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# Comments on "A Linear Solution to the Kinematic Parameter Identification of Robot Manipulator"

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## ABSTRACT

The paper by Zhuang and Roth [7] presents a linear solution to the kinematic parameter identification of robot manipulator. With their method, the orientation parameters for all joints are solved, before solving for the translation parameters altogether. Our main contribution here is to decompose the kinematic parameters estimation problem into many subproblems of single joint axis, such that the complexity is reduced and an easier implementation is derived. In addition, it leads to a general solution for any robot with arbitrary combination of prismatic and revolute joints. Our modified solution also provides better robustness and intuition.

## I. INTRODUCTION

Zhuang and Roth [7] proposes a linear method for the identification of the unknown kinematic parameters of robot manipulator from end-effector pose measurement and robot joint position readings at some configurations. Their result is of great value for many applications (e.g., [2]). While the calibration method is clearly illustrated with the use of an all-revolute robot, a Cartesian robot, and a Stanford-arm-type robot, no general procedure is given for dealing with arbitrary type of robot. In the derivation of their linear solution, the rotation matrix and translation vector were considered separately. When solving for the translation vector, the transition vectors of all joints have to be solved simultaneously which makes the implementation very complicated, e.g. [5, eqs. (16), (19)]. Moreover, if a robot contains some prismatic joints, then the directions of the prismatic joint axes have to be solved together with the transition vectors, e.g. [5, eqs. (20)], which means that some nonlinear constraints should be involved to force the direction vectors to be unit vectors. However, the constraints were not discussed in [7]. We shall show that due to the nice structure of the complete and parametrically continuous (CPC) kinematic model [6], the kinematic parameter identification problem can be decomposed into many kinematic parameter calibration problems of each individual prismatic or revolute joint. With this kind of decomposition, the calibration procedure can be applied to any robot which is composed of prismatic or revolute joints. Since the scale of the problem is reduced to be of single joint, not only the exact closed-form solution of the direction of the prismatic joint axis estimation, with nonlinear constraint, can be found, but the calibration method can be implemented in an easier way.

## II. PROBLEM FORMULATION

The CPC kinematic model for a revolute or prismatic joint is as follows (refer to [6])

$${}^{i-1}T_i = Q_i V_i, \quad (1)$$

where

$$Q_i = \begin{cases} Rot_z(q_i), & \text{for revolute joint,} \\ Trans\left(\begin{bmatrix} 0 & 0 & q_i \end{bmatrix}^T\right), & \text{for prismatic joint,} \end{cases} \quad (2)$$

$$q_i = s_i q'_i, s_i \in \{+1, -1\}, \quad (3)$$

$q'_i$  is the  $i$ th joint value,

$$V_i = R_i \text{Rot}_z(\beta_i) \text{Trans}\left(\begin{bmatrix} l_{ix} & l_{iy} & l_{iz} \end{bmatrix}^T\right), \quad (4)$$

and

$$R_i = \begin{bmatrix} 1 - \frac{b_{ix}^2}{1 + b_{iz}} & \frac{-b_{ix}b_{iy}}{1 + b_{iz}} & b_{ix} & 0 \\ \frac{-b_{ix}b_{iy}}{1 + b_{iz}} & 1 - \frac{b_{iy}^2}{1 + b_{iz}} & b_{iy} & 0 \\ -b_{ix} & -b_{iy} & b_{iz} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (5)$$

Notice that the CPC convention requires that any two consecutive joint axes should have nonnegative inner product, i.e.,  $b_{iz} \geq 0$ . In general, this requirement can be achieved by changing the sign of one of the joint values of consecutive joints. This is because changing the sign of the joint value is equivalent to reversing the joint axis for both revolute and prismatic joints. Therefore, we have slightly modified the convention of the CPC model by including a sign parameter,  $s_i$ , as shown in equation (3).

