Predicting Travel Time through a Congested Road Segment using Regression in Time-Space Diagram

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Abstract

Travel time estimation is a basic component in many applications of an intelligent transportation system (ITS). We consider the travel time estimation of passing through a congested road segment and present a novel travel time prediction algorithm. The dynamic nature of our congestion model can be characterized by the propagation of its head and tail boundaries, which are determined by shock wave equations. We show that the propagation of the tail boundary can be effectively approximated if the inflow to the congestion is given, while the head boundary can be approximated as a line in a time-space diagram. We demonstrate the effectiveness of our algorithm by measuring its performance using the Nagel-Schreckenberg (CA) model. Simulation results from varying the probe car density are also given.

1   Introduction

Intelligent transportation systems (ITS) have attracted much interest. Hardware components such as on-board computers, communication equipment, and automation have commonly been included in an ITS. However, important software components for dynamic traffic choice and network modeling; accident detection and management; and dynamic traffic control are not commonly included in an ITS. Travel time estimation is one of the basic software components for ITS [1, 2, 3, 5, 7, 14]. Usually, statistics of the previous travel time of a road segment can yield a satisfactory reference for predicting the future travel time of that same segment. This approach smoothes short term traffic variations, which gives a smooth estimation of travel time [18, 16]. However, traffic variations incurred by congestion and accidents may introduce a sharp change in traffic parameters, so they cannot be effectively predicted by using smoothing techniques [11].

Congestion may cause serious delays and its effect on travel time should be incorporated for reliable prediction. We study the travel time prediction for a vehicle passing through a congested road segment. We assume that a probe car is on a one-lane highway and is heading towards a road segment. With a flow detector placed before the congested segment, and with all the probe cars reporting their positions to a center station, the travel time of the car through the congested segment can be predicted. A framework of the proposed system is shown in Figure 1. Because of the dynamic sensors and a small amount of bi-dimentional information exchange between the probe car and the center station, our approach is ecomonic and feasible for estimating the dynamic variations of congestion.

Our approach is based on extending the time-space pattern of the congestion beyond the current time, with the help of the flow detector data, to obtain an extended time-space diagram. From the extended diagram, we characterize two curves; one corresponds to the shock wave at the head boundary of the congestion, and the other to the shock wave at the tail boundary of the congestion. We show analytically that the travel time through the congestion can be estimated from the two curves in the extended diagram, plus some moderate assumptions about the traffic parameters, which are discussed in Section 2.2.

This scenario assumes that all the cars are probe cars, which is impractical. In practice,

probe cars would only be a percentage of all the cars. This percentage is measured by probe car density, a number always between 0 and 1. We thus investigate the effect of probe car density on the performance of our algorithm by a simulation, in which we assume that probe cars are uniformly distributed among all cars. In our derivation, the purpose of the flow detector is to provide the congestion inflow in order to estimate the propagation of the tail curve. This flow detector can be replaced by any sensors, if they are able to provide congestion inflow data.

Our notations and assumptions are given at the end of Section 2, in which we also review the shock wave propagation equation and propose our model. Section 3 describes our method to extend the time-space pattern of the congestion and characterizes the dynamics of the congestion. Our travel time estimation algorithm is given in Section 4. The performance of our algorithm using a CA model is described in Section 5 [12]. Finally, in Section 6, we present our conclusions.

2   The Formulation of the Problem

The important parameters of congestion in travel time estimation are the propagation of the head and tail boundaries of the congestion. Because of the congestion, the traffic parameters at the boundaries of the congestion are discontinuous, but propagation of that boundaries can be modeled by a shock wave equation. Based on the shock wave propagation, we propose our congestion model for travel time estimation. We review the shock wave equation first, and then propose our congestion model.

2.1   Shock Wave Equation

A shock wave shows how the discontinuities of traffic parameters in a boundary are developed into the propagation of the boundary [8, 10, 13, 17]. If there is congestion on a road segment, both the head and tail of the congestion usually move backwards, as illustrated by the dashed lines in Figure 2(a). An example of a time-space diagram of congestion by simulation using a CA model is given in Figure 2(b), in which the head and tail of the congestion are moving backwards. The boundary w in Figure 2(c) divides the road segment into distinct

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homogeneous regions in density. When w is the head boundary, the segment to the left of it is the congested region, but when w is the tail boundary, the segment to the right of it is the congested region. Let ρ_l and v_l be the respective density and average velocity of the region to the left of w; and ρ_r and v_r be the respective density and average velocity of the region to the right of w. Then, the velocity of the boundary is the shock wave $\frac{dw}{dt}$, which is propagating along the segment according to the flow conservation equation:

$$\rho_l(v_l - \frac{dw}{dt}) = \rho_r(v_r - \frac{dw}{dt}).$$

Thus, we have

$$\frac{dw}{dt} = \frac{\rho_l v_l - \rho_r v_r}{\rho_l - \rho_r}.$$

2.2   Congestion Model for Travel Time Estimation

The congestion pattern in the time-space diagram is enclosed by two curves corresponding to the evolution of the head and tail boundaries of the congestion. Figure 2(b) gives a typical time-space pattern of congestion. The time-space region of the congestion is enclosed by the curves produced by shock waves corresponding to the congestion boundaries. There are other time-space congestion patterns [6, 9, 15] that differ in details, but in general have a common structure. Assume that our car is heading toward the congestion, but is not in it yet. The amount of time our car will be in the congestion can be estimated if we know when our car will encounter it, and when our car will exit from it.

In Figure 3, the point of entering the congestion corresponds to meeting the tail curve $(t, x(t))$ at time t_a, and the point of leaving it corresponds to meeting the head curve $(t, y(t))$, at time t_b. If the complete curves were given, and some moderate assumptions about the traffic parameters were made, then there is sufficient information to estimate the passing time through the congestion. However, when there is a need to estimate the passing time of the congestion, there is only a part of time-space diagram of the congestion pattern available. Thus, an important component of the prediction of the travel time through congestion is to solve the problem of extending the time-space boundaries of the congestion beyond the current time.

The extension of the two curves is analytically possible. The tail curve can be extended to the future ($t > t_c$) with the help of the inflow to the congestion. Also, the head curve can be effectively approximated by a line in the time-space diagram. Before proposing our algorithm for curve extension, we introduce some notations and make assumptions to simplify the subsequent analysis.

