# Robust EZW Image Compression using Rate-Distortion Analysis

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#### Abstract

The EZW(Embedded Zerotree Wavelet)-like image compression algorithms lack the error-resilience ability in noisy transmission environments. We propose the channel-optimized source coding scheme to improve the robustness of them. First, a block-based method is adopted to localize the error effects. Then we assign bits to each block by applying dynamic bit allocation to the block-based EZW algorithm based on the rate-distortion functions computed from the channel noise models. The performance of our method was evaluated on both the binary symmetric channel and burst noise channel models.

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# 1 Introduction

A modern communication system consists of the source coding and the channel coding. These two parts are usually designed separately because of Shannon's *separation principle* [1]. A design separating source coding from channel coding can potentially be inefficient in practice. For example, the Embedded Zerotree Wavelet (EZW) coder [2] is known to be a state-of-art image compression algorithm, but even a single bit error occurring in the transmitted bitstream will drastically affect the overall decoded image quality. In this case, channel coding has to be designed so as to guarantee almost error-free performance, which will result in many redundant bits. Therefore, the joint consideration of source and channel coding may be necessary and useful [3]. One class of joint source-channel coding is *channel-optimized source coding*, which considers the channel properties in the source coding design. Based on the idea of channel-optimized source coding, we propose a channel-optimized source coding adaptation for EZW-like progressive codec using the rate-distortion functions. Our innovation lies in considering the statistics of the channel noise in rate-distortion analysis.

Recently, the best low bit-rate image compression method has been known to be the Embedded Zerotree Wavelet (EZW) algorithm [2]. Although several improved methods for EZW [4] have been proposed, their basic ideas are similar. Therefore, we have concentrated on improving the robustness of the EZW algorithm. Our proposed method can be extended to the other EZW-like progressive codec with minor modification. The main problem with EZW over a noisy transmission channel is that once a single bit error occurs in the encoded bitstream, the bits before the error can be correctly decoded, but all the bits after the error are useless, which can drastically affect the overall image quality if they are decoded. The other problem with EZW is that we do not know whether errors have occurred. Compared to EZW, the block-based image compression methods, like JPEG, have the advantage that one single error in the bitstream only affects the quality of the block in which the error occurs. For the above reasons, blockbased EZW has been proposed [7].

The idea of block-based EZW is to divide the original image into groups of square blocks, and each block is coded using the EZW method. Like JPEG, this block-based EZW method limits error propagation to a single block and, hence, improves the robustness of the EZW algorithm. However, the PSNR(Peak Signal-to-Noise Ratio) performance of the block-based EZW method in noiseless condition will be slightly less than that of the original EZW method. Thus, an optimal bit allocation is usually conducted to assign variable bits to each block and improves the performance of the block-based codec. We will show that the performance of block-based EZW can be further improved if channel noise statistics are used in bit allocation which assigns bits to each block. In other words, the effects of channel errors are considered in our ratedistortion analysis for all blocks, and these rate-distortion functions are used in the dynamic bit allocation procedure.

One interesting question is whether the progressive property is still satisfied in each block in our block-based scheme. That is, whether the number of bits allocated to each block increases as the total number of bits assigned to the whole image increases. We will show that the answer to this question is positive.

The performance of our proposed method was evaluated in the binary symmetric channel and burst noise channel. Our method outperforms the original EZW method in both noise models about 5 dB when the image is coded at 1bpp(bit per pixel) rate and the average bit error rate of the transmission channel is  $10^{-3}$  and  $10^{-4}$ .

In Section 2, we introduce the proposed block-based method for EZW-like progressive codec and discuss the implementation of the block-based wavelet transform and the dynamic bit allocation. In Section 3, we discuss the dynamic bit allocation with rate-distortion constraints and analyze the rate-distortion functions with binary symmetric channel and burst noises. In Section 4, simulation and experiment results are discussed. Finally, conclusions are given in the last section.

# 2 The Block-Based Channel-Optimized Source Coding

In this section, we will propose a framework of block-based channel-optimized source coding based on EZW. Although we will mainly discuss EZW, our method is applicable to other wavelet-based progressive codec with minor modifications. This framework is designed to improve the robustness of the EZW algorithm in noisy channel environments.

