisfies the select condition of $\varphi)\}$, and

$Unsat(\boldsymbol{t}) = \{(t_i, \mu_r(t_i)) \mid (\forall t_i)((t_i, \mu_r(t_i)) \in (\boldsymbol{t}, \mu_r(\boldsymbol{t})) \wedge (t_i, \mu_r(t_i)) \notin Sat(\boldsymbol{t}))\}$,

where $Sat(\boldsymbol{t})$ and $Unsat(\boldsymbol{t})$ represent the satisfied and unsatisfied parts of $(\boldsymbol{t}, \mu_r(\boldsymbol{t}))$ for query σ_φ, respectively. ❑

As stated in section 3.1, a method for determining the matching strengths of answers to a query is required. In the work of Chiang et al. [5], all the sub-tuples in $Sat(\boldsymbol{t})$ are considered as indefinite maybe answers to a query, and they have defined matching information and extra information which can be used to evaluate the matching strength of each answer to a query. The *matching information* refers to the matching degree provided by $Sat(\boldsymbol{t})$ of tuple $(\boldsymbol{t}, \mu_r(\boldsymbol{t}))$; and the *extra information* refers to the uncertainty degree needed by $Unsat(\boldsymbol{t})$ to make tuple $(\boldsymbol{t}, \mu_r(\boldsymbol{t}))$ a sure answer to the query. However, the quality of each tuple is not only dynamically dependent on the query but also statically dependent on the length of the tuple itself. We employ two measures, *matching information* and *extra information*, to determine the quality of answers as explained in the following sections.

3.3 Matching Information

For each answer to a query, matching information is defined as the average matching degree of the answer to a query.

Definition 6 Let *r* be an extended fuzzy relation, and let σ_φ be a query. The matching information is defined as follows:

(1) When $(\boldsymbol{t}, \mu_r(\boldsymbol{t})) = (t_1, \mu_r(t_1)) \vee ... \vee (t_k, \mu_r(t_k))$ and $(\boldsymbol{t}, \mu_r(\boldsymbol{t}))$ is a sure answer to a query, σ_φ, the matching information of $(\boldsymbol{t}, \mu_r(\boldsymbol{t}))$ for the query is

$$I_{mat}(\boldsymbol{t}) = \sum_{i=1}^{|Sat(t)|} I_{mat}(t_i) \Big/ k,$$

(2) When (t, $\mu_r(t)$) is a maybe answer to a query, σ_φ, the matching information of (t, $\mu_r(t)$) is

$$I_{mat}(t) = \mu_r(t),$$

where $I_{mat}(t_i)$ is the membership value of sub-tuple (t_i, $\mu_r(t_i)$ imposed by query σ_φ and (t_i, $\mu_r(t_i)$) $\in$ $Sat(\boldsymbol{t})$, $\sum_{i=1}^{|Sat(t)|} I_{mat}(t_i)$ is the total matching information of the tuple ($\boldsymbol{t}$, $\mu_r(\boldsymbol{t})$) with respect to a query σ_φ. ❑

3.4 Extra Information

Extra information is used to measure the uncertainty degree of maybe answers. Two kinds of extra information must be considered in query processing: the first kind is dynamically dependent on a query, and the second kind is statically dependent on the length of the tuple [5, 9, 14, 15].

Definition 7 Let ($\boldsymbol{t}$, $\mu_r(\boldsymbol{t})$) = (t_1, $\mu_r(t_1)$) $\vee$ ... $\vee$ (t_k, $\mu_r(t_k)$) be a tuple in the extended fuzzy relation r, let $\sigma_{\varphi(A)=\{b\}}$ be a query, let A be an attribute, let α_A be the associated threshold value of attribute A, and let $Sat(\boldsymbol{t}) \neq \emptyset$ and (t_i, $\mu_r(t_i)$) $\in$ $Unsat(\boldsymbol{t})$. Then, the independent extra information of (t_i, $\mu_r(t_i)$) with respect to $\sigma_{\varphi(A)}$ is

