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SHAPE FROM SHADING: A NONLINEAR APPROACH

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1. INTRODUCTION

Shape from shading is crucial for computer vision research simply because many objects have smooth and featureless regions that constitute the three dimensional shapes, and the only one way to inference these shapes is by the analysis of shading information since the classical stereo vision is helpless here[1]. Obviously to understand the shape from shading needs the understanding of light intensity which leads to Horn's method of shape from shading [2-5]. The trouble of Horn's method is that to estimate the shape made by a particular material requires the reflectance map of that material. In general applications we do not have prior information of the material,and hence the reflectance map of that material is not available or not well chosen from vast of reflectance maps. Thus many researchers try to limit the problem to some proper domain such as Lambertian surface[6-8], and some mathematical results are derived and their interpretations are excellent.

However, I consider the natural surface's characteristics probably can be inferred from the quantum nature of light. From the lighting characteristics one can teach computer to learn and to understand the relationship between the surface's characteristics and the lighting characteristics. Then we'll be able to solve the shape from shading problem without the need of reflectance map, perhaps by some new stereo vision analysis.

In this report I consider the Lambertian case and have developed some new methods for accessing shape from shading. Although Pentland [7] and, Lee and Rosenfeld [6] have also derived many meaningful results by assuming Lambertian surface, they actually approximate a surface patch by a sphere which has center at origin. This approximation is too strong. Here Te relax this approximation by an ellipsoid and a saddle surface with arbitrary orientation and also arbitrary center point. By using more than ten points in a small image patch Te can estimate its surface orientation, its center point, its three principle parameters, and also the light direction. The major technique used here is the nonlinear least squares, which is difficult to programming and testing. However some meaningful results are obtained through several experiments. The generalizations to specular and glossy surfaces[3,9] are also discussed here in detail.

2. IMAGE FORMATION

Let a small patch of a surface be illuminated by a distant point-source light with direction L as shown in Fig. 1. Let the surface normal be N and the viewer direction be V. Assuming the patch surface is Lambertian, which is the idealization of rough, matte surface, a surface with a Lambertian reflectance function scatters incident light isotropically. Under these assumptions the image intensity I(x,y) is given by

$$I(x,y) = \rho \lambda \; (N \cdot L),$$

where $N = (x_N, y_N, z_N)$, $L = (\ell_1, \ell_2, \ell_3)$, and ρ is the albedo (or reflectivity) of the surface and λ is the intensity of the illuminant. The assumption of Lambertian surface is valid for a variety of surfaces. Similarly the distant point-source light assumption is not too restrictive; we know that any constant distribution of illumination is equivalent to a single distant point-source illumination; this follows from application of mean-value theorem as pointed out by Pentland[7].

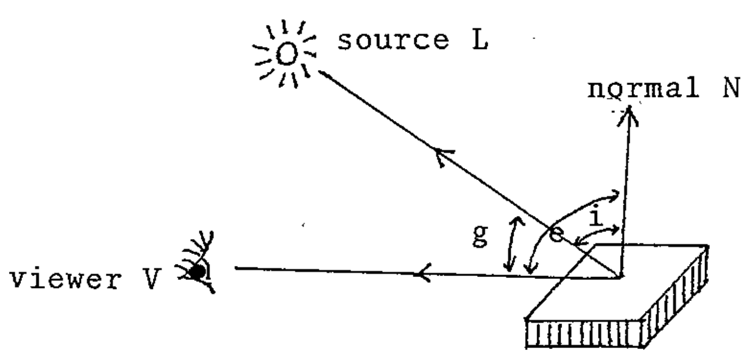


Fig. 1. A simple model of image formation.

If the surface is not Lambertian, then the reflectivity ρ depends on the incidence angle i, emittance angle e and the phase angle g as shown in Fig. 1. A general model of reflectivity for specular and glossy surfaces is given by Horn[3] :

$$I(x,y) = \lambda\rho\, k(N\cdot L) + \tfrac{1}{2}\lambda\rho\,(1-k)(n+1)[2(N\cdot L)(N\cdot V) - L\cdot V]^{n} \quad (1)$$

where k is in the range [0,1] representing the relative weights of the Lambertian and specular components, and n determines the sharpness of the specularity. The first term of Eq.(1) gives the Lambertian component and the second term gives the specular component. Another model given by Pentland[9] is

$$I(x,y) = \lambda\rho\, k(N\cdot L) + \lambda\rho\,(1-k)\left(\frac{2N - V}{\|2N - V\|}\cdot L\right)^{n} \quad (2)$$

If the difference between two times the surface normal and the viewer direction is perpendicular to the light direction then the specular term is zero. If the difference has the same direction as L then the specular term has maximum.