#### 2.1 A block-based EZW scheme

As we stated earlier, we will adopt the block-based modification for EZW to improve the robustness of transmission against channel noises. This means that an image is first divided into blocks, then the wavelet coefficients of each block are obtained and organized into an individual EZW bitstream. One can obtain the wavelet coefficients of an image block, by a similar ideal from [5] and [7], from relocation of the corresponding wavelet coefficients of the image. See Fig. 1 for the correspondence between the wavelet decomposition of an image and the block formation. By relocation the wavelet coefficients of the image in Fig. 1(a), one will obtain the wavelet coefficients of the block indicated in Fig. 1(b). In general, for a  $2^m \times 2^m$  image, one can divide it into totally  $(2^{m-k} \cdot 2^{m-k})$  blocks by applying k wavelet decompositions of it and associate each wavelet coefficient in the DC block with its children in the three orientations. These relocated wavelet coefficients are equivalent to the wavelet coefficients of each divided block of size  $2^k \times 2^k$  in the original image.

In [5], the authors assign equal bits to each block. However, it is more reasonable to assign different bits to each block according to its rate-distortion function, which is calculated based on the channel statistics, which will be discussed later. Therefore, we have variable-length bitstreams for all blocks after conducting unequal bit allocation. Suppose we concatenate by cascading these variable-length blocks into a single bitstream. One error bit in some block will affect all the bits in the bitstream after the error bit because of the loss of synchronization information about the block lengths. Therefore, we propose a scheme similar to EREC(Error-Resilient Entropy Code) [6] which aligns these variable-length blocks into equal-length slots. These slots are then interleaved to form a single bitstream. EREC has been shown to be effective when more important information is transmitted near the beginning of each variable-length block [6], which is similar to EZW compression. In addition, we must add an EOB(end of block) symbol to the EZW coding of each block to signal the end of a block. The other benefit of adding an EOB symbol is that an error can be detected earlier. In fact, EZW has an additional ability of error detection in its significance map decoding process, which is not mentioned in the original work of EZW. When we decode a POS or a NEG symbol for a coefficient which was already found to be significant in the previous pass, some errors will certainly occur. This knowledge can enable early detection of errors.

### 2.2 The optimized bit allocation for block-based source coding

The block-based EZW method as stated in the previous subsection produces individual bitstreams for all blocks. Then, these coded bitstreams are sent through noisy channels with specific noisy properties. In our channel-optimized source coding framework, the number of bits from a given bit budget allocated to each block will be assigned according to the corresponding channel statistics to achieve the least overall distortion. We will discuss the bit allocation considering the channel effects in the next section. For the moment, we will have a brief discussion of the bit allocation formula without considering of channel noises.

Let  $D_i(n)$  denote the distortion in block *i* with *n* bits correctly decoded.  $D_i(n)$  can be calculated based on the MSE (mean square error) between the original and decoded wavelet coefficients. The bit allocation problem involved in assigning  $b_i$  bits to block *i*, i = 1...K, with a total bit budget R, can be formulated as follows:

$$min \qquad \sum_{i=1}^{K} D_i(b_i), \tag{1}$$
$$subject to \qquad \sum_{i=1}^{K} b_i \le R,$$

Many conventional dynamic programming skills can be used to solve the above problem [10].

A desired property of the above bit allocation procedure is that, as the total bit budget R increases, the number of bits allocated to each block also monotonically increases. This property is especially important for a progressive coder since it preserves the progressive property in each block after the bit allocation. To prove this fact, we refer to two results in [9]. The first theorem is a known result of the Lagrange multiplier method [9].

Theorem 1: For any  $\lambda \geq 0$ , the solution  $b_i^*(\lambda), i = 1...K$ , to the unconstrained problem

$$\min_{\{b_i, i=1...K\}} \sum_{i=1}^{K} D_i(b_i) + \lambda \sum_{i=1}^{K} b_i$$
(2)

is also the solution to the constrained problem (1), with the constraint  $R = \sum_{i=1}^{K} b_i^*(\lambda)$ .

For a given  $\lambda$ , the solution to (2) can be obtained by minimizing each term of the sum in (2) separately. We also need the following lemma from [9] to complete our theorem.