$$I_{ext}(t_i) = I_{min}(t_i) I_\sigma(t_i),$$

where $I_{min}(t_i) = \mu_{EQ}(\mu(t_i[A]), \mu(b))$ denotes the minimal amount of information required by the database system to ensure that tuple (t_i, $\mu_r(t_i)$) satisfies $\sigma_{\varphi(A)}$, and $I_\sigma(t_i) = \alpha_A$ denotes the maximum amount of available information concerning tuple (t_i,$\mu_r(t_i)$) for the query $\sigma_{\varphi(A)}$. Here, the fuzzy resemblance relation EQ is defined as

$$\mu_{EQ}(x,y) = \begin{cases} 0 & x \neq y \\ (1 - abs(\mu(x) - \mu(y))) & x = y \end{cases}. \quad ❑$$

Let ($\boldsymbol{t}$, $\mu_r(\boldsymbol{t})$) be an sure tuple in the extended fuzzy relation. Then, only when all the sub-tuples of ($\boldsymbol{t}$, $\mu_r(\boldsymbol{t})$) exist at the same time, and all the sub-tuples of ($\boldsymbol{t}$, $\mu_r(\boldsymbol{t})$) satisfy the query can the tuple ($\boldsymbol{t}$, $\mu_r(\boldsymbol{t})$) be said to definitely be true. Otherwise, there exists a maybe answer to a query. Restated, for each sub-tuple of ($\boldsymbol{t}$, $\mu_r(\boldsymbol{t})$), the sub-tuple still requires $1-(1/|\boldsymbol{t}|)$ extra information to ensure that ($\boldsymbol{t}$, $\mu_r(\boldsymbol{t})$) is definitely true in the extended fuzzy relation. Consequently, the extra information of each sub-tuple for a query is defined as follows:

Definition 8 Let ($\boldsymbol{t}$, $\mu_r(\boldsymbol{t})$) = (t_1, $\mu_r(t_1)$) $\vee$ ... $\vee$ (t_k, $\mu_r(t_k)$) be a tuple in the extended fuzzy relation r, let σ_φ be a query, and let $|Unsat(\boldsymbol{t})| = n$. Then, the extra information of a maybe tuple, (t', $\mu_r(t')$), with respect to a query, σ_φ, is

$$I_{ext}(t') = \sum_{i=1}^{n} I_{ext}(t_i) + (1 - 1/|\boldsymbol{t}|),$$

where $(t', \mu_r(t')) \in$ *Sat*(***t***) and $(t_i, \mu_r(t_i)) \in$ *Unsat*(***t***), $i = 1,\ldots, n$. ❑

In the above definition, the value $\sum_{i=1}^{n} I_{ext}(t_i)$ depends dynamically on the query, and $(1-1/|\boldsymbol{t}|)$ depends statically on the length of the tuple. These two values are called the dynamic and static extra information of a tuple, respectively. The proposed extra information satisfies Buckles's view [2] of fuzzy relational databases.

3.5 The Qualities of Answers

The following theorems state that the quality of answers to a query can be measured by means of matching information and extra information.

Theorem 1 Let $(\boldsymbol{t}_1, \mu_r(\boldsymbol{t}_1))$ and $(\boldsymbol{t}_2, \mu_r(\boldsymbol{t}_2))$ be two sure tuples in the extended fuzzy relation *r* that satisfy query, σ_φ. Then:

(1) When $|\boldsymbol{t}_1| > |\boldsymbol{t}_2|$, the quality of $(\boldsymbol{t}_2, \mu_r(\boldsymbol{t}_2))$ is better than that of $(\boldsymbol{t}_1, \mu_r(\boldsymbol{t}_1))$.
(2) When $|\boldsymbol{t}_1| = |\boldsymbol{t}_2|$ and $I_{mat}(\boldsymbol{t}_1) < I_{mat}(\boldsymbol{t}_2)$, then the quality of $(\boldsymbol{t}_2, \mu_r(\boldsymbol{t}_2))$ is better than that of $(\boldsymbol{t}_1, \mu_r(\boldsymbol{t}_1))$. ❑