Notations and Assumptions

$p(t)$: the position of our car at time t

t_c: the current time

$f(t)$: flow measured by the loop detector at time t (a slowly varying function)

v_1: average velocity of traffic before entering the congestion (a constant)

v_2: average velocity of cars in the congestion (a constant)

v_3: average velocity of traffic after leaving the congestion (a constant)

$x(t)$: distance from the flow detector to the tail of the congestion

$y(t)$: distance from the flow detector to the head of the congestion

$\rho_1(t)$: average density of cars on the road segment before entering the congestion (a slowly varying function)

ρ_2: average car density in the congestion (a constant)

ρ_3: average density of cars on the road segment after leaving the congestion (a constant)

$l(t)$: number of cars in the congestion at time t

$\mu(t)$: the instant flow leaving the congestion

ρ_p: probe car density

3 Head and Tail Boundaries of the Congestion

We use the shock wave propagration equation to derive the propagation of the head and tail congestion boundaries and extend them to the future in the time-space diagram.

3.1 Tail Shock Wave

From the tail boundary curve $(t, x(t))$ in the time-space diagram, we can estimate the travel time of a car, which is not yet in the congestion, but is heading towards it. The top subfigure in Figure 4 gives the schematic diagram for our estimation of the tail curve. The left region of $x(t)$, $[x(t) - \delta, x(t))$, in the subfigure is a non-congested segment, while the right region (marked by gray dots) is a congested segment. According to the shock wave propagation equation, the velocity of $x(t)$ can be measured by

$$\rho_1(t)(v_1 - \frac{dx(t)}{dt}) = (v_2 - \frac{dx(t)}{dt})\rho_2, \qquad (1)$$

where $\rho_1(t)$ is the average density at the space interval $[x(t) - \delta, x(t))$; v_1 is the average car velocity in the interval; and v_2 and ρ_2 are respectively the average car velocity and average car density within the congestion at time t. According to our assumptions, v_1, v_2, and ρ_2 are constants. The first term to the left of Equation (1) is $\rho_1(t)v_1$. If δ is small, then this term represents the instantaneous inflow of the congestion at $x(t)$.

We further assume that the flow measured by the flow detector at time $t - \frac{x(t)}{v_1}$ is equal to the instantaneous inflow entering the tail of the congestion at time t. Therefore, if $f(.)$ is the flow of the flow detector, then

$$f(t - \frac{x(t)}{v_1}) = \rho_1(t)v_1.$$

Substituting the simplified parameters into Equation (1) for $\rho_1(t)v_1$ and rearranging the terms, we obtain the tail boundary velocity, $\frac{dx(t)}{dt}$, as follows:

$$\rho(t)\frac{dx(t)}{dt} = -f(t - \frac{x(t)}{v_1}) + \rho_2 v_2, \qquad (2)$$

where $\rho(t) = \rho_2 - \rho_1(t)$ is the difference between the densities in the congested and non-congested regions divided by $x(t)$. We have assumed that $\rho_1(t)$ is a slowly time varying function. By dividing the time into segments of interval T and assuming that within each internal $\rho_1(t)$ is a constant, we can derive a recursive estimation algorithm for $x(t)$:
By integrating both sides of the above equation on a small time interval $[t_c + (n-1)T, t_c + nT)$, we have

$$\int_{t_c+(n-1)T}^{t_c+nT} \rho(\tau)\frac{dx(\tau)}{d\tau}d\tau = -\int_{t_c+(n-1)T}^{t_c+nT} f(\tau - \frac{x(\tau)}{v_1})d\tau + \rho_2 v_2 T.$$

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Furthermore, assuming that $\rho(t)$ in $[t_c + (n-1)T, t_c + nT)$ is constant and denoted as $\bar{\rho}_n$, we get

$$\bar{\rho}_n[x(t_c + nT) - x(t_c + (n-1)T)] = -\int_{t_c+(n-1)T}^{t_c+nT} f(\tau - \frac{x(\tau)}{v_1})d\tau + \rho_2 v_2 T, \qquad (3)$$

for $n = 1, 2, \ldots, N$. The two terms $x(t_c + nT)$ and $\bar{\rho}_n$ cannot be uniquely solved from Equation (3). The right side of this equation can be obtained, provided that the flow of the flow detector is measured and extrapolated. Because of the slow variation of $\rho(t)$, we thus use the previous value $\bar{\rho}_{n-1}$ to approximate $\bar{\rho}_n$. Substituting $\bar{\rho}_{n-1}$ for $\bar{\rho}_n$ in Equation (3), we obtain the following approximation for $x(t_c + nT)$:

$$x(t_c + nT) = x(t_c + (n-1)T) - \frac{1}{\bar{\rho}_{n-1}}[\int_{t_c+(n-1)T}^{t_c+nT} f(\tau - \frac{x(\tau)}{v_1})d\tau + \rho_2 v_2 T], \qquad (4)$$

where

$$\bar{\rho}_{n-1} = \frac{-\int_{t_c+(n-2)T}^{t_c+(n-1)T} f(\tau - \frac{x(\tau)}{v_1})d\tau + \rho_2 v_2 T}{x(t_c + (n-1)T) - x(t_c + (n-2)T)}. \qquad (5)$$

We use the inflow to the congestion in time interval $[t_c + (n-2)T, t_c + (n-1)T)$ for $\bar{\rho}_{n-1}$ estimation, and the inflow in the interval $[t_c + (n-1)T, t_c + nT)$, for an approximation of the tail boundary $x(t_c + nT)$. The tail boundary of the congestion can thus be estimated by the following regressing algorithm.

1. Measuring the flow $f(.)$ of the flow detector. If necessary, the flow can be obtained by extrapolation.

2. $n = 0$, we measure $\bar{\rho}_0$ from Equation (5).

3. For $n = 1, 2, \cdots, N$; We estimate $x(t_c + nT)$ using Equation (4) and then obtain $\bar{\rho}_n$ from Equation (5).

An interesting special case of the above derivation is the inflow to the congestion being a constant flow. Because of the constant inflow to the congestion, we have

$$\int_{t_c+(n-2)T}^{t_c+(n-1)T} f(\tau - \frac{x(\tau)}{v_1})d\tau = \int_{t_c+(n-1)T}^{t_c+nT} f(\tau - \frac{x(\tau)}{v_1})d\tau. \qquad (6)$$

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Then, substituting $\bar{\rho}_{n-1}$ in Equation (5) for that in Equation (4), and using Equation (6), we obtain the constant tail boundary velocity:

$$x(t_c + nT) - x(t_c + (n-1)T) = x(t_c + (n-1)T) - x(t_c + (n-2)T).$$

With constant inflow to the congestion, the tail shock wave has a constant velocity and the tail boundary is a linear function of t. In the time-space diagram, the evolution of the tail boundary corresponds to a line.

