

**TR-91-007**

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# Modeling Deformable Objects Using a Linkage Model <sup>1</sup>

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## Abstract:

We present a *linkage model* to model the 2D and 3D deformable objects under externally applied forces. In this method, an object is modeled by a linkage structure and forces and displacements for each link are solved to generate the deformation. It is simple, yet it can effectively model realistic deformed shapes.

**Key Words:** Solid modeling, Computer Graphics, Animation, Scientific Visualization.

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<sup>1</sup>Research supported in part by National Science Council, Taiwan, Republic of China, under grant NSC-79-0408-E-001-01

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## 1. Introduction

In recent years, solid modeling has received a lot of attention due to its great application potential and the advances of computer technology. It acts as the basis for fields such as computer graphics, computer aided design, computer animation and simulation, scientific visualization, etc. The goal of solid modeling is to represent the geometric features of a solid object effectively.

Until now, there are two basic approaches for modeling 3-D objects: the volumetric approach and the boundary approach[1]. The volumetric approach describes an object as a combination of primitive volumes. This class includes all the decomposition techniques related to the underlying space representation. Octree encoding[2] and constructive solid geometry technique[3] are typical examples of this approach. On the other hand, the boundary approach describes the solid volumes using the enclosing surfaces. Such models contain both the individual oriented surfaces of the object and the topological relationships between the surfaces. One example of this approach is the face-adjacency graph developed by Floriani and Falcidieno[4].

Although the researchers have proposed various solid models, most of them are used to represent rigid solid objects. Recently, some ambitious researchers start to aim at representing deformable objects[5,9]. To name a few examples: Platt et al.[5] developed a general deformable solid model based on the theory of elasticity; Waters[6] developed a muscle model for animating human facial expression; Miller[7] studied the motion of snakes and worms; and Terzopoulos et al.[8] tried to model complex behaviors of the solid based on the theories of viscoelasticity, plasticity, and fracture.

In this study, we use a linkage model to represent a deformable solid. This model applies to 3D cases as well as 2D cases. With relatively simple data structure, realistic deformed shapes can be generated using this model. By assigning different stiffness to the axial members of the linkage model, we can even model materials of different characteristics and composite materials.

In the following, Section 2 describes the linkage model and the corresponding data structure. The formulation of the element stiffness matrix of a single axial member is described in Section 3. The method for assembling the element stiffness matrices to get the global stiffness matrix of the linkage

model is described in Section 4. To gain a resolvable, there must exist some boundary conditions. The method of applying the boundary conditions is given in Section 5. Two examples, one is a 2D case and the other is a 3D case, are shown and described in Section 6. Conclusions and Discussions can be found in the final section.

## 2. Linkage Model for Deformable Solids

From microscopic point of view, there are forces acting between any two atoms in a solid and the forces are balanced with the externally applied forces to reach the final deformed shape. The structure of the solid can be viewed as a set of axial members and connection points where the axial members take the forces acting between atoms and the connection points represent the atoms. In this study, we use such a linkage model to represent a solid in which each axial member is constrained to extend or shrink in the axial direction. No bending effect of the member is allowed. The connection point of several members is called a node. Fig. 1 shows the linkage model of a circular disk with 16 members and 9 nodes. The data structure of this model is very simple. Two files are required for a 2D case. The first file stores the 2D coordinates of the nodes and the second file stores the node number pairs of the linkages. Each node number pair has two node numbers which are the node numbers of the nodes at the two ends of a linkage. For 3D cases, a third file is necessary to store node number triplets of the surface triangular patches. In this study, all surface patches of a deformable body are triangular with each side being a linkage. With this information, we can compute the outward normals at the nodes of a patch and hence can perform shading to get a realistic view of the deformation process on the screen.