Theorem 2 Let $(t_1, \mu_r(t_1))$ and $(t_2, \mu_r(t_2))$ be two maybe tuples in the extended fuzzy relation *r* that satisfy query, σ_φ. Then:

(1) When $I_{ext}(t_1) > I_{ext}(t_2)$, then the quality of $(t_2, \mu_r(t_2))$ is better than $(t_1, \mu_r(t_1))$.
(2) When $I_{ext}(t_1) = I_{ext}(t_2)$ and $I_{mat}(t_1) < I_{mat}(t_2)$, the quality of $(t_2, \mu_r(t_2))$ is better than that of $(t_1, \mu_r(t_1))$. ❑

4. REDUNDANT-FREE FUZZY RELATIONS

The key to preserving the desirable properties of the classical fuzzy relational database model in the proposed extended fuzzy relational database model is to remove redundancies from the database. We use the concepts "the quality of answers" and "minimal answers" to eliminate redundant tuples from the extended fuzzy relation. As shown in Algorithm 1, the *REDUCE** operation is used to remove redundancies between sub-tuples in a sure tuple.

Algorithm 1 *REDUCE** operation
Input: An extended fuzzy relation *r*.
Output: The reduced extended fuzzy relation *r*, where *r* = *REDUCE**(*r*).
Method:

```
Let (t, μr(t)) = (t1, μr(t1)) ∨ … ∨ (tk, μr(tk)) be a sure tuple and k > 1;
For each sure tuple (t, μr(t)) in r Do
    If (ti, μr(ti)) ∈ (t, μr(t)) and (tj, μr(tj)) ∈ (t, μr(t)) and (ti = tj) Then
        If μr(ti) > μr(tj) Then
            delete (tj, μr(tj)) from (t, μr(t))
```

else delete (t_i, $\mu_r(t_i)$) from ($\boldsymbol{t}$, $\mu_r(\boldsymbol{t})$);
End *REDUCE** operation.

Definition 9 Let ($\boldsymbol{t}_1$, $\mu_r(\boldsymbol{t}_1)$) be a sure tuple in the extended fuzzy relation r. Then, tuple ($\boldsymbol{t}_1$, $\mu_r(\boldsymbol{t}_1)$) is redundant if and only if another sure tuple ($\boldsymbol{t}_2$, $\mu_r(\boldsymbol{t}_2)$) exists in r such that $\boldsymbol{t}_2 \subseteq \boldsymbol{t}_1$. When ($\boldsymbol{t}_1$, $\mu_r(\boldsymbol{t}_1)$) does not contain any redundant information, tuple ($\boldsymbol{t}_1$, $\mu_r(\boldsymbol{t}_1)$) is said to be *minimal* in the extended fuzzy relation r.

Definition 10 Let (t_1, $\mu_r(t_1)$) be a maybe tuple in the extended fuzzy relation r. Then, (t_1, $\mu_r(t_1)$) is redundant if and only if a sure tuple ($\boldsymbol{t}$, $\mu_r(\boldsymbol{t})$) exists in r such that $t_1 \in \boldsymbol{t}$ or another maybe tuple (t_2, $\mu_r(t_2)$) exists in r such that $t_1 = t_2$, and the quality of (t_1, $\mu_r(t_1)$) is worse than that of (t_2, $\mu_r(t_2)$). When (t_1, $\mu_r(t_1)$) does not contain any redundant information, tuple (t_1, $\mu_r(t_1)$) is said to be *minimal* in the extended fuzzy relation r. ❑

As shown in Algorithm 2, the *REDUCE* operation eliminates redundancies over an extended fuzzy relation.