Suppose we have a robot with  $n$  joints. Its world-to-end-effector transformation matrix can be expressed as

$${}^wT_n = {}^wT_0 \cdot \dots \cdot ({}^{n-1}T_n). \quad (6)$$

Without loss of generality, we assume that the kinematic parameters of the joints from the end-effector to the  $(i+1)$ th joint have been known, and that the unknowns to be estimated are the world-to-base transformation,  ${}^wT_0$ , and the kinematic parameters of joints  $1, \dots, i$ . Also, we assume that the world-to-end-effector transformation matrix can be measured. Same as the calibration procedure described in Zhuang and Roth [7], when calibrating the  $i$ th joint, only those joints with known kinematic parameters plus the  $i$ th joint itself are permitted to be moved. By moving those joints (from the end-effector to the  $i$ th joint) to two different configurations and recording their corresponding world-to-end-effector transformation matrices, we have

$${}^wT_{n1} = {}^wT_i Q_{i1} V_i ({}^{i+1}T_{n1}), \quad (7)$$

and

$${}^wT_{n2} = {}^wT_i Q_{i2} V_i ({}^{i+1}T_{n2}), \quad (8)$$

where  ${}^w T_{n1}$  and  ${}^w T_{n2}$  are the measured world-to-end-effector transformation matrices,  ${}^{(i+1)1} T_{n1}$  and  ${}^{(i+1)2} T_{n2}$  can be computed from the kinematic model since their kinematic parameters have been known already. Instead of separating equation (6) into two parts: rotation matrix and translation vector equations, as the methods used in [7] and [2], we decompose the problem into many kinematic parameter estimation sub-problems of single joint axis.

By multiplying  ${}^w T_{n1}^{-1}$  and  ${}^w T_{n2}^{-1}$  to both sides of equations (7) and (8), respectively, we have the following equality

$${}^w T_i Q_{i1} V_i {}^{(i+1)1} T_{n1} {}^w T_{n1}^{-1} = I_{4 \times 4} = {}^w T_i Q_{i2} V_i {}^{(i+1)2} T_{n2} {}^w T_{n2}^{-1}, \quad (9)$$

or

$$Q_{i1} V_i {}^{(i+1)1} T_{n1} {}^w T_{n1}^{-1} = Q_{i2} V_i {}^{(i+1)2} T_{n2} {}^w T_{n2}^{-1}. \quad (10)$$

Rearranging equation (10), we have

$$\Delta Q V_i = V_i \Delta T, \quad (11)$$

where  $\Delta Q \equiv Q_{i1}^{-1} Q_{i2}$ ,  $\Delta T \equiv {}^{(i+1)1} T_{n1} {}^w T_{n1}^{-1} {}^w T_{n2} {}^{(i+1)2} T_{n2}^{-1}$  and  $V_i$  is the unknown homogeneous transformation matrix to be estimated. Equation (11) in this form of representation is very similar to the equation for the hand/eye calibration problem. Unfortunately, the solution obtained in the hand/eye calibration problem cannot be directly applied to this problem. This is because the hand/eye calibration techniques proposed by Shiu and Ahmad [3], Tsai [4] and Wang [5], all need at least two equations as follows

$$A_1 X = X B_1, \quad (12)$$

and

$$A_2 X = X B_2, \quad (13)$$

where  $X$  is the unknown hand/eye transformation matrix,  $A_1$ ,  $A_2$ ,  $B_1$  and  $B_2$  are the measured homogeneous transformation matrices, and the rotation axes of the rotation matrices of  $A_1$  ( $B_1$ ) and  $A_2$  ( $B_2$ ) should be neither parallel nor anti-parallel. But in the single joint calibration problem, it is obvious that there is at most one effective rotation axis, i.e., the rotation axis of  $\Delta Q$  (there is even no rotation axis if the joint is prismatic). However, due to the special structure of the CPC kinematic model, we have the following linear solution.

Notice that equation (11) can be separated into two equations, one is the rotation matrix equation and the another is the translation vector equation, i.e.,

$$R_{\Delta Q} R_{V_i} = R_{V_i} R_{\Delta T}, \quad (14)$$

and

$$R_{\Delta Q} t_{V_i} + t_{\Delta Q} = R_{V_i} t_{\Delta T} + t_{V_i}, \quad (15)$$

where

$$R_{V_i} = R_i \text{Rot}_z(\beta_i), \quad (16)$$

$$t_{V_i} = R_{V_i} \begin{bmatrix} l_{ix} & l_{iy} & l_{iz} \end{bmatrix}^T, \quad (17)$$

$R_{\Delta Q}$ ,  $R_{\Delta T}$ ,  $R_{V_i}$ ,  $R_i$  and  $\text{Rot}_z(\beta_i)$  are 3x3 rotation matrices of  $\Delta Q$ ,  $\Delta T$ ,  $V_i$ ,  $R_i$  and  $\text{Rot}_z(\beta_i)$ , respectively, and  $t_{\Delta Q}$ ,  $t_{\Delta T}$  and  $t_{V_i}$  are 3x1 transition vector of  $\Delta Q$ ,  $\Delta T$ ,  $V_i$ , respectively. In the following sections, we shall show how to solve the kinematic parameters of the prismatic and revolute joints from the above equations.