3.2 Head Shock Wave

Here, we derive the head boundary from the shock wave equation. From the head boundary curve $(t, y(t))$, the amount of time for a car in the congestion to pass out of it can be obtained. The velocity of $y(t)$ can be derived by a shock wave propagation equation. The schematic diagram is given in the bottom subfigure of Figure 4. Let the mean density and velocity in the non-congested segment be denoted as ρ_3 and v_3, respectively. We then select a relatively large segment in the non-congested road segment such that we can assume both means are constants. According to the shock wave equation, we have outflow $\mu(t)$ of the congestion as

$$\mu(t) = \rho_2(v_2 - \frac{dy(t)}{dt}) = \rho_3(v_3 - \frac{dy(t)}{dt}). \tag{7}$$

Rearranging the above equation, we attain the shock wave for $y(t)$:

$$\frac{dy(t)}{dt} = \frac{\rho_2 v_2 - \rho_3 v_3}{\rho_2 - \rho_3}. \tag{8}$$

Note that the head shock wave $\frac{dy(t)}{dt}$ is a constant velocity, irrespective of time. By integrating Equation (8) from any reference time t_r, we have the head curve

$$y(t) = y(t_r) + \frac{\rho_2 v_2 - \rho_3 v_3}{\rho_2 - \rho_3}(t - t_r). \tag{9}$$

Thus, $(t, y(t))$ is a line in time-space. Substituting Equation (8) into Equation (7), we conclude that the flow departing from the congestion $\mu(t) = \mu$ is a constant.

4 Travel Time Estimation

The travel time of a car to its destination is composed of the time for the car to meet the congestion, the time the car is in the congestion, and the time to reach its destination after

it leaves the congestion. We estimate the travel time of each component as follows.

When Our Car Will Enter the Congestion

Assume that our car is not in the congestion, but is heading towards it. The time t_a when our car will meet the congestion can be obtained by solving

$$\frac{x(t) - p(t_c)}{v_1} = t - t_c,$$

where $p(t_c)$ is the current car position. The predicted elapsed time Δt_a for our car to meet the congestion is

$$\Delta t_a = t_a - t_c. \tag{10}$$

When Our Car Will Leave the Congestion

According to the constraint of a one-lane highway, our car can only leave the congestion when all the cars in front of it have left. The number of cars in front of our car when we enter the congestion at time t_a is

$$l(t_a) = [y(t_a) - x(t_a)]\rho_2, \tag{11}$$

where the length of the congestion at time t_a is $y(t_a) - x(t_a)$. Let the time our car leaves the congestion be t_b. Then, according to our first-in-first-out rule, we have

$$\int_{t_a}^{t_b} |\mu(t)| dt = l(t_a).$$

Since $\mu(t) = \mu$ is a constant, we have the duration of time Δt_{ab} that elapsed while our car was in the congestion

$$\Delta t_{ab} = t_b - t_a = \frac{l(t_a)}{|\mu|}. \tag{12}$$

Thus, our car will leave the congestion at time

$$t_b = t_a + \frac{l(t_a)}{|\mu|}. \tag{13}$$

The point where our car will leave the congestion is $y(t_b)$. It can be obtained either from calculating car velocity inside the congestion, $x(t_a) + v_2(t_b - t_a)$, or from $y(t_b)$.

The End of Congestion

Precisely identifying the beginning and end of congestion is not easy. Here, we present a convenient definition of the end of congestion. We define the end of it as the time-space point at which the head curve $(t, y(t))$ crosses the tail curve $(t, x(t))$ of the congestion. Let the end time be t^*, where is the solution of $x(t) = y(t)$. Knowing the end time aids in estimating travel time. Even if there is congestion ahead of us, we may not encounter it, as it may disappear before we reach its tail boundary. This is the condition when

$$t^* \leq t_a,$$

where t_a is the time when our car will meet the congestion. In this case, our car will maintain an approximately constant speed in traveling to its destination - as if the congestion had not happened. The travel time Δt_c will be

$$\Delta t_c = t - t_c = \frac{x_{end} - p(t_c)}{v_1}. \tag{14}$$

Within the Congestion

If our car is in the congestion, then we do not need $x(t)$ for travel time prediction. The head boundary of the congestion, $y(t)$, can be obtained in a similar way as that described previously. The number of cars jammed in front of us at current time, t_c, is

$$l(t_c) = \frac{y(t_c) - p(t_c)}{\rho_2},$$

where $p(t_c)$ is the current position of our car, and ρ_2 is the average density in the congestion. This result is derived from Equation (11) with t_a being replaced by t_c. In this case, traveling from $p(t_c)$ to the destination x_{end} will take

$$\frac{l(t_c)}{|\mu|} + \frac{x_{end} - y(t_c)}{v_3}. \tag{15}$$

Travel Time Prediction Algorithm

Summarizing these components, the total travel time through the congestion is estimated by the aforementioned equations. Our algorithm is simple and straightforward. It is comprised of the following steps:

Algorithm

1. Calculate $x(t)$ and $y(t)$ for $t \geq t_c$.

2. Determine whether our car is approaching, or within the congestion.

3. If our car is approaching the congestion, determine the time t_a when our car will meet the tail curve $(t, x(t))$. Determine the end time t^* of the congestion.

   3.1 If $t^* \leq t_a$, then our car will not encounter the congestion, as the congestion will disappear before we reach it. The travel time is estimated by Equation (14).

   3.2 If we will encounter the congestion, determine the amount of time up to meeting the head curve, $(t, y(t))$, and calculate the travel time using

$$\Delta t_a + \Delta t_{ab} + \frac{x_{end} - y(t_b)}{v_3}, \tag{16}$$

where Δt_a and Δt_{ab} are given in Equations (10) and (12), respectively. The last term in the above equation is the time it takes for our car to reach its destination after leaving the congestion.

4. If our car is in the congestion, use Equation (15) for travel time estimation.

5   Simulation and Performance Evaluation

We use a CA model to simulate the motion of cars on a circular single-lane road. A diagram of a CA model with parameters is given in Figure 5. In the model, the road is segmented into sites; each site is either empty, or occupied, by one car with velocity $v(t)$, $0 \leq v(t) \leq v_{max}$. At each time step, $v(t)$ is updated by the following rules:

$$v(t+1) = \min\{v(t) + a, v_{max}, gap\},$$

where gap is the distance (number of empty sites) to the front car, and a is the acceleration of the cars. Besides this updated rule for velocity, there is a probability of brake p_b to decrease the $v(t+1)$ so that

$$v(t+1) = \max\{v(t+1) - 1, 0\}.$$

The position of a car at time step $t+1$ is

$$p(t+1) = p(t) + v(t+1).$$

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In general, the model can be used to simulate actual highway traffic situations with parameters $v_{max} = 5$, $a = 1$, and $p_b = 0.5$. With these parameters, each cell corresponds to about $7.5m$ on a road. In our simulation, the circular road is comprised of $2,000$ sites, and the density of cars on the road is 0.2. Note that choosing a cell site of more than $2,000$ can effectively eliminate the circle boundary artifacts of our CA model [4].