Algorithm 2 *REDUCE* operation
Input: An extended fuzzy relation *r*.
Output: The reduced extended fuzzy relation r, where r = *REDUCE*(r).
Method:
For each sure tuple ($\boldsymbol{t}_1$, $\mu_r(\boldsymbol{t}_1)$) in r_{sure} Do
{ If ($\boldsymbol{t}_1$, $\mu_r(\boldsymbol{t}_1)$) is not minimal and there exists another tuple ($\boldsymbol{t}_2$, $\mu_r(\boldsymbol{t}_2)$) in r_{sure} such that
$\boldsymbol{t}_2 \subseteq \boldsymbol{t}_1$ Then
{ If $|\boldsymbol{t}_1| = |\boldsymbol{t}_2|$ Then {
If the quality of ($\boldsymbol{t}_1$, $\mu_r(\boldsymbol{t}_1)$) is better than that of ($\boldsymbol{t}_2$, $\mu_r(\boldsymbol{t}_2)$) Then
delete ($\boldsymbol{t}_2$, $\mu_r(\boldsymbol{t}_2)$) from r_{sure}; else
delete ($\boldsymbol{t}_1$, $\mu_r(\boldsymbol{t}_1)$) from r_{sure}; }
else {
For each sub-tuple (t_i, $\mu_r(t_i)$) of ($\boldsymbol{t}_1$, $\mu_r(\boldsymbol{t}_1)$), $t_i \in \boldsymbol{t}_1 \tilde{-} \boldsymbol{t}_2$ Do
{ $I_{ext}(t_i) = 1 - 1/|\boldsymbol{t}|$;
add (t_i, $\mu_r(t_i)$) into r_{maybe}; }
delete ($\boldsymbol{t}_1$, $\mu_r(\boldsymbol{t}_1)$) from r_{sure};}
}
}
For each maybe tuple (t_1, $\mu_r(t_1)$) in r_{maybe} Do
{ If (t_1, $\mu_r(t_1)$) is not minimal and there exists another maybe tuple (t_2, $\mu_r(t_2)$) such that
$t_2 = t_1$ Then
{ If the quality of (t_1, $\mu_r(t_1)$) is better than that of (t_2, $\mu_r(t_2)$) Then
delete (t_2, $\mu_r(t_2)$) from r_{maybe}; else
delete (t_1, $\mu_r(t_1)$) from r_{maybe}; }
}
End *REDUCE* Operation.

Let n_{sure} and n_{maybe} be the number of tuples in r_{sure} and r_{maybe}, respectively. Then, the time complexity of the *REDUCE* operation is $O((n_{sure}*k + n_{maybe})^2)$, where k is the average length of the sure tuples. In this paper, we are only concerned with the semantics of the *REDUCE* operation. The performance of this algorithm can be further improved.

5. EXTENDED FUZZY RELATIONAL ALGEBRA

This paper claims that the quality of answers can be determined during query processing. If the values of the *original* length, the *dynamic* extra information, and the *total* matching information of a tuple are known, then the quality of each answer can be determined directly from the resultant tuple by means of the definitions states discussed in section 3. We use $length(\boldsymbol{t})$, $ext(\boldsymbol{t})$, and $total(\boldsymbol{t})$ to represent the original length, dynamic extra information, and total matching information of tuple $(\boldsymbol{t}, \mu_r(\boldsymbol{t}))$, respectively.

5.1 Extended Fuzzy Project and Select Operation

The extended fuzzy project operation is used to select certain attributes of interest over an extended fuzzy relation. Although some sub-tuples of a tuple will be removed by the *REDUCE** operation, the quality of answers will not change after projection. Restated, the quality of each answer is inherited from the original tuple.

Definition 11 (Extended fuzzy Project Operation) Let r be an extended fuzzy relation. Then, $\pi_A(r) = r_1$, where

$$r_{1,sure} = \{(\boldsymbol{t}, \mu_r(\boldsymbol{t})) \mid (\boldsymbol{t}_1, \mu_r(\boldsymbol{t}_1)) \in r_{sure} \wedge (\boldsymbol{t}, \mu_r(\boldsymbol{t})) = REDUCE^*(\pi'_A(\boldsymbol{t}_1, \mu_r(\boldsymbol{t}_1))) \wedge length(\boldsymbol{t}) = length(\boldsymbol{t}_1) \wedge total(\boldsymbol{t}) = total(\boldsymbol{t}_1)\},$$

$$r_{1,maybe} = \{(t, \mu_r(t)) \mid (t_1, \mu_r(t_1)) \in r_{maybe} \wedge (t, \mu_r(t)) = \pi'_A(t_1, \mu_r(t_1)) \wedge length(t) = length(t_1) \wedge ext(t) = ext(t_1)\},$$

where A denotes a set of attributes and π'_A is the conventional relational project operation. ❑