### III. CALIBRATION OF A PRISMATIC JOINT

The redundant parameters and the unknowns of a prismatic joint are first listed below for clarity of our derivation:

- i). Four given redundant parameters (they are typically set to zero if not used):  $\beta_i$  and  $t_{V_i}$ .
- ii). The unknowns:  $R_i$  and the sign parameter,  $s_i$ .

From equation (2),  $\Delta Q = \text{Trans}([0 \ 0 \ \Delta q]^T)$ , or more specifically,  $R_{\Delta Q} = I_{3 \times 3}$  and  $t_{\Delta Q} = [0 \ 0 \ \Delta q]^T$ . By substituting  $R_{\Delta Q}$  and  $t_{\Delta Q}$  into equation (15), we have

$$t_{\Delta Q} = R_{V_i} t_{\Delta T}. \quad (18)$$

Substituting equation (16) into (18), we have

$$R_i^T t_{\Delta Q} = \tilde{t}_{\Delta T}, \quad (19)$$

where  $\tilde{\mathbf{t}}_{\Delta T} \equiv \text{Rot}_z(\beta_i) \mathbf{t}_{\Delta T}$  and  $\beta_i$  is the given redundant parameters. Note that in the above equation,  $\mathbf{t}_{\Delta Q} = [0 \ 0 \ \Delta q]^T$ , therefore,

$$\mathbf{b}'_i \Delta q = \tilde{\mathbf{t}}_{\Delta T}, \quad (20)$$

where  $\mathbf{b}'_i = [-b_{ix} \ -b_{iy} \ b_{iz}]^T$  is the third column vector of the rotation matrix  $R_i^T$ .

Suppose we have  $M$  observations, i.e.,  $\Delta q_j$  and  $\Delta T_j, j = 1, 2, \dots, M$ . We can solve  $\mathbf{b}'_i$  by minimizing the following error using the least square method.

$$e \equiv \sum_{j=1}^M \|\mathbf{b}'_i \Delta q_j - \tilde{\mathbf{t}}_{\Delta T_j}\|^2, \quad (21)$$

where  $\mathbf{b}'_i^T \mathbf{b}'_i = 1$ . To solve the above equation, we first form the lagrangian

$$l \equiv \sum_{j=1}^M \left\{ \mathbf{b}'_i^T \mathbf{b}'_i \Delta q_j^2 - 2\mathbf{b}'_i^T \tilde{\mathbf{t}}_{\Delta T_j} \Delta q_j + \tilde{\mathbf{t}}_{\Delta T_j}^T \tilde{\mathbf{t}}_{\Delta T_j} \right\} + \lambda (1 - \mathbf{b}'_i^T \mathbf{b}'_i). \quad (22)$$

The gradient of equation (22) is

$$\nabla l \equiv 2 \sum_{j=1}^M \left\{ \mathbf{b}'_i \Delta q_j^2 - \tilde{\mathbf{t}}_{\Delta T_j} \Delta q_j \right\} - 2 \lambda \mathbf{b}'_i. \quad (23)$$

By letting  $\nabla l = 0$ , we have

$$\mathbf{b}'_i = \sum_{j=1}^M (\tilde{\mathbf{t}}_{\Delta T_j} \Delta q_j) / \left[ \sum_{j=1}^M (\Delta q_j^2) - \lambda \right], \quad (24)$$

where  $\lambda$  can be determined such that  $\mathbf{b}'_i$  is a unit vector. Consequently,

$$\mathbf{b}'_i = \sum_{j=1}^M (\tilde{\mathbf{t}}_{\Delta T_j} \Delta q_j) / \left\| \sum_{j=1}^M (\tilde{\mathbf{t}}_{\Delta T_j} \Delta q_j) \right\|, \quad (25)$$

which means that the least-square-error solution is just the weighted average of the translations vectors,  $\tilde{\mathbf{t}}_{\Delta T_j}$ .