## 3. Formulation of Element Stiffness Matrix

The basic principle for computing the displacements of a solid is equilibrium. From a local point of view, the resultant force acting on one end of an axial member and the resultant force acting on the other end should be equal in magnitude and opposite in direction. Furthermore, both of them should act along the axial direction of the member. This is due to the assumption that the member can only deform along the axial direction. Fig. 2 shows

this situation of a member. The force  $F_2$  applied by the wall on the member is equal to  $-F_1$  and the displacement of the free end  $D$  can be found to be

$$D = F_1 L / AE \quad (1)$$

where  $A$  is the cross-sectional area of the member,  $L$  is the original length of the member, and  $E$  is a material property called Young's modulus. The formula of  $D$  can be easily found in any textbook of mechanics.

To relate the end forces and the end displacements of the member, an element stiffness matrix equation can be formed. To unify our approach, we define a global coordinate system  $x - y$  and decompose each of the end forces and end displacements into their  $x$  component and  $y$  component. Fig. 3 shows this case. By letting  $U_1$  to be positive and the other three displacement components to be zero, the axial shortage of the member is  $U_1 \cdot \cos \phi$  and the axial force can be found to be  $AEU_1 \cdot \cos \phi / L$ . According to Eq. (1), the force components  $F_{x1}$ ,  $F_{y1}$ ,  $F_{x2}$ , and  $F_{y2}$  can easily be found to be  $AEU_1 \cos^2 \phi / L$ ,  $AEU_1 \cdot \cos \phi \cdot \sin \phi / L$ ,  $-AEU_1 \cdot \cos^2 \phi / L$ , and  $-AEU_1 \cdot \cos \phi \cdot \sin \phi / L$  respectively. These comprise the first column of the element stiffness matrix. In a similar way, we can derive other elements of the element stiffness matrix and the final form of element stiffness matrix equation is as follows

$$\begin{Bmatrix} F_{x1} \\ F_{y1} \\ F_{x2} \\ F_{y2} \end{Bmatrix} = \begin{bmatrix} A & B & -A & -B \\ B & C & -B & -C \\ -A & -B & A & B \\ -B & -C & B & C \end{bmatrix} \begin{Bmatrix} U_1 \\ V_1 \\ U_2 \\ V_2 \end{Bmatrix} \quad (2)$$

where  $A = \cos^2 \phi$ ,  $B = \sin \phi \cdot \cos \phi$ ,  $C = \sin^2 \phi$ .

#### 4. Assemblage of Global Stiffness Matrix

From a global point of view, the forces acting on each node, both externally applied forces and internal forces provided by the members, should be summed to zero. Otherwise, the node will accelerate away under the unbalanced force according to the second law of Newton. Using the element stiffness matrices, we can assemble the global stiffness matrix. Fig. 4 shows a square solid with four nodes and eight degree of freedoms. We use this example to explain the assemblage of global stiffness matrix. The displacement

of a node can be decomposed into two components along the two degrees of freedom associated with that node. Moreover, the resultant force acting on a node can also be decomposed into its  $x$  component and  $y$  component according to the principle of vector decomposition. The final global matrix equation can be represented in the following form

$$\begin{Bmatrix} F_{x1} \\ F_{y1} \\ \cdot \\ \cdot \\ F_{y4} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & \cdots & S_{18} \\ S_{21} & S_{22} & S_{23} & \cdots & S_{28} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ S_{81} & S_{82} & S_{83} & \cdots & S_{88} \end{bmatrix} \begin{Bmatrix} U_1 \\ V_1 \\ \cdot \\ \cdot \\ V_4 \end{Bmatrix} \quad (3)$$

or in a more uniform form like

$$\begin{Bmatrix} F_1 \\ F_2 \\ \cdot \\ \cdot \\ F_8 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & \cdots & S_{18} \\ S_{21} & S_{22} & S_{23} & \cdots & S_{28} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ S_{81} & S_{82} & S_{83} & \cdots & S_{88} \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ \cdot \\ \cdot \\ U_8 \end{Bmatrix} \quad (4)$$

where coefficient  $S_{ij}$  can be assembled from the related local matrices of the members. Take member #1 for example where the related degrees of freedom are 1, 2, 7, and 8. The 16 elements of the matrix in Eq. (2) should be added to the 16 elements in the matrix of Eq. (4) which correspond to the four target degrees of freedom. Thus,  $\cos^2 \phi$  will be added into  $S_{11}$  and  $S_{77}$ ,  $-\cos^2 \phi$  will be added into  $S_{17}$  and  $S_{71}$ , etc.