As stated by Buckles and Petry [2], a tuple's membership value with respect to a query is a measure of the appropriateness of the tuple to the query. Therefore, query evaluation is the processes to determine the truth value of a tuple to a query. However, in many situations, a tuple may not exactly match the select condition of a query. Therefore, specifying a threshold value, α_j, as the minimum degree of matching between the selected tuples and the selected condition is necessary. When the matching degree of a tuple is less than the threshold value, that tuple is assumed to not satisfy the selected condition. Consequently, the extended fuzzy selection operation is defined as follows.

Definition 12 (Extended fuzzy Selection Operation) Let r be an extended fuzzy relation, and let $\sigma_{\varphi(A)}$ be an extended fuzzy selection operation, where $\varphi(A)$ is a selection condition over an attribute A. Then, $\sigma_{\varphi(A)}(r) = r_1$, where

$$r_{1,sure} = \{(\boldsymbol{t}, \mu_r(\boldsymbol{t})) \mid (\boldsymbol{t}, \mu_r(\boldsymbol{t})) \in r_{sure} \wedge (\forall t_i)((t_i, \mu_r(t_i)) \in (\boldsymbol{t}, \mu_r(\boldsymbol{t})) \wedge \varphi(t_i[A])) \wedge total(\boldsymbol{t}) = \sum_{i=1}^{length(\boldsymbol{t})} \mu_r(t_i)\},$$

$$r_{1,maybe} = \{(t, \mu_r(t)) \mid ((t, \mu_r(t)) \in r_{maybe} \wedge \varphi(t[A])) \vee (\exists \boldsymbol{t}_1)((\boldsymbol{t}_1, \mu_r(\boldsymbol{t}_1)) \in r_{sure} \wedge (t, \mu_r(t)) \in (\boldsymbol{t}_1, \mu_r(\boldsymbol{t}_1)) \wedge (t, \mu_r(t)) \in Sat(\boldsymbol{t}_1) \wedge (t_i, \mu_r(t_i)) \in Unsat(\boldsymbol{t}_1) \wedge Unsat(\boldsymbol{t}_1) \neq \emptyset \wedge length(t) = length(\boldsymbol{t}_1) \wedge ext(t) = \sum_{\forall t_i} I_{ext}(t_i)\}.$$ ❑

Definition 13 Let $(\boldsymbol{t},\mu_r(\boldsymbol{t})) = (t_1,\mu_r(t_1)) \vee ... \vee (t_k,\mu_r(t_k))$ be a tuple in the extended fuzzy relation r, and let $\varphi(\boldsymbol{A})$ be a general select condition. Then, the following calculations can be applied:

(1) When $(t_i,\mu_r(t_i)) \in Sat(\boldsymbol{t})$:

(a) when $\varphi(\boldsymbol{A}) = \varphi(A_1) \wedge \ldots \wedge \varphi(A_m)$, $\mu_r(t_i) = min_{j=1}^{m}(\mu(t_i[A_j]), \mu_r(t_i))$;

(b) when $\varphi(\boldsymbol{A}) = \varphi(A_1) \vee \ldots \vee \varphi(A_m)$, $\mu_r(t_i) = max_{j=1}^{m}(min(\mu(t_i[A_j]), \mu_r(t_i)))$.

(2) When $(t_i,\mu_r(t_i)) \in Unsat(\boldsymbol{t})$:

(a) when $\varphi(\boldsymbol{A}) = \varphi(A_1) \wedge \ldots \wedge \varphi(A_m)$, $I_{ext}(t_i) = \sum_{j=1}^{m}(\alpha_j - \mu(t_i[A_j]))$;

(b) when $\varphi(\boldsymbol{A}) = \varphi(A_1) \vee \ldots \vee \varphi(A_m)$, $I_{ext}(t_i) = min_{j=1}^{m}(\alpha_j - \mu(t_i[A_j]))$.