Moreover, by intuition, if the difference of the joint values, i.e.,  $\Delta q_j$  (or equivalently, the length of  $\tilde{\mathbf{t}}_{\Delta T_j}$ ), can be made larger, then the S/N ratio will be larger too, therefore, larger difference of the joint values would lead

to more accurate results. Notice that if the third component of  $b'_i$  is negative, in order to be consistent with the CPC convention, we should change the sign of  $b'_i$  and let  $s_i = -1$ ; otherwise, let  $s_i = +1$ . Once the unit vector  $b'_i$  is obtained, the rotation matrix,  $R_i$ , can be computed with equation (5). Then, we can transform the redundant translation vector,  $t_{V_i}$ , into the CPC parameter format, i.e.,

$$\begin{bmatrix} l_{ix} & l_{iy} & l_{iz} \end{bmatrix}^T = R_{V_i}^T t_{V_i} \quad (26)$$

#### IV. CALIBRATION OF A REVOLUTION JOINT

The redundant parameters and the unknowns of a revolute joint are first listed below for clarity of the derivation:

- (1). Two given redundant parameters (they are typically set to zero if not used):  $\beta_i$  and the z-component of  $t_{V_i}$ .
- (2). The unknowns:  $R_i$ , the sign parameter,  $s_i$ , and the first two components of  $t_{V_i}$ .

Note that for a revolute joint,  $\Delta Q = Rot_z(\Delta q)$ , i.e.,

$$R_{\Delta Q} = Rot_z(\Delta q), \quad (27)$$

and

$$t_{\Delta Q} = \mathbf{0}_{3 \times 1} \quad (28)$$

Substituting equations (27) and (28) into equations (14) and (15), we have

$$Rot_z(\Delta q) R_{V_i} = R_{V_i} R_{\Delta T}, \quad (29)$$

and

$$Rot_z(\Delta q) t_{V_i} = R_{V_i} t_{\Delta T} + t_{V_i}, \quad (30)$$

By taking the transpose of both sides of equation (29), and then multiplying the unit vector,  $z = [0 \ 0 \ 1]^T$  on the right, we have (by noticing that  $Rot_z(-\Delta q) z = z$ ),

$$R_{V_i}^T z = R_{\Delta T}^T R_{V_i}^T z. \quad (31)$$

By use of equation (16), we have

$$b'_i = D b_i, \quad (32)$$



where  $\mathbf{b}'_i$  is the same vector as in equation (20), and  $D \equiv Rot_z(\beta_i) R_{\Delta T}^T Rot_z(-\beta_i)$ . Since in equation (32),  $D$  is a rotation matrix,  $\mathbf{b}'_i$  can be found, up to its sign, by computing the rotation axis of  $D$  with the following method. The correct sign of  $\mathbf{b}'_i$  can be determined such that the third component of  $\mathbf{b}'_i$  is nonnegative.

If we have  $M$  observations, then we will have the following homogeneous equation (overdetermined if  $M > 1$ )

$$\begin{bmatrix} D_1^T - I_{3 \times 3} \\ \vdots \\ D_M^T - I_{3 \times 3} \end{bmatrix} \mathbf{b}'_i \equiv \mathbf{E} \mathbf{b}'_i = \boldsymbol{\varepsilon} \approx \mathbf{0}, \quad (33)$$

where  $\boldsymbol{\varepsilon}$  is the error vector induced by the observation noise. The parameter vector  $\mathbf{b}'_i$  can be estimated by minimizing  $\|\boldsymbol{\varepsilon}\|^2$  subject to  $\|\mathbf{b}'_i\|^2 = 1$ . It can be shown that the solution for  $\mathbf{b}'_i$  is the unit eigenvector of  $\mathbf{E}^T \mathbf{E}$  corresponding to the smallest eigenvalue.