## 5. Applying the Boundary Conditions

Once the global stiffness matrix equation is found, we can compute the desired displacements after the known boundary conditions are applied. Fig. 5 shows a set of boundary conditions applying to the example of square solid. node 3 is constrained not to move in both degrees of freedom, and node 4 is constrained not to move in the  $y$  direction. Furthermore, an external force is applied to node 2 in the  $x$  direction with magnitude 200 Newton. Thus, the unknown force components include  $F_5$ ,  $F_6$ ,  $F_8$ , and the unknown displacements include  $U_1$ ,  $U_2$ ,  $U_3$ ,  $U_4$ , and  $U_7$ . We can rearrange Eq. (4) so that the

components corresponding to the supports and the other components can be separated. The rearranged global matrix equation is as

$$\left\{ \begin{matrix} F_f & F_s \end{matrix} \right\} = \begin{bmatrix} K_{ff} & K_{fs} \\ K_{sf} & K_{ss} \end{bmatrix} \left\{ \begin{matrix} U_f \\ U_s \end{matrix} \right\} \quad (5)$$

where vectors  $U_f$  and  $F_s$  are unknowns to be found. Eq. (5) include two set of equations as

$$F_f = K_{ff}U_f + K_{fs}U_s, \quad (6)$$

$$F_s = K_{sf}U_f + K_{ss}U_s. \quad (7)$$

Since  $U_s$  is a zero vector, Eq. (6) reduces to

$$F_f = K_{ff}U_f \quad (8)$$

or

$$U_f = K_{ff}^{-1}F_f. \quad (9)$$

Substituting Eq. (9) into Eq. (7), we have

$$F_s = K_{sf}K_{ff}^{-1}F_f. \quad (10)$$

Solving Eq. (9) and Eq. (10) give the unknown displacement components  $U_f$  and force components  $F_s$  respectively.

## 6. Examples

Two cases have been implemented and displayed to show the deformation process of a deformable solid using the linkage model. The first example is a 2D solid as shown in Fig. 6. The lower left node of the solid is constrained not to move and the lower right node is constrained not to move in the vertical direction. These supports are represented as short black line segments. A horizontal force is applied at the upper right node which varies in magnitude. The force is represented by a blue line segment and the length of the line segment represents its magnitude. During the deformation process, some of

the axial members will be loaded too much and will exceed the strength limit. We did not simulate the fractured process but the members are marked as black once they exceed the strength limit. Fig. 6 shows a sequence of pictures which represent different steps in the deformation process.

The second example draws the deformation process of a 3D cylindrical shell structure. One end of the shell is completely fixed and will not move. The other end of the shell is subjected to an external force which varies in magnitude. Shading is also applied so as to display the realistic deformed shape. Fig. 7 shows a sequence of pictures of this deformation process.

Although we did not draw the deformation of a 3D solid, it is rather easy to prepare data and compute the deformed shapes using the same procedures developed by us.

## 7. Conclusions and Discussions

In this study, we have proposed a linkage to represent deformable solid objects. Due to its simplicity and flexibility, it can effectively model a lot of realistic deformed shapes of solids under externally applied forces. A 2D example and a 3D example are implemented to test the effect of our model. Realistic deformation process has been produced and the feasibility of this method has also been proved.

Several problems found in this study will be discussed here. One problem is the computation time. If we want the deformed shape to be real, the number of members and nodes in the model must be large. Real time computation and display of the deformation process may not be possible for three-dimensional cases. Results of the deformation steps should be stored in files and then displayed in sequence so as to produce the deformation process. Another problem with this model is its inability to model the fracture phenomenon. To model the process of crack formation and crack propagation using this model is difficult. Suitable modification of the model may be necessary for fracture simulation. Still another problem is when there exists several deformable solids simultaneous. In this situation, they may collide with each other and deform. For purpose of collision detection, the surface information of the solid object is necessary. Extra data about surface patches must be provided by the user so that collision can be tested for node-patch



pairs[10]. These problems will be the topics of our further study about drawing deformed bodies. For complex shaped objects, to prepare the necessary data automatically is also a challenge of interest.