Here, α_i is the pre-specified threshold value for attribute A_i. ❑

5.2 Extended Fuzzy Set Operations

Three set operations are used to merge the elements of two sets in various ways, including union, intersection, and difference operations, which are all treated as binary operations. The quality of any resultant tuple obtained by the union operation, is inherited from the original tuple.

Definition 14 (Extended fuzzy union operation) Let r_1 and r_2 be two union-compatible extended fuzzy relations. Then, $r_1 \cup r_2 = r$, where

$$r_{sure} = \{(\boldsymbol{t}, \mu_r(\boldsymbol{t})) \mid (\boldsymbol{t}, \mu_r(\boldsymbol{t})) \in r_{1,sure} \vee (\boldsymbol{t}, \mu_r(\boldsymbol{t})) \in r_{2,sure}\},$$
$$r_{maybe} = \{(t, \mu_r(t)) \mid (t, \mu_r(t)) \in r_{1,maybe} \vee (t, \mu_r(t)) \in r_{2,maybe}\}.$$ ❑

Next, the extended fuzzy intersection operation between two union-compatible extended fuzzy relations, r_1 and r_2, will be defined.

Definition 15 (Extended fuzzy Intersection Operation) Let r_1 and r_2 be two union-compatible extended fuzzy relations. Then, $r_1 \cap r_2 = r$, where p, q = 1,2, and $p \neq q$:

$$r_{sure} = \{ (\boldsymbol{t},\mu_r(\boldsymbol{t})) \mid (\boldsymbol{t},\mu_r(\boldsymbol{t})) \in r_{p,sure} \wedge (\forall t_i)(\exists \boldsymbol{t}_1)((t_i,\mu_r(t_i)) \in (\boldsymbol{t},\mu_r(\boldsymbol{t})) \wedge (\boldsymbol{t}_1,\mu_r(\boldsymbol{t}_1)) \in r_{q,sure} \wedge |\boldsymbol{t}_1| = 1 \wedge t_i = \boldsymbol{t}_1 \wedge \mu_r(t_i) = min(\mu_r(\boldsymbol{t}_1),\mu_r(t_i))) \},$$

$$
\begin{aligned}
r_{maybe} = \{\, & (t,\mu_r(t)) \mid (\exists \boldsymbol{t}_1)(\exists \boldsymbol{t}_2)(\, (\boldsymbol{t}_1,\mu_r(\boldsymbol{t}_1)) \in r_{p,sure} \wedge (\boldsymbol{t}_2,\mu_r(\boldsymbol{t}_2)) \in r_{q,sure} \wedge \\
& (t_i,\mu_r(t_i)) \in (\boldsymbol{t}_1,\mu_r(\boldsymbol{t}_1)) \wedge (t_j,\mu_r(t_j)) \in (\boldsymbol{t}_2,\mu_r(\boldsymbol{t}_2)) \wedge t = t_i \wedge \\
& t = t_j \wedge \mu_r(t) = min(\mu_r(t_i),\mu_r(t_j)) \wedge ext(t) = 0 \wedge \\
& length(t) = max(length(\boldsymbol{t}_1), length(\boldsymbol{t}_2))) \vee \\
& (\exists \boldsymbol{t}_1)(\exists t)(\, (\boldsymbol{t}_1,\mu_r(\boldsymbol{t}_1)) \in r_{p,sure} \wedge (t,\mu_r(t)) \in r_{q,maybe} \wedge \\
& (t_i,\mu_r(t_i)) \in (\boldsymbol{t}_1,\mu_r(\boldsymbol{t}_1)) \wedge t = t_i \wedge \mu_r(t) = min(\mu_r(t_i),\mu_r(t)) \wedge \\
& ext(t) = ext(t) \wedge length(t) = max(length(\boldsymbol{t}_1), length(t))) \vee \\
& (\exists t_1)(\exists t_2)(\, (t_1,\mu_r(t_1)) \in r_{p,maybe} \wedge (t_2,\mu_r(t_2)) \in r_{q,maybe} \wedge t = t_1 \\
& \quad \wedge t = t_2 \wedge \mu_r(t) = min(\mu_r(t_1),\mu_r(t_2)) \wedge ext(t) = ext(t_1) + ext(t_2) \\
& \quad \wedge length(t) = max(length(t_1), length(t_2)))\, \}. \qquad \square
\end{aligned}
$$