Figure 6 illustrates an example of predicting the parameters in congestion. A flow detector is deployed at cell site 740. In the figure, the locus of our car is marked by $\triangle$. Figure 6(a) shows the predicted head and tail curves from the time at 43 (present time) until the time at 55 (the point at which our car enters the congestion). Figure 6(b) shows samples of pairs of points that record the predicted time-space coordinates of when our car enters and leaves the congestion. Figure 7 shows the comparison of the predicted travel time that we will be in congestion to that of the actual travel time.

Figure 8 shows simulation results of estimating travel time through congestion using various probe car densities. Our probe begins at cell 733 at time 43 and our destination is cell 800. The actual travel time of this trip is 80, which is shown in the figure by a horizontal dashed line. In our simulation, $\mu = -0.45$ is given after preprocessing. The vertical axis gives the predicted travel time of the trip at each time instance. One can observe that when the probe car density is 0.1 or 0.2, the mean of our predicted travel time at the beginning of our trip is lower than the actual travel time. Figure 9 gives an example of a time-space diagram for probe car density 0.2, which explains this under-estimated travel time. The locus of our car is marked by $\triangle$ and our trip begins at time 43. Note that the length of the congestion enclosed by the points c and d at time 43 is smaller than the actual congestion length at that time. Point c is extrapolated from point a, since a is in the closest trace of a probe car leaving the congestion. It is clear that c occurs earlier than the actual head of the congestion. Similarly, point d is extrapolated from point b, as it is the closest trace of a probe car entering the congestion. Because we under-estimate the congestion length, we predict a shorter travel time to the destination. A better estimation of the congestion length is obtained after time 48, when a probe car leaves the congestion and reports the correct head location e of the congestion. After this occurs, the estimation error is corrected. In other examples, extrapolating the tail of the congestion may be the cause of the error. Errors in

estimating congestion length depend on probe car density and on the distance of the closest probe cars from our car. The relative error at each time of varied ρ_p is given in Figure 10. The vertical axis gives the percentage of relative errors which is computed by

$$\frac{\sqrt{\frac{\sum(T_p(t)-T_t)^2}{N}}}{T_t} \times 100$$

where N is the number of simulated observations for a time instant, T_t is the true travel time, and $T_p(t)$ is the predicted travel time at time t. Note that by increasing the probe car density, the relative prediction error will be reduced.

Constant Flow Assumption

In this simulation, we demonstrate the effect of a constant inflow assumption on the travel time estimation by our algorithm. To eliminate the effect of the lengths of the non-congested road segments on the final result, we only compare our estimated time within the congestion with varying inflows. We assume that the probe car density is 1 and the inflow to the congestion is a constant. We then use this constant inflow in our algorithm to estimate the duration of time within the congestion at any one instance. Since the inflow only affects the travel time estimation of our algorithm at the period when a probe car is approaching the tail of the congestion, our results are based on simulations during this period of time. Figure 11(a) superimposes the various estimation results. From the results, we found that all measurements yield similar curves, and that the estimated time within the congestion varies with the inflow estimation in such a way that if the inflow is under-estimated, then the passing time is under-estimated, and vice versa.

Figure 11(a) compares the variations of the instantaneous estimation, while Figure 11(b) compares the average estimation error of "within-the-congestion time" to the inflow error. The average estimation error, relative to the actual within-the-congestion time, increases as the inflow error is more than 10%, relative to actual inflow to the congestion. Figure 11(b) shows that our method can tolerate a relative inflow estimation error of about 10%.

Compare with Another Estimation Method

We compared our method with a scenario in which the most recent travel time of the road

segment is reported by a previous car, as illustrated in Figure 12. Let the most recent travel time be t_I; the length of the road segment is L. The current car in this scenario estimates the travel time by summing up its own travel time to the current point p, $t_c(p)$, and the estimated travel time of the previous car from the current position to the destimation. The latter is $\frac{L-p}{L}t_I$, which is a simple linear estimation. Thus, the estimated travel time of the current car is

$$t_e(p) = t_c(p) + \frac{L-p}{L}t_I.$$

Figure 13 is the plot of this estimated time as a function of the current car position, p. From the figure, the current car position is divided into three segments, $[730, 765]$, $[766, 770]$, and $[771, 803]$. The estimated time t_e in each segment can be approximated as a line. The slopes of these lines can be derived by taking the derivative of the above equation with respect to p:

$$\frac{dt_e(p)}{dp} = \frac{dt_c(p)}{dp} - \frac{t_I}{L},$$

where $\frac{t_I}{L}$ is a constant which is irrelevant to p; and $\frac{dt_c(p)}{dp}$ is the inverse of the velocity of the current car at p. The velocities of a car heading towards the congestion, within the congestion, and leaving the congestion have different constants - corresponding to the car in different segments. As shown in Figure 13, the proposed estimation method yields a better travel time estimaiton.

6   Conclusion

The dynamic traffic parameters of a congested road segment can be characterized by the boundaries of the congestion. The flow detector provides data about the congestion inflow. We show that we can extrapolate the tail curve of the congestion from the inflow to the congestion. We also show that the head curve of the congestion can be effectively approximated as a line in a time-space diagram. Any sensor that can assess the inflow to the congestion is effective for our algorithm. Our algorithm is straightforward and can be easily implemented in a control station, or a probe car. The performance of the algorithm is measured using

a CA simulation model with different probe car densities and its accuracy is demonstrated. A probe car makes a trace in a time-space diagram. Whether there is a sampling theorem relating the probe car density to a reliable travel time estimation is not known and is worth further investigation.

References

[1] M. Van. Aerde, B. Hellinga, L. Yu, and H. Rakha, "Vehicle Probes as Real-Time ATMS Sources of Dynamic O-D Travel Time Data," *Proceedings of the ATMS Conference*, St. Petersburg, Florida. pp. 207- 230, 1993.

[2] R. Bin, N. M. Pouphail, A. P. Tarko, and D. E. Boyce, "Toward a Class of Link Travel Time Function for Dynamic Assignment Models on Signalized Networks," *Transportation Research Part B*, Vol. 31, No. 4, pp. 277-290, 1997.