Lemma 1: Let D(b) be a real-valued function over some bounded and closed domain Z in the real line. Let  $b_1$  be a solution to

$$\min_{b\in Z} \{D(b) + \lambda_1 b\},\$$

and let  $b_2$  be a solution to

$$\min_{b\in Z} \{D(b) + \lambda_2 b\};$$

then,

$$(\lambda_1 - \lambda_2)(b_1 - b_2) \le 0$$

for any function D(b).

We prove our theorem in the following.

Theorem 2: For bit budgets  $R_1$  and  $R_2$ , let their corresponding optimal solutions to (1) be  $\{b_{11}, b_{12}, \dots, b_{1K}\}$  and  $\{b_{21}, b_{22}, \dots, b_{2K}\}$ . If  $R_1 < R_2$ , then  $b_{1i} \leq b_{2i}$ , i = 1...K.

*Proof:* Let the solution  $\lambda$  corresponding to  $R_1$  and  $R_2$  be  $\lambda_1$  and  $\lambda_2$ , respectively. Since  $R_1 < R_2$ , there exists some block m, such that  $b_{1m} \leq b_{2m}$ . From Lemma 1,

$$(\lambda_1 - \lambda_2)(b_{1m} - b_{2m}) \le 0,$$

we derive  $\lambda_1 \geq \lambda_2$ . Applying Lemma 1 and  $\lambda_1 \geq \lambda_2$  to other blocks, we derive

$$b_{1i} \leq b_{2i}, i = 1...K.$$

The above theorem indicates that the number of bits allocated to each block will be increased once the total bit budget is increased, and will be decreased if the amount of bit budget is decreased. This theorem provides the progressive property for each block after the bit allocation procedure is conducted.

## 2.3 An extension to the multi-channel framework

The block-based EZW scheme stated in the previous section can be extended to a multi-channel framework, where multiple channels with different noise characteristics are used to transmit the coded bitstreams.

In the block-based EZW scheme, the whole image is divided into small square blocks and these blocks are coded with EZW method. We can further regroup these blocks into partitions, producing separated bitstreams for the partitions and transmitting them through channels with different noise characteristics. The block organization and assignment to channels are also interesting issues. For example, blocks containing visual significant features can be grouped and assigned to a more reliable channel.

The overall flow diagram of our block-based EZW method is depicted in Fig. 2. The extension to the multi-channel framework is depicted in Fig. 3.

The dynamic bit allocation formula in (1) can be modified as follows:

$$\min \sum_{i=1}^{K} D_i(b_i) + k_i \cdot C_i(b_i), \qquad (3)$$

$$subject \ to \qquad \sum_{i=1}^{K} b_i \le R,$$

where  $C_i(b_i)$  is the cost function for channel *i*, and  $k_i$  is used to balance the distortions and the cost functions. Note that the cost function  $C_i(b_i)$  is the cost of sending  $b_i$ bits through the channel responsible for block *i*, where the cost can be money, time, or other meaningful units. For example, a high quality transmission line usually costs more money than a noisy channel. The multi-channel extension shows an application of our proposed method.

# 3 Rate-Distortion Analysis Considering Channel Errors

In this section, we will analyze the rate-distortion functions of a bitstream generated by a progressive coder when it is sent through a noisy channel. We will consider the rate-distortion functions calculated in the noiseless channel, called the original ratedistortion functions, and the rate-distortion functions calculated in the noisy channel, called the expected rate-distortion functions. Since the channel is contaminated with noise, the expected rate-distortion functions will be calculated statistically. Two kinds of channel errors will be examined: the binary symmetric channel(BSC) noise and burst noise.

Let D(n) be the distortion of an image block when n bits are assigned to encode the image block by a progressive coder and decoded without any errors. Consider transmitting these n bits through a noisy channel. The expected rate-distortion function is formulated as follows:

$$D^{*}(n) = \mathbf{E}\{D(n)\} = \sum_{i=1}^{n} P(i)D(i-1) + P^{0}(n)D(n),$$
(4)

where P(i) is the probability that the first error occurs at the *i*-th bit, and  $P^0(n)$  is the probability that all *n* bits are error free. P(i) and  $P^0(n)$  are dependent on the channel's noise model. We will describe how to calculate P(i) and  $P^0(n)$  for the BSC and burst noise models in the following paragraphs.