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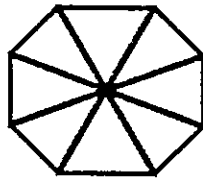


Fig. 1 The linkage model of a circular disk.

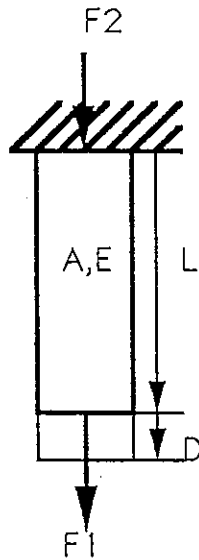


Fig. 2 An axial member under applied force  $F_1$ .

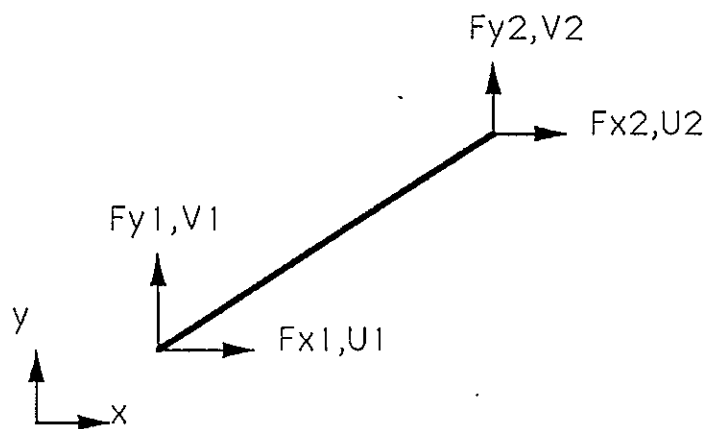


Fig. 3 End force components and end displacement components of a member.

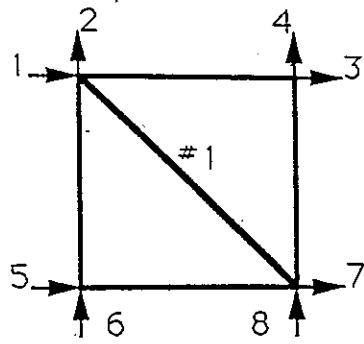


Fig. 4. The linkage model of a square solid.

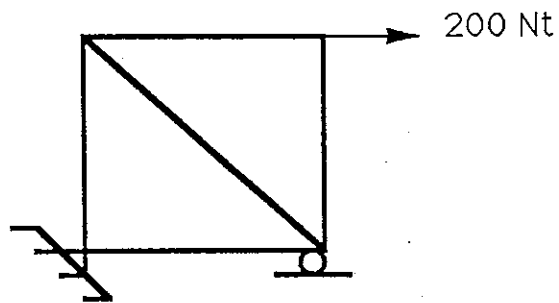
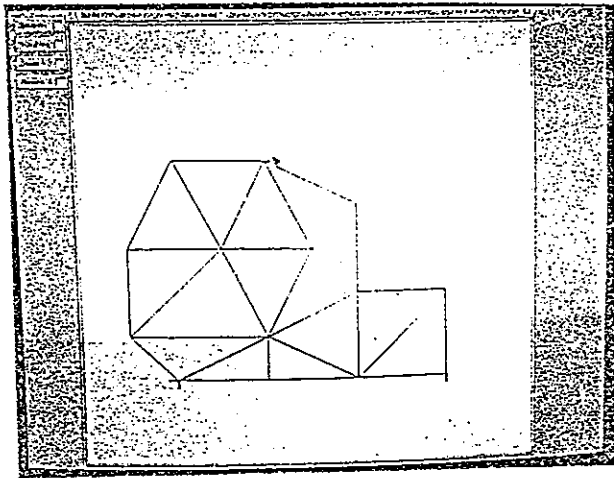
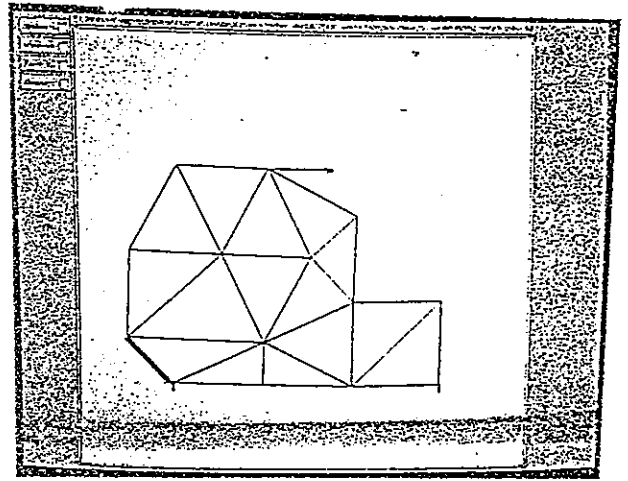