[3] F. W. Cathy and D. J. Dailey, "Transit Vehicles as Traffic Probe Sensors," *IEEE International Transporation Systems Conference Proceedings*, Aug. 25-29, 2001, Oakland, USA.

[4] S. P. Chen and D. W. Huang, "Noise Properties in Nagel-Schreckenberg Traffic Model," *Physical Review E*, Vol. 63, 2001.

[5] H. K. Chen, "Dynamic Travel Choice Models," *Springer*, 1999.

[6] B. Coifman, "Time Space Diagrams for Thirteen Shock Waves," *PATH Project Program of University of California*, http://www.cs.berkeley.edu/ zephyr, 1997.

[7] C. F. Daganzo, "Properties of Link Travel Time Functions under Dynamic Loads," *Transportation Research Part B*, Vol. 29, No. 2, pp. 95-98, 1995.

[8] P. Kachroo and K. Ozbay, "Feedback Control Theory for Dynamic Traffic Assignment," *Springer*, 1999.

[9] B. Kerner, "Traceing and Forecasting of Congested patterns for Highway traffic Management," *IEEE International Systems Conference Proceedings*, pp.88-93, Oakland, Aug. 25-29, 2001.

[10] M. J. Lighthill and G. B. Whitham, "On Kinematics Waves II. A Theory of Traffic Flow on Long Crowded Road," *London, Proceedings Royal Society*, A229, pp. 317-345, 1956.

[11] T. Lomax, S. Tuner, G. Shunk, H. Levinson, R. Pratt, P. N. Bay, and G. B. Douglas, "Quantifying Congestion," *National Coorperative Highway Research Program*, Report 398, 1997.

[12] K. Nagel and M. Schreckenberg, "A celluar automaton model for freeway traffic," *J. Phys. I*, 2, 1992, pp. 221-2229.

[13] P. I. Richard, "Shock Waves on the Highway," *Operation Research*, Vol. 4, pp. 42-51, 1956.

[14] A. Sen, P. G. Thakuriah, X. Q. Zhu, and A. Karr, "Variances of link travel time estimates: implications for optimal routes," *International Transactions in Operational Research*, Vol. 6, pp.75-87 1999.

[15] M. Treiber, A. Hennecke, and D. Helbing, "Congested Traffic States in Empirical Observations and Microscopic Simulations," *Phycical Review E*, Vol. 62(2), pp.1805-1824, 1999.

[16] H. Sun, H. X. Liu, H. Xiao, and B. Ran, "Short Term Traffic Forecasting Using the Local Linear Regression Model," *UCI-ITC-WP-02-2*, Jul., 2002.

[17] S. C. Wirasinghe, "Determination of Traffic Delay from Shock Wave Analysis," *Transportation Research*, Vol. 12, pp. 343-348, 1978.

[18] X. Zhang and J. Rice, "Short-Term Travel time prediction Using A time-Varying Coefficient Linear Model," http://stat-www.berkeley.edu/users/rice/xTTfinal.pdf, 2001.
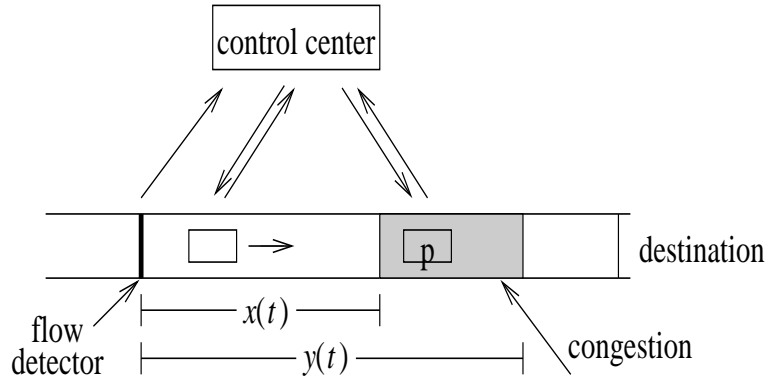
Figure 1: The framework of our travel time estimation system. The flow detector performs a one-way communication, while each probe car performs a two-way communication. A probe car sends its position to the central station, which then transmits to the car the estimated travel time from its current positon to its destination.
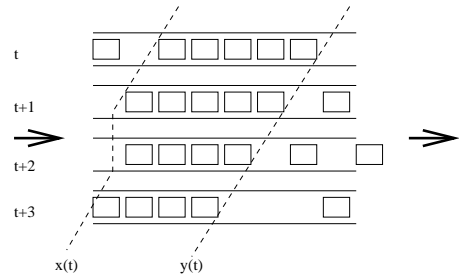
(a)
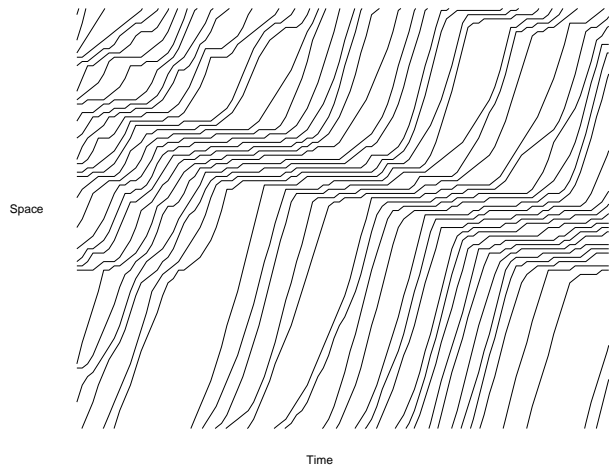
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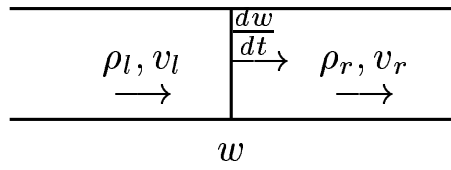
(c)