3. SHAPE FROM SHADING FOR SMOOTH OBJECTS

Because of simplicity we only consider the smooth (may not be convex) surfaces. Instead of approximating each surface point by a sphere centered at the origin, we approximate each surface patch by an ellipsoid or a saddle surface. Let the approximation surface be

$$Z = C\sqrt{1 - \frac{X^2}{a} - \frac{Y^2}{b}}$$

where a, b, c may be negative; in case a, b, c all positive, the surface is upper ellipsoid. Now for the consideration of surface orientation, we rotate the principle axis by an angle θ :

$$\begin{pmatrix} X' \\ Y' \end{pmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} .$$

Then the better approximation surface is

$$Z = C\sqrt{1 - \frac{(x\cos\theta + y\sin\theta)^2}{a} - \frac{(-x\sin\theta + y\cos\theta)^2}{b}}$$

with unknown parameters a, b, c, θ. Since the center of the approximation surface may not be at origin, we can shift the center to some unknown new point (X_0, Y_0, Z_0):

$$Z = Z_0 + C\sqrt{1 - \frac{((x-x_0)\cos\theta+(y-y_0)\sin\theta)^2}{a} - \frac{(-(x-x_0)\sin\theta+(y-y_0)\cos\theta)^2}{b}} \quad \ldots\ldots(3)$$

with unknown parameters a, b, c, θ, x_0, y_0, z_0.

Now we assume the surface is Lambertian, hence the shading equation becomes: for image intensity I(x,y),

$$I(x,y) = \rho\lambda\, N\cdot L$$

$$= \rho\lambda \frac{-l_1 z_x - l_2 z_y + l_3}{\sqrt{z_x^2 + z_y^2 + 1}},$$

where, from (3),

$$Z_x = -\frac{C}{\sqrt{T}}\left\{[(x-x_0)\cos\theta+(y-y_0)\sin\theta]\cos\theta/a + [(x-x_0)\sin\theta-(y-y_0)\cos\theta]\sin\theta/b\right\},$$

$$Z_y = \frac{-C}{\sqrt{T}}\left\{[(x-x_0)\cos\theta+(y-y_0)\sin\theta]\sin\theta/a + [-(x-x_0)\sin\theta+(y-y_0)\cos\theta]\cos\theta/b\right\},$$

and $T = 1 - ((x-x_0)\cos\theta+(y-y_0)\sin\theta)^2/a - (-(x-x_0)\sin\theta+(y-y_0)\cos\theta)^2/b$.

After simplification,

$$Z_x = \frac{-C}{\sqrt{T}}\left\{(x-x_0)\left(\frac{\cos^2\theta}{a} + \frac{\sin^2\theta}{b}\right) + (y-y_0)\frac{\sin 2\theta}{2}\left(\frac{1}{a} - \frac{1}{b}\right)\right\},$$

$$Z_y = \frac{-C}{\sqrt{T}}\left\{(x-x_0)\frac{\sin 2\theta}{2}\left(\frac{1}{a} - \frac{1}{b}\right) + (y-y_0)\left(\frac{\sin^2\theta}{a} + \frac{\cos^2\theta}{b}\right)\right\},$$

and $Z_x^2 + Z_y^2 + 1 = 1 +$

$$\frac{C^2}{T}\Big\{(x-x_0)^2\left[\left(\frac{\cos^2\theta}{a} + \frac{\sin^2\theta}{b}\right)^2 + \frac{\sin^2 2\theta}{4}\left(\frac{1}{a} - \frac{1}{b}\right)^2\right] +$$

$$(y-y_0)^2\left[\left(\frac{\sin^2\theta}{a} + \frac{\cos^2\theta}{b}\right)^2 + \frac{\sin^2 2\theta}{4}\left(\frac{1}{a} - \frac{1}{b}\right)^2\right] +$$