## 3.1 BSC model

Assume that a bitstream is transmitted over a binary symmetric, memoryless channel (BSC), and that the channel bit error rate (BER) is  $P_b$ . The BSC noise can be modeled as shown in Fig. 4.

If we allocate n bits for encoding and transmission over this noisy channel, then we

have the following rate-distortion function:

$$D^*(n) = \sum_{i=1}^n (1 - P_b)^{i-1} P_b D(i-1) + (1 - P_b)^n D(n).$$
(5)

Since  $\sum_{i=1}^{n} (1-P_b)^{i-1} P_b + (1-P_b)^n = 1$ , the above equation calculates the expected distortion when *n* bits are sent over a noisy channel with a BER of  $P_b$ . Comparing (5) with (4), we find that  $P(i) = (1-P_b)^{i-1} P_b$  and  $P^0(n) = (1-P_b)^n$ .

Some properties of equation (5) should be discussed. First, if the original ratedistortion function D(i) is monotonically decreasing, then it is obvious that  $D^*(n) > D(n)$ . This implies that we will have more expected distortion while transmitting a bitstream through a noisy channel. Second, by calculating the difference between the expected distortion with n bits and n + 1 bits, we will obtain

$$\Delta D^* = D^*(n) - D^*(n+1)$$

$$= (1 - P_b)^{n+1} [D(n) - D(n+1)].$$

$$= (1 - P_b)^{n+1} \Delta D.$$
(6)

From (6),  $n \to \infty \Rightarrow \Delta D^* = 0$ , which means that the expected distortion converges with large n. Moreover,  $\Delta D^*$  is smaller with a larger value of  $P_b$ , which means that the expected rate-distortion function converges faster as the BER increases. Fig. 5 shows examples of the original rate-distortion function D(i) (with BER=0) and the expected rate-distortion functions  $D^*(i)$  with different BERs. One can observe that the ratedistortion curves become flatter when the BERs increase. Also, one can observe from the figure that for a fixed number of bits, the expected distortion increases when the BER increases.

Based on the definition of the expected rate-distortion function  $D^*(n)$ , we can denote  $D_i^*(b_i)$  as the expected distortion involved in allocating  $b_i$  bits to block *i* under a channel noise BER of  $P_i$ . We now have a new formula for dynamic bit allocation as follows:

$$\min \sum_{i=1}^{K} D_i^*(b_i)$$

$$subject \ to \ \sum_{i=1}^{K} b_i \le R.$$
(7)

### 3.2 Burst noise model

The other type of channel error is burst noise, which may cause errors to occur continuously in the bitstream. Let us use X to denote the random variable of the burst noise. We adopt the Gilbert-Elliot(G-E) model [8] for burst noise simulation. The G-E model depicted as Fig. 6 has two states: the G(Good) state is almost error free, and has probability of error (1 - k); the B(Bad or Burst) state has burst errors, and has probability of error (1 - h). The transition probabilities P(B|G) and P(B|G) are pand q, respectively. Based on the properties of the Markov chain, we can derive the steady-state probabilities in states G and B as

$$P(G) = \frac{q}{p+q}, P(B) = \frac{p}{p+q}.$$
(8)

Moreover, we can derive the *average* BER as

$$P_b(p,q,k,h) = \frac{1}{p+q} \{q(1-k) + p(1-h)\}.$$
(9)

The average BER measure provides us a concept about the randomness of the created burst noise. As shown in Fig. 7, we plot the expected rate-distortion curves of a block in BSC noise with BER = 0.01, 0.005, 0.001 and 0.0005, and that of the same block in burst noise with average BER = 0.0025. The rate-distortion curve of the burst noise just lies between the curves of the BSC noise with BER = 0.005 and 0.001.