The sign parameter,  $s_i$ , can be determined as follows. From equation (5), we can compute  $\hat{R}_i$  from  $\mathbf{b}'_i$ . Substituting the estimated  $\hat{R}_i$  into equation (29), we have

$$Rot_z(s_i \cdot \Delta q'_i) = \hat{R}_{V_i} R_{\Delta T} \hat{R}_{V_i}^T, \quad (34)$$

and the sign parameter,  $s_i$ , can be obtained from the following equation

$$s_i = \arg \left\{ \min_{s_i = +1, -1} \sum_{j=1}^M \left\| Rot_z(s_i \cdot \Delta q'_{ij}) - \hat{R}_{V_i} R_{\Delta T_j} \hat{R}_{V_i}^T \right\|^2 \right\} \quad (35)$$

Notice that in order to estimate  $\mathbf{b}'_i$  more accurately, the difference of the joint angle, i.e.,  $\Delta q$ , should be made larger. Otherwise, the  $D$  matrix, in equation (32), will approach a unit matrix when  $\Delta q$  is very small, which will cause the estimation of the rotation axis very sensitive to noise.

After  $\mathbf{b}'_i$  is obtained, we can compute the rotation matrix  $R_i$  and then post multiply it by  $Rot_z(\beta_i)$  to obtain  $R_{V_i}$ . Next, by substituting  $R_{V_i}$  into equation (30) and rearranging it, we have

$$\begin{bmatrix} \cos(\Delta q) - 1 & -\sin(\Delta q) & 0 \\ \sin(\Delta q) & \cos(\Delta q) - 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} t_{V_i} = R_{V_i} t_{\Delta T}. \quad (36)$$

Note that on the left hand side, the third row is exactly zero, while on the right hand side,  $t_{\Delta T}$  is the translation vector observed at the  $i$ th link frame, and  $Rot_z(q_i) R_{V_i} t_{\Delta T}$  is the corresponding translation vector observed at  $(i-1)$ th frame, therefore, the third component of the translation vector  $Rot_z(q_i) R_{V_i} t_{\Delta T}$  is zero. This is because the z-axis of frame  $(i-1)$  is the rotation axis, and rotating around the rotation axis will not cause any motion along the direction of the rotation axis. Since the rotation matrix  $Rot_z(q_i)$  will not change the third component of  $R_{V_i} t_{\Delta T}$ , the third component of  $R_{V_i} t_{\Delta T}$  is zero. Hence, equation (36) is consistent with our intuition.

Because the third entry of the translation vector  $t_{V_i}$  is a redundant parameter, we have the following two equations for two unknowns (from equation (36))

$$\begin{bmatrix} \cos(\Delta q) - 1 & -\sin(\Delta q) \\ \sin(\Delta q) & \cos(\Delta q) - 1 \end{bmatrix} \begin{bmatrix} t_{i,x} \\ t_{i,y} \end{bmatrix} = \bar{R}_{V_i} t_{\Delta T}, \quad (37)$$

where  $t_{i,x}$  and  $t_{i,y}$  are the first two entries of  $t_{V_i}$ , and  $\bar{R}_{V_i}$  is a 2x3 matrix obtained by deleting the third row of  $R_{V_i}$ . The determinant of the 2x2 matrix on the left hand side of equation (37) is  $(2 - 2\cos(\Delta q))$ . Therefore, equation (37) has unique solution if and only if  $\Delta q \neq 0$ . This condition for uniqueness is always satisfied since we always move the joint to be calibrated in the kinematic identification process. Again, we observed that larger amount of  $\Delta q$  will improve the robustness for the estimation of the kinematic parameters. Of course, if more than one observations are available, we can use least square method to solve the two unknown translation components.

After all the unknown parameters are obtained, we can transform the translation vector,  $t_{V_i}$ , into the CPC parameter format by using equation (26). If it is necessary to let  $l_{i,z} = 0$ , then the following equation can be used to determine the redundant translation parameter, i.e., the third entry of  $t_{V_i}$ ,

$$t_{i,z} = - (b_{i,x} t_{i,x} + b_{i,y} t_{i,y}) / b_{i,z}, \quad (38)$$

providing that  $b_{i,z}$  is not equal to zero.