$$(x-x_0)(y-y_0)\sin 2\theta\left(\frac{1}{a^2} - \frac{1}{b^2}\right),$$

$$= 1 + \frac{C^2}{T}\left[\frac{(x-x_0)^2}{2}\left\{\frac{1}{a^2} + \frac{1}{b^2} + \cos 2\theta\left(\frac{1}{a^2} - \frac{1}{b^2}\right)\right\}\right.$$

$$+ \frac{(y-y_0)^2}{2}\left\{\frac{1}{a^2} + \frac{1}{b^2} - \cos 2\theta\left(\frac{1}{a^2} - \frac{1}{b^2}\right)\right\}$$

$$\left. + (x-x_0)(y-y_0)\sin 2\theta\left(\frac{1}{a^2} - \frac{1}{b^2}\right)\right], \qquad \ldots\ldots(4)$$

where $T = 1 - \left\{(x-x_0)^2\left(\frac{\cos^2\theta}{a} + \frac{\sin^2\theta}{b}\right) + (y-y_0)^2\left(\frac{\sin^2\theta}{a} + \frac{\cos^2\theta}{b}\right) + (x-x_0)(y-y_0)\sin 2\theta\left(\frac{1}{a} - \frac{1}{b}\right)\right\}$.

Hence $Z_x{}^2 + Z_y{}^2 + 1 = \frac{S}{T}$

where $S = (x-x_0)^2\left[\frac{-\cos^2\theta}{a} - \frac{\sin^2\theta}{b} + \frac{c^2\cos 2\theta}{2}\left(\frac{1}{a^2} - \frac{1}{b^2}\right) + \frac{c^2}{2}\left(\frac{1}{a^2} + \frac{1}{b^2}\right)\right]$

$$+ (y-y_0)^2\left[\frac{c^2}{2}\left(\frac{1}{a^2} + \frac{1}{b^2}\right) - \frac{c^2}{2}\cos 2\theta\left(\frac{1}{a^2} - \frac{1}{b^2}\right) - \frac{\sin^2\theta}{a} - \frac{\cos^2\theta}{b}\right]$$

$$+ (x-x_0)(y-y_0)\sin 2\theta\left[c^2\left(\frac{1}{a^2} - \frac{1}{b^2}\right) - \frac{1}{a} + \frac{1}{b}\right] + 1$$

The final form for $I(x,y)$ becomes:

$$I(x,y) = \rho\lambda\frac{\sqrt{T}}{S}\left\{l_1 c\left[(x-x_0)\left(\frac{\cos^2\theta}{a} + \frac{\sin^2\theta}{b}\right) + (y-y_0)\frac{\sin 2\theta}{2}\left(\frac{1}{a} - \frac{1}{b}\right)\right]\right.$$

$$+ l_2 c\left[(x-x_0)\frac{\sin 2\theta}{2}\left(\frac{1}{a} - \frac{1}{b}\right) + (y-y_0)\left(\frac{\sin^2\theta}{a} + \frac{\cos^2\theta}{b}\right)\right]$$

$$\left. + l_3\sqrt{T}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

Now for a small neighborhood or small patch of the surface, we can consider each image point (x_i, y_i) $i = 1, 2, \ldots, n$ and its intensity $I(x_i, y_i)$,

$$I(x_i, y_i) = \rho\lambda \left(\frac{-z_{xi} l_1 - z_{yi} l_2 + l_3}{\sqrt{z_{xi}^2 + z_{yi}^2 + 1}} \right), \quad \ldots\ldots\ldots(6)$$

which has simplification as (5). We can estimate all unknown parameters ($\rho\lambda$ is treated as one parameter) by nonlinear regression if n is large enough. Since $l_1^2 + l_2^2 + l_3^2 = 1$ and $\rho\lambda$ is treated as one unknown, we have $\rho\lambda$, l_1, l_2, a, b, c, θ, x_0, y_0, z_0 totally ten unknowns; i.e. $n > 10$.

In fact, we can devide the estimation procedure into two step: the first step is linear regression and the second step is nonlinear regression.