Next, the extended fuzzy difference operation between two union-compatible extended fuzzy relations, r_1 and r_2, will be defined.

Definition 16 (Extended fuzzy difference operation) Let r_1 and r_2 be two union-compatible extended fuzzy relations. Then, $r_1 \,\tilde{-}\, r_2 = r$, where

$$
\begin{aligned}
r_{sure} = \{ & (\boldsymbol{t}, \mu_r(\boldsymbol{t})) \mid (\boldsymbol{t}, \mu_r(\boldsymbol{t})) \in r_{1,sure} \wedge \neg(\exists \boldsymbol{t}_1)((\boldsymbol{t}_1, \mu_r(\boldsymbol{t}_1)) \in r_2 \wedge \boldsymbol{t} \cap \boldsymbol{t}_1 \neq \emptyset)\}, \\
r_{maybe} = \{ & (t, \mu_r(t)) \mid (\neg\exists t_1)(t_1 \in r_{2,sure} \wedge t = t_1 \wedge |t_1| = 1) \wedge (\, (t, \mu_r(t)) \in r_{1,maybe}\,) \vee \\
& (\exists \boldsymbol{t}_2)(\exists \boldsymbol{t}_3)((\boldsymbol{t}_2, \mu_r(\boldsymbol{t}_2)) \in r_{1,sure} \wedge (t,\mu_r(t)) \in (\boldsymbol{t}_2, \mu_r(\boldsymbol{t}_2)) \wedge \\
& (\boldsymbol{t}_3, \mu_r(\boldsymbol{t}_3)) \in r_2 \wedge \boldsymbol{t}_2 \cap \boldsymbol{t}_3 \neq \emptyset \wedge length(t) = |\boldsymbol{t}_2| \wedge ext(t) = 0))\}. \qquad \square
\end{aligned}
$$

Let $(\boldsymbol{t}, \mu_r(\boldsymbol{t}))$ be obtained by taking the extended fuzzy Cartesian product from two tuples, $(\boldsymbol{t}_1, \mu_r(\boldsymbol{t}_1))$ and $(\boldsymbol{t}_2, \mu_r(\boldsymbol{t}_2))$. Then, the information concerning the quality of this resultant tuple is updated as $ext(\boldsymbol{t}) = ext(\boldsymbol{t}_1)*length(\boldsymbol{t}_2) + ext(\boldsymbol{t}_2)*length(\boldsymbol{t}_1)$ and $length(\boldsymbol{t}) = length(\boldsymbol{t}_1)*length(\boldsymbol{t}_2)$. When tuple $(\boldsymbol{t}_i, \mu_r(\boldsymbol{t}_i))$, i =1, 2, is a sure tuple or a sub-tuple of a sure tuple, its dynamic extra information equals zero; that is, $ext(\boldsymbol{t}_i) = 0$. Therefore, the extended fuzzy Cartesian product operation is defined as follows.