We repeat the expected rate-distortion function in (4) here:

$$D^*(n) = \sum_{i=1}^n P(i)D(i-1) + P^0(n)D(n).$$
(10)

For the burst noise case, the P(i), i = 0...n - 1, and  $P^0(n)$  terms can be derived recursively. We introduce two intermediate terms,  $P_G^0(i)$  and  $P_B^0(i)$ , which represent the probabilities that the first *i* generated bits of the burst noise *X* are correct, and the random variable *X* enters the G state and the B state at the *i*-th bit, respectively. The recursive formulas for  $P_G^0(i)$  and  $P_B^0(i)$  are

$$P_{G}^{0}(i) = P_{G}^{0}(i-1)P(G|G)k + P_{B}^{0}(i-1)P(G|B)k,$$

$$P_{B}^{0}(i) = P_{G}^{0}(i-1)P(B|G)h + P_{B}^{0}(i-1)P(B|B)h.$$
(11)

We then have  $P^0(n) = P^0_G(n) + P^0_B(n)$  and

$$P(i) = \{P_G^0(i-1)P(G|G) + P_B^0(i-1)P(G|B)\}(1-k) + \{P_G^0(i-1)P(B|G) + P_B^0(i-1)P(B|B)\}(1-h).$$

Once we know how to calculate P(i) and  $P^0(n)$  for the G-E model, we can calculate the expected rate-distortion function  $D^*(n)$  in (10) and then apply it to the dynamic bit allocation scheme in (7) for robust compression.

## 4 Experiment Results

We will demonstrate the performance of our proposed block-based channel-optimized source coding framework for BSC and burst noise separately.

## 4.1 BSC model

For BSC noise, we evaluated our proposed algorithms by encoding the  $512 \times 512$  Lena image using different target bit rates and block sizes. The resultant bitstreams were then sent to the simulated channel with the desired BER and decoded to evaluate their PSNRs(Peak Signal-to-Noise Ratios). The average performance was obtained from 32 random noise sequences for each BER. In Fig. 8, we compare the performance of our method with that obtained using the method in [5] and with that obtained using the original EZW method over noisy channels with the same BER. The encoded bit rate was 1*bpp*, and we divided the original image into  $32 \times 32$  blocks. It is obvious that our method is better than the others. For very large BER, such as  $10^{-2}$ , errors often occur at the front of the encoded bitstream in each block. All the bits after that error are useless, so there is no significant performance gain in this situation.

Fig. 9 shows the results obtained using our proposed dynamic bit allocation method and the expected rate-distortion constraints, with the same  $32 \times 32$  block division and different BERs. We find also that in a transmission environment with very large BER, the bit allocation is useless even under high bit rate conditions because of the high bit error rate.

Fig. 10(a) and Fig. 10(b) show the sample images obtained when the coded 1*bpp* images were transmitted through BSC noise with BER =  $10^{-3}$ . Fig. 10(a) shows the result of coding with the original EZW method, and Fig. 10(b) shows the result of coding with our block-based EZW method, where the image was divided into  $32 \times 32$  blocks with the BER known in a *priori*. Both images were thresholded to pixel values

ranging from 0 to 255; otherwise, errors would affect the contrast in the images. We can see clearly that the bit errors affected the overall image quality under the original EZW method, but that only some blocks were blurred when our block-based EZW method was used. Both images require post-processing for better visual quality.

### 4.2 Burst noise model

We also applied our method to a burst noise channel. We used the G-E model to simulate burst noise as shown in Fig. 6 with 4 parameters. It was difficult and less meaningful to control these 4 parameters and to describe the results obtained using these parameters. We adopted the simplification of the G-E model given in [11].

In the simplified G-E model, the original 4 parameters were replaced with 3 more meaningful parameters. The first parameter was  $\overline{\varepsilon}$ , the *average BER* of the channel. Note that  $\overline{\varepsilon}$  is just the same as the equivalent BER  $P_b(p, q, k, h)$  stated in equation (9).