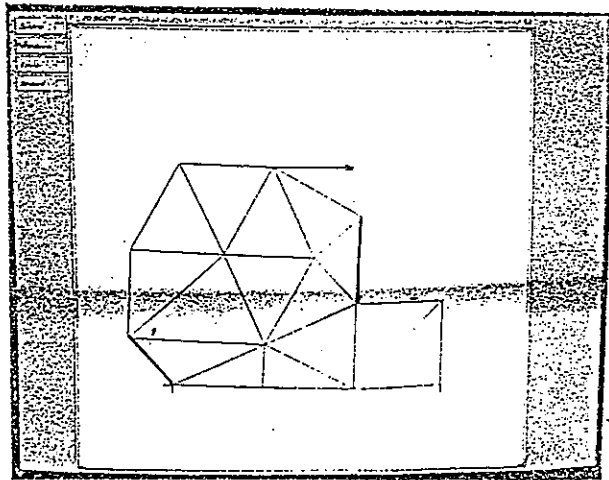
Fig. 5 The supported square solid.



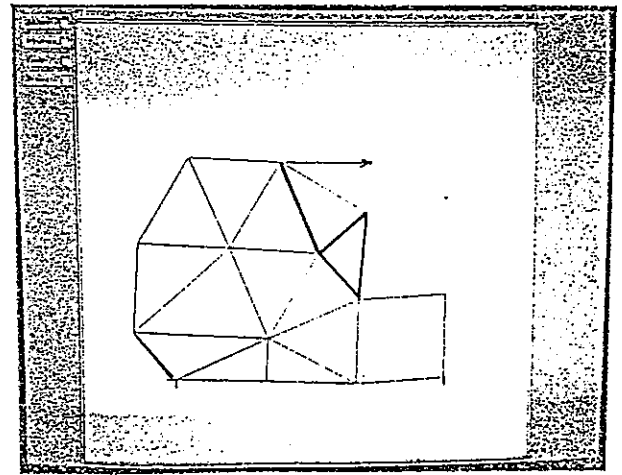
(a)



(b)



(c)