Definition 17 Let $(\boldsymbol{t}_1, \mu_r(\boldsymbol{t}_1))$ and $(\boldsymbol{t}_2, \mu_r(\boldsymbol{t}_2))$ be two tuples, where $(\boldsymbol{t}_1, \mu_r(\boldsymbol{t}_1)) = \{(t_{11}, \mu_r(t_{11})),\ldots, (t_{1n}, \mu_r(t_{1n}))\}$ and $(\boldsymbol{t}_2, \mu_r(\boldsymbol{t}_2)) = \{(t_{21}, \mu_r(t_{21})),\ldots, (t_{2m}, \mu_r(t_{2m}))\}$. Then, the $\times'$ operation is defined as

$$
\begin{aligned}
\boldsymbol{t}_1 \times' \boldsymbol{t}_2 = \{ & (t_{11}t_{21}, min(\mu_r(t_{11}),\mu_r(t_{21}))), \ldots,(t_{11}t_{2m}, min(\mu_r(t_{11}),\mu_r(t_{2m}))),\ldots, \\
& (t_{1n}t_{21}, min(\mu_r(t_{1n}),\mu_r(t_{21}))), \ldots,(t_{1n}t_{2m}, min(\mu_r(t_{1n}),\mu_r(t_{2m})))\}.
\end{aligned}
$$

Definition 18 (Extended fuzzy Cartesian Product Operation) Let r_1 and r_2 be two extended fuzzy relations. Then , $r_1 \times r_2 = r$, where

$$
\begin{aligned}
r_{sure} = \{ & (\boldsymbol{t}, \mu_r(\boldsymbol{t})) \mid (\exists \boldsymbol{t}_1)(\exists \boldsymbol{t}_2)((\boldsymbol{t}_1, \mu_r(\boldsymbol{t}_1)) \in r_{1,sure} \wedge (\boldsymbol{t}_2, \mu_r(\boldsymbol{t}_2)) \in r_{2,sure} \wedge \\
& \boldsymbol{t} = \boldsymbol{t}_1 \times' \boldsymbol{t}_2 \wedge length(\boldsymbol{t}) = length(\boldsymbol{t}_1)*length(\boldsymbol{t}_2)\, \}, \\
r_{maybe} = \{ & (t, \mu_r(t)) \mid (\exists t_1)(\exists t_2)(\, (t_1, \mu_r(t_1)) \in r_{1,maybe} \wedge (t_2, \mu_r(t_2)) \in r_{2,maybe} \wedge \\
& t = t_1 \times' t_2 \wedge ext(t) = ext(t_1)*length(t_2) + ext(t_2)*length(t_1)
\end{aligned}
$$

$\wedge$ *length*(t) = *length*(t_1)**length*(t_2)) $\vee$
$(\exists \boldsymbol{t}_1)(\exists t_2)(\exists t_i)(\ (\boldsymbol{t}_1, \mu_r(\boldsymbol{t}_1)) \in r_{1,sure} \wedge (t_2, \mu_r(t_2)) \in r_{2,maybe}$
$\wedge\ (t_i, \mu_r(t_i)) \in (\boldsymbol{t}_1, \mu_r(\boldsymbol{t}_1)) \wedge t = t_i \times' t_2 \wedge$
$ext(t) = ext(t_2) * length(\boldsymbol{t}_1) \wedge length(t) = length(\boldsymbol{t}_1) * length(t_2)\ ) \vee$
$(\exists t_1)(\exists \boldsymbol{t}_2)(\exists t_i)(\ (t_1, \mu_r(t_1)) \in r_{1,maybe} \wedge (\boldsymbol{t}_2, \mu_r(\boldsymbol{t}_2)) \in r_{2,sure}$
$\wedge\ (t_i, \mu_r(t_i)) \in (\boldsymbol{t}_2, \mu_r(\boldsymbol{t}_2)) \wedge t = t_1 \times' t_i \wedge$
$ext(t) = ext(t_1) * length(\boldsymbol{t}_2) \wedge length(t) = length(t_1) * length(\boldsymbol{t}_2))\ \}$. ❑

6. CONCLUSIONS

This paper has presented an effective means of measuring the quality of answers to queries of extended fuzzy relational databases with fuzzy inclusive-or disjunctive information. Matching information and extra information are used to measure the imprecision and uncertainty of each answer to a query. The proposed method depends not only on the semantics of tuples but also on the criteria of the query. The method can not only select the more likely answers to a query but also distinguish between sure answers and maybe answers to the query.

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