The second parameter was  $\overline{b}$ , the *average burst length*, i.e., the average number of times that the random variable of the burst noise staying in the B(Bad) state. Note that

$$\overline{b} = \frac{1}{q}.$$

The third parameter was  $p_1$ , the *duty cycle* or the steady-state probability of being in the B state. Note that

$$p_1 = \frac{p}{p+q}.$$

These three parameters  $\overline{\varepsilon}$ ,  $\overline{b}$ , and  $p_1$  are more meaningful and are better to characterize the burstiness of the channel than the original four parameters. Since we had one less degree of freedom than the original G-E model, we had to introduce the following relation:

$$1-k=\overline{\varepsilon}p_1.$$

This relation ensured that the simplified G-E model was able to describe dense(low duty cycle and high intensity, i.e., high  $\frac{1-h}{1-k}$ ), and diffuse(large duty cycle and low intensity) conditions [11].

All the experiments below are conducted at 1bpp bit rate. In Fig. 11, we compare the performance our method with that of the equal bit allocation method [5] and the original EZW method over burst noise channels with fixed  $p_1$  and  $\overline{b}$ . Note that the conditions  $p_1 = 0.5$  and  $\overline{b} = 2$  represent a case with fast fading burst noise. It is obvious that our method achieved better performance than the other two methods. The difference in performance was insignificant at BER= $10^{-2}$  because of the high bit error rates.

In Fig. 12, we show the performance results obtained with fixed  $p_1 = 0.5$  and  $\overline{b} = 12.5$ , which represent the slow fading burst noise. Our method still achieved better performance than the others did.

In Fig. 13, the performance results obtained using our method are shown with fixed  $p_1 = 0.5$ .  $\overline{b}$  was changed from 2 to 5 and to 12.5 to represent the change from slow fading to fast fading. We can observe that our method achieved stable performance under these different burst noise conditions.

As an example, we show in Fig. 14 the image samples decoded following transmission through the burst noise channel. It is obvious that transmission errors affected the overall image when the original EZW method was used, while our method limited the affected regions to local blocks.

# 5 Conclusion

We have proposed a block-based channel-optimized source coding modification for EZW-like progressive codec to enhance its error-resilience under channel noise. A blockbased scheme was adopted to localize the error effects. Then, we used the dynamic bit allocation strategy to optimally allocate the bit budget to each block. The dynamic bit allocation scheme employed rate-distortion functions, computed based on the channel noise models, as constraints. Thus, the channel noise statistics were used to improve the performance of the EZW-like image compression method in a noisy transmission environment.

Experiment results show that our proposed method has better error-resilience ability compared to the original EZW method and the other proposed method [5]. Both the BSC and burst noise models were used to evaluate the performance.

We expect to add channel coding to our framework in the future.

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(b)

Figure 1: The correspondence between wavelet decomposition and block formation. (a) Original wavelet decomposition, (b) the corresponding block.



Figure 2: The block-based EZW flow diagram.



Figure 3: The multi-channel channel-optimized source coding framework.



Figure 4: The binary symmetric channel.



Figure 5: The expected rate-distortion functions of a given block with different BERs.



Figure 6: The Gilber-Elliot burst noise model.



Figure 7: The expected rate-distortion functions of a given block in BSC noise with different BERs and in burst noise with average BER = 0.0025.



Figure 8: The PSNR performance of various compression schemes. Solid '-' is the result of our proposed block-based EZW using statistical rate-distortion constraints. '-.' is the result of our proposed block-based EZW using original rate-distortion constraints. '- -' is the result of equal length allocation to each block [5]. '..' is the result of original EZW over noisy channels.



Figure 9: The PSNR performance of our proposed block-based EZW scheme using expected rate-distortion constraints. Different bit rates are given. '..', '-.', and '- -' lines denote the results of 1bpp, 0.75bpp, 0.5bpp and 0.25bpp, respectively.



(a)



(b)

Figure 10: The reconstructed images through the BSC noise with  $BER = 10^{-3}$ . (a) Original EZW method, (b) the proposed block-based EZW with expected ratedistortion constrained bit allocation.



Figure 11: The PSNR performance of various compression schemes in fast fading burst noise channel.



Figure 12: The PSNR performance of various compression schemes in slow fading burst noise channel.



Figure 13: The PSNR performance of our robust compression schemes in burst noise channels with different fading characteristics.



(a)



(b)

Figure 14: The reconstructed images through the G-E burst noise with average BER  $= 10^{-3}$ . (a) Original EZW method, (b) the proposed block-based EZW with expected rate-distortion constrained bit allocation.