## V. DETERMINATION OF THE WORLD-TO-BASE TRANSFORMATION MATRIX

After all the revolute and prismatic joints were calibrated, then we can compute the world-to-base transformation matrix,  ${}^wT_0$ , defined in equation (6). Suppose we have  $m$  observations, i.e.,  $j = 1, 2, \dots, M$ . By separating equation (6) into rotation matrix and translation vector equations, we have

$${}^wR_{nj} = {}^wR_0 {}^0R_{nj}, \quad (39)$$

and

$${}^wt_{nj} = {}^wR_0 {}^0t_{nj} + {}^wt_0, \quad (40)$$

where  ${}^0R_{nj}$  and  ${}^0t_{nj}$  are respectively the 3x3 rotation matrix and 3x1 translation vector of the transformation matrix,  ${}^0T_{nj} \equiv {}^0T_{1j} {}^1T_{2j} \dots {}^{(n-1)j}T_{nj}$ . From (39), we have the following matrix equations

$$A = {}^wR_0 B, \quad (41)$$

where  $A \equiv [{}^wR_{n1} \dots {}^wR_{nj} \dots {}^wR_{nM}]$  and  $B \equiv [{}^0R_{n1} \dots {}^0R_{nj} \dots {}^0R_{nM}]$ . By solving the following "rotation of subspecies" problem [1]:

$$\text{minimize } \|A^T - B^T {}^wR_0^T\|_F \text{ subject to } {}^wR_0^T {}^wR_0 = I_{3 \times 3}, \quad (42)$$

we have the following procedures for finding the closed-form solution for  ${}^wR_0$ .

Step 1: Compute the matrix  $C \equiv B A^T$ .

Step 2: Compute the singular value decomposition  $C = U S V^T$ .

Step 3: Compute  ${}^wR_0 = V U^T$ .

After the rotation matrix  ${}^wR_0$  is obtained, by substituting  ${}^wR_0$  into equation (40), we have

$${}^wt_0 = \frac{1}{M} \sum_{j=1}^M ({}^wt_{nj} - {}^wR_0 {}^0t_{nj}), \quad (43)$$

which completes the calibration procedure.

#### IV. EXPERIMENT

To test our modified method for kinematic parameter identification, we use a six-revolute-joint Kawasaki Js-10 robot arm. Instead of measuring the positions and orientations of the end-effector with a CMM (Coordinate Measuring Machine), we simply record both the joint values and the position and Euler angles of the end-effector provided by the controller. The position and Euler angles of the end-effector are then used to compute the world-to-end-effector transformation matrix, where we assume the world coordinate system is aligned with the base reference frame of the robot. We took 7 observations for each joint for the calibration (42 poses in total), and 32 additional poses for testing. The kinematic parameters of the Kawasaki Js-10 shown in Table-1 is obtained from the calibration process (rounded to the fourth digit after point). Using the estimated kinematic parameters for forward kinematic computation, the RMSE position error, comparing to the position given by the controller, is about 0.01 millimeter with the 32 testing poses.

#### V. SUMMARY

In this report, we have shown that the kinematic parameter identification problem can be decomposed into many kinematic parameter calibration problems of each individual prismatic or revolute joint. This will not only reduce the complexity of the identification problem, but also provide a calibration method that is

Table-1.  
The Estimated CPC Parameters of the Kawasaki Js-10

$i$	$b_{i,x}$	$b_{i,y}$	$b_{i,z}$	$l_{i,x}(mm)$	$l_{i,y}(mm)$	$l_{i,z}(mm)$	$\beta_i(rad)$	$s_i$
0	0.0000	0.0000	1.0000	0.0013	0.0020	-0.0075	0.00	
1	-1.0000	0.0000	0.0000	0.0002	100.0116	0.0152	0.00	-1
2	0.0000	0.0000	1.0000	650.00	-0.0153	-0.0068	0.00	+1
3	1.0000	0.0000	0.0000	0.0028	0.0001	600.0014	0.00	-1
4	1.0000	0.0000	0.0000	0.0000	0.0005	0.0000	0.00	+1
5	-1.0000	0.0000	0.0000	-0.0005	-0.0022	125.0021	0.00	+1
6	0.0000	0.0000	1.0000	0.0026	0.0018	0.0000	0.00	+1

universal to any kind of robot composed of prismatic and/or revolute joints. Due to the decomposition of the problem, the closed-form solution to the direction unit vector of a prismatic joint axis is more robust because a nonlinear constraint is included. It also provides the intuition for reducing the estimation error via choosing larger magnitude of the difference of the joint values when constructing the calibration equations. This method has been tested by a real experiment, which shows that if the measurements of the world-to-end-effector transformation matrices is accurate, the calibration results can also be very accurate. Although, the problem is not resolved, that the kinematic parameters may not be parametrically continuous when the two consecutive joints are perpendicular to each other. It will not cause any problem if we solve this kinematic parameter identification problem by using any closed-form solution (including our solution).

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