Figure 2: (a) Congestion boundaries move backward. (b) A time-space pattern of the congestion. (c) Traffic shock wave $\frac{dw}{dt}$.
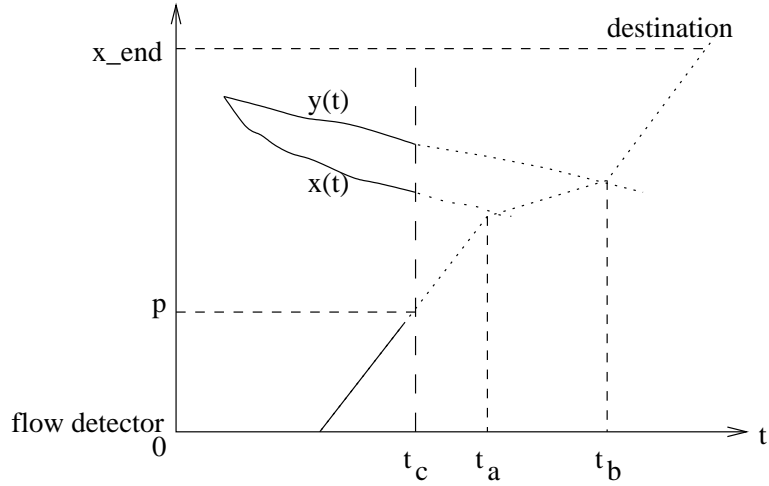
Figure 3: Our problem is to extrapolate the time-space of the congestion beyond the current time t_c. The segment of time-space to the right of the dashed line at t_c will be extrapolated. $x(t)$ is the curve of the tail boundary, and $y(t)$ is the head boundary of the congestion. t_a and t_b predict when our car will meet and leave the congestion, respectively.
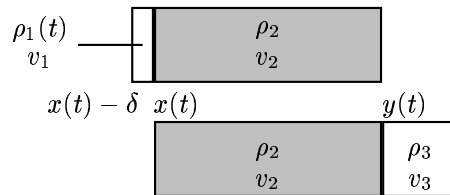
Figure 4: The top subfigure is the diagram for estimating the tail curve $x(t)$, and the bottom subfigure is the estimation of the head curve $y(t)$.
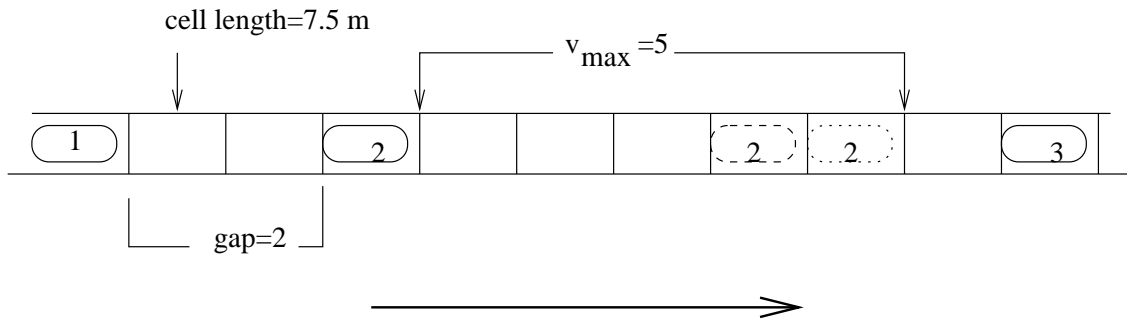
Figure 5: In a CA model, a car moves from one cell to the next, according to a given maneuvering rule.

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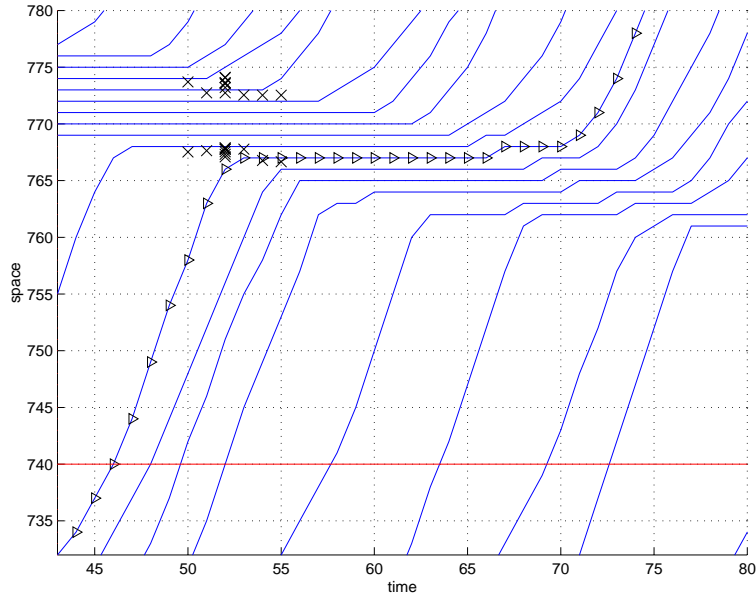
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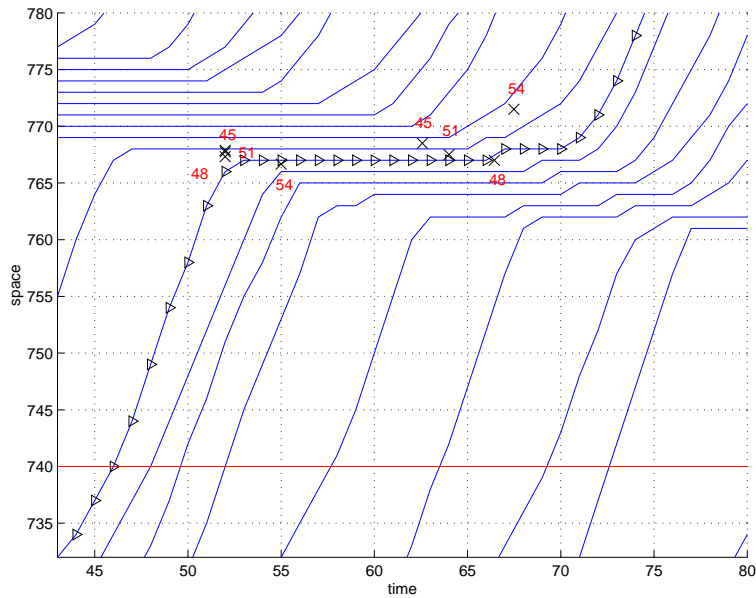
(b)