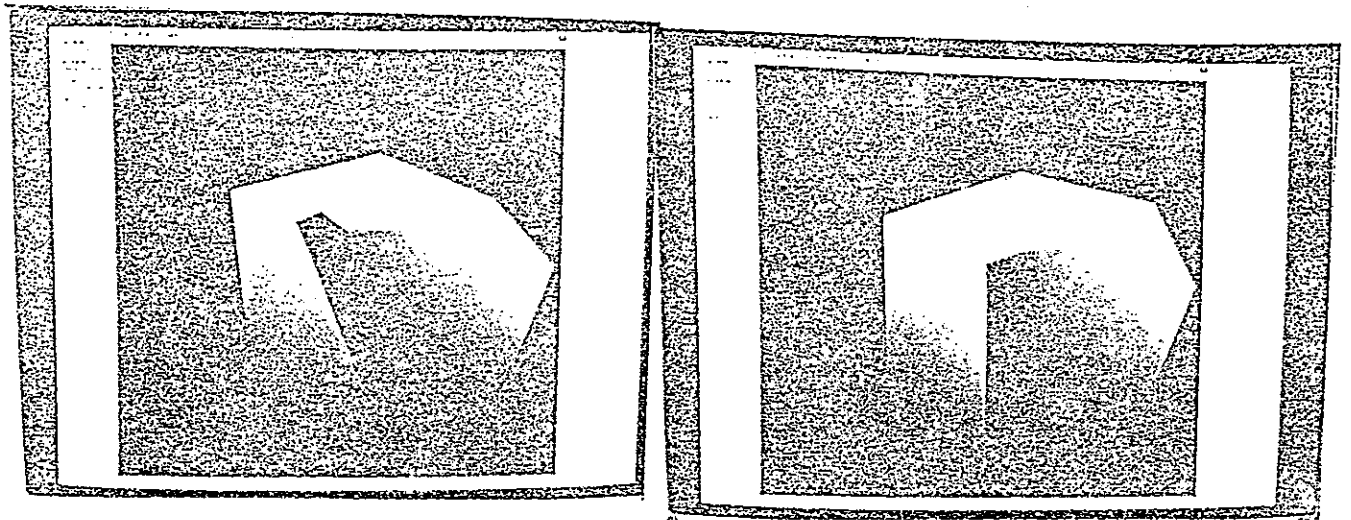


(d)

Fig. 6 Deformation of the 2D case

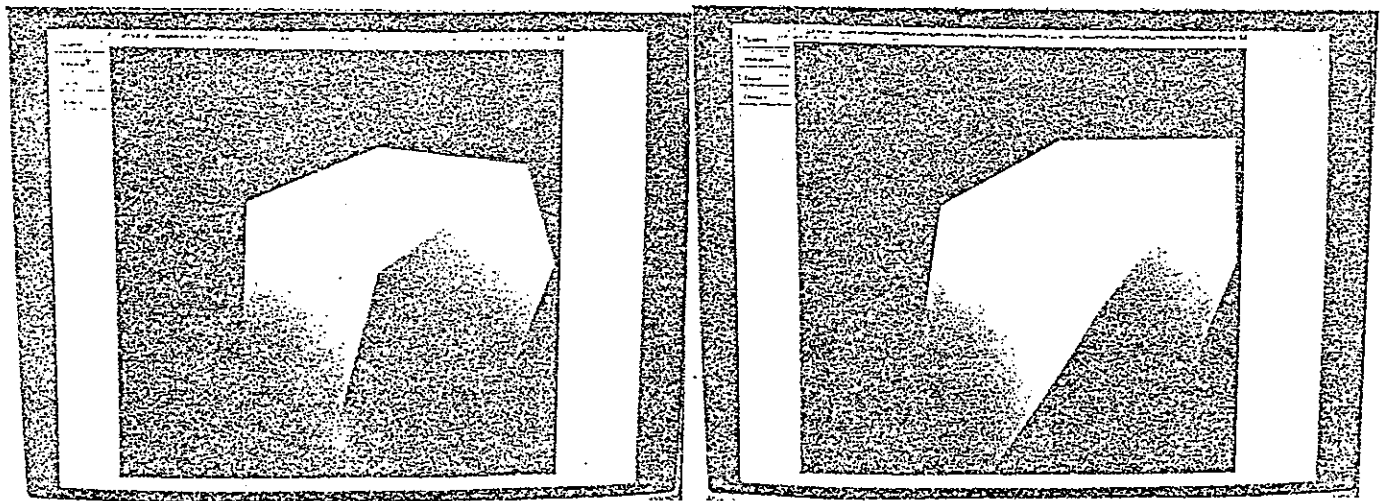
right arrow is the force

bold line segments are overloaded segments



(a)

(b)



(c)

(d)

Fig. 7 Deformation of the 3D case