Let (6) be rewritted into

$$-Z_{xi} l_1 - Z_{yi} l_2 + l_3 = \frac{I(x_i, y_i) Q(x_i, y_i)}{\rho\lambda} + \epsilon_i, \quad i = 1, 2, \ldots, n$$

where ϵ_i represents the sum of measurement errors and approximation errors and $Q(x_i, y_i) = \sqrt{z_{xi}^2 + z_{yi}^2 + 1}$. Then forming this system of equations by

$$A \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix} = B + \underline{\epsilon}$$

where

$$A = \begin{bmatrix} -z_{x_1} & -z_{y_1} & 1 \\ -z_{x_2} & -z_{y_2} & 1 \\ \vdots & \vdots & \vdots \\ -z_{x_n} & -z_{y_n} & 1 \end{bmatrix} \quad \text{and} \quad B = \frac{1}{\rho\lambda} \begin{bmatrix} I(x_1, y_1) Q(x_1, y_1) \\ I(x_2, y_2) Q(x_2, y_2) \\ \vdots \\ I(x_n, y_n) Q(x_n, y_n) \end{bmatrix},$$

We get estimates of l_1, l_2, l_3 by

$$\begin{bmatrix} \hat{l}_1 \\ \hat{l}_2 \\ \hat{l}_3 \end{bmatrix} = (A'A)^{-1}A'B = \frac{1}{\rho\lambda} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \quad \text{since B has a factor } \frac{1}{\rho\lambda} .$$

Now since $l_1^2 + l_2^2 + l_3^2 = 1$ we have

$$(\rho\lambda)^2 = d_1^2 + d_2^2 + d_3^2,$$

or

$$l_i = d_i \Big/ \sqrt{d_1^2 + d_2^2 + d_3^2} \ , \ i = 1, 2, 3$$

Substituting this into (6) we can use nonlinear regression to solve the estimation problem.

4. GENERALIZATION TO GENERAL REFLECTIVITY MODEL

The previous discussion is based on the Lambertian model, which is more or less restrictive in real applications. The generalization to the more general cases can be done by using equations (1) and (2). Let $V = (v_1, v_2, v_3)$ be the viewer direction, where $v_1^2 + v_2^2 + v_3^2 = 1$. Then

$$N \cdot V = \frac{-v_1 z_x - v_2 z_y + v_3}{\sqrt{z_x^2 + z_y^2 + 1}} ,$$

where z_x and z_y are given in section 3. Also

$$L \cdot V = l_1 v_1 + l_2 v_2 + l_3 v_3 .$$

Thus the image model is given by, from (1),

$$I(x,y) = \lambda \rho k(-l_1 z_x - l_2 z_y + l_3)\Big/\sqrt{z_x^2 + z_y^2 + 1}$$

$$+ \frac{1}{2} \lambda \rho \, (1-k)(n+1) \left[\frac{2(-l_1 z_x - l_2 z_y + l_3)(-v_1 z_x - v_2 z_y + 1)}{z_x^2 + z_y^2 + 1} \right.$$

$$\left. - l_1 v_1 - l_2 v_2 - l_3 v_3 \right]^n, \quad 0 \leq k \leq 1.$$

This model is given by Horn[3] in 1977. A later revised model given by Pentland [9] in 1983 is, from (2),

$$I(x,y) = \lambda \rho k \left(\frac{-l_1 z_x - l_2 z_y + l_3}{\sqrt{z_x^2 + z_y^2 + 1}} \right)$$

$$+ \lambda \rho \, (1-k) \left(\frac{-(2z_x + v_1) l_1 - (2z_y + v_2) l_2 + (2 - v_3) l_3}{\sqrt{(2z_x + v_1)^2 + (2z_y + v_2)^2 + (2 - v_3)^2}} \right) .$$

The above two models are nonlinear in 11 parameters, assuming viewer coordinate system, i.e. $V = (1, 0, 0)$, and these two models can be solved by nonlinear regression method.

4. EXPERIMENTAL RESULTS

The nonlinear regression is a tough computational problem in statistical literatures. The available computing package we found is BMDP: Biomedical Data Processing software package, which contains a program named PAR for computing derivative free nonlinear regression. Because the models we discussed are quite complex and the functional derivatives would be too complicated to be derived. Thus we need derivative free computational program PAR. The first experimental image is a cylinder shown in Figure 2, with light direction around (7, -4, 6) in viewer centered coordinate system. In the beginning we have troubles for taking a good image data. Because the background of