Figure 6: (a) The predicted head and tail curves from time 43 (present time) up to time 55 (the point where our car enters the congestion). Note that many predicted points are located at time 53. (b) A pair of points recording the predicted time-space coordinates of entering and leaving the congestion. Note that many points are located at time 53. The number beside × is the time when × is predicted.
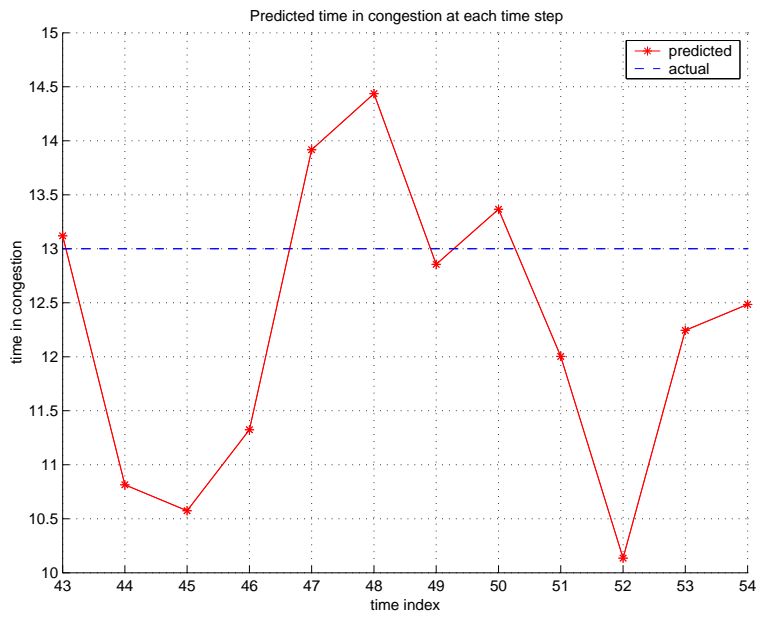
Figure 7: Comparison of predicted travel time to the actual travel time in congestion when the probe car density is 1. The actual travel time is represented by the horizontal dashed line. The vertical axis indicates the predicted travel time in the congestion. The horizontal axis is the time when the prediction is made.
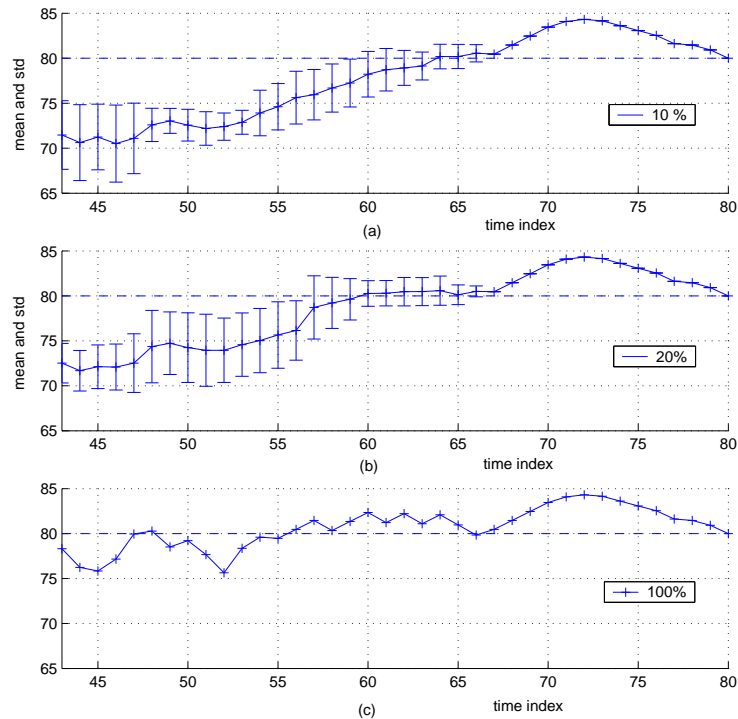
Figure 8: The predicted travel time with different probe car densities. The trip begins at time 43 and the true travel time is 80. The vertical bars in the top and middle subfigures give the range of one standard deviation. (a) The probe car density is 0.1. (b) The probe car density is 0.2. (c) The probe car density is 1. Increasing the probe car density gives a better estimation of the travel time through the congestion.
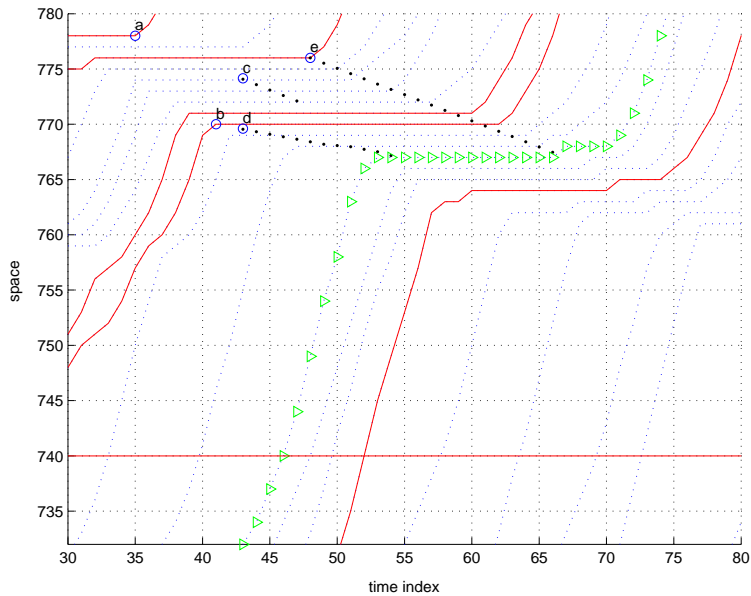
Figure 9: An example of the time-space diagram for probe car density 0.2. Our trip begins at time 43. The traces of probe cars are represented by solid lines, and those of blind cars by dashed lines. × represents the predicted head and tail of the congestion. Point c is extrapolated from point a, and d is extrapolated from b. b and a are the respective points of the closest probe cars entering and leaving the congestion. Congestion lengths are under-estimated from time 43 to 48.
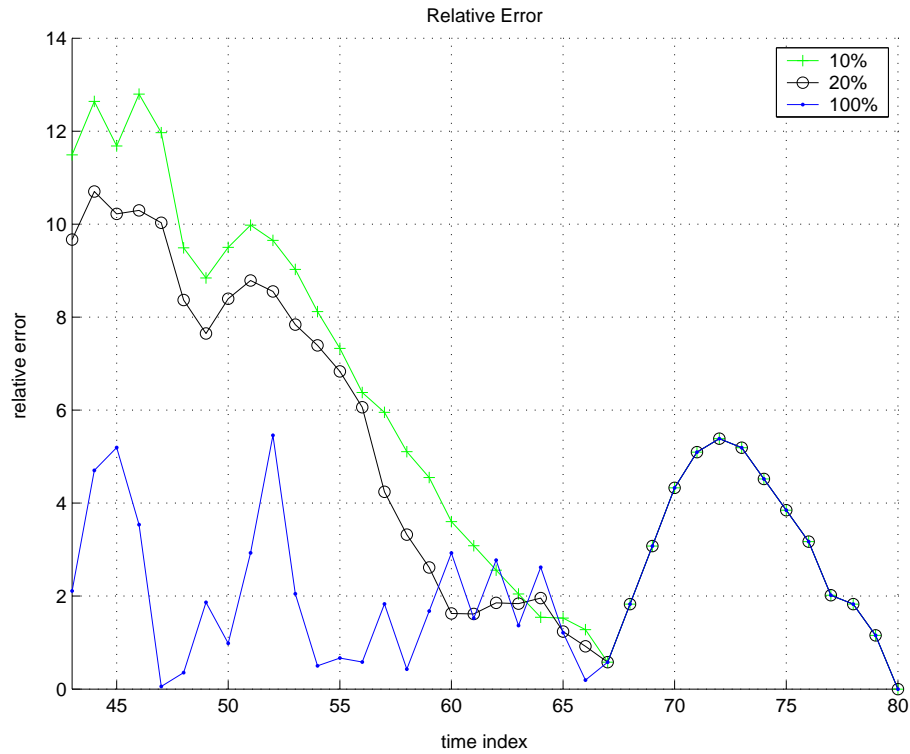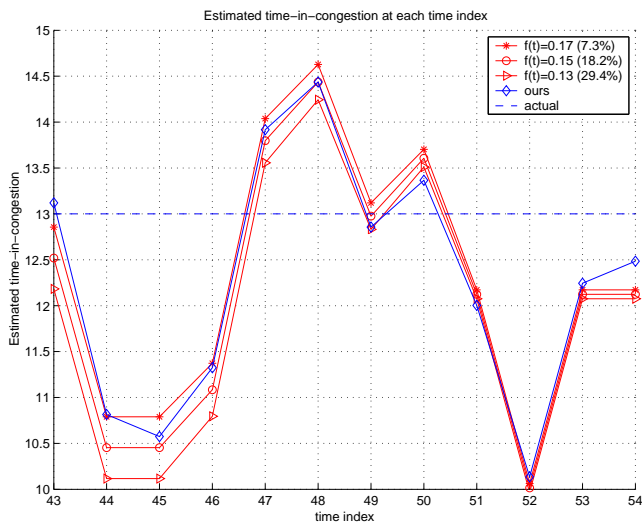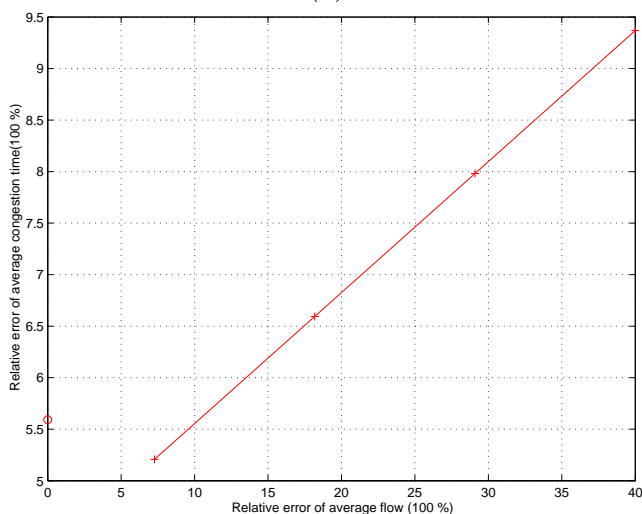
Figure 10: The percentage of the relative error of the predicted travel time with different probe car densities with values 0.1, 0.2 and 1. The relative errors after time 65 for all densities are the same, since our car has left the congestion. Any errors after time 65 arise from the estimation of velocity v_3 after the congestion.

(a)

(b)

Figure 11: (a) From time 43 to 54, our simulation car is approaching the tail boundary of the congestion. The vertical axis is the estimated time for the car within the congestion. The parameter of the plots is the inflow to the congestion. In the legend, the number in each set of parentheses is the relative inflow error to the congestion between the time from 43 to 54. (b) The relative error of the within-the-congestion time is plotted against the relative inflow error to the congestion. When the relative inflow error is above 10%, the relative error of within-the-congestion time estimation increases as well. The circle on the vertical axis corresponds to the measurement using the measured inflow, whereas the rest of the points on the line are obtained by using the assumed constant inflow.
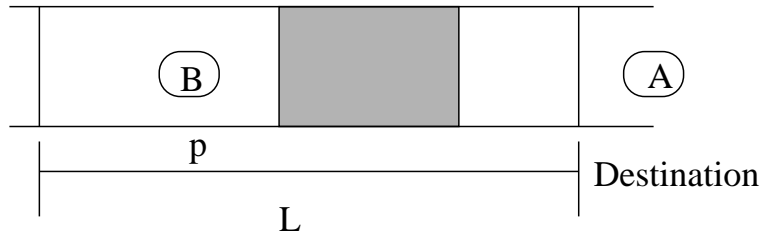
Figure 12: The travel time of the most recent car (A) is used for car (B) to estimate its travel time. The current position of car B is at p. The length of the road segment is L.
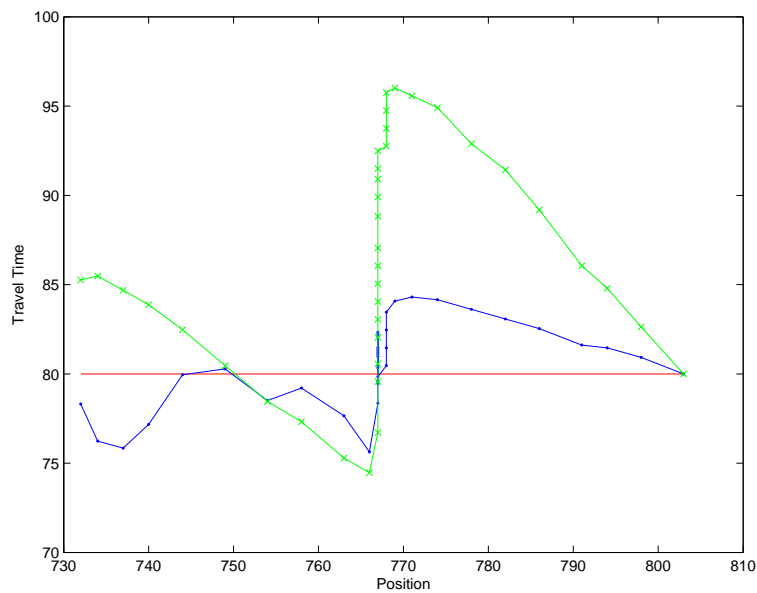

Figure 13: Travel time estimation comparison. The curve with the larger variations (marked by "×") is obtained by using the scenario illustrated in Figure 12, while the other curve with smaller variations (marked by ".") is obtained by using our method. The curve of flat travel time is the actual travel time of car B, which is a reference to compare with different methods. At any time instance, our method has a better estimation time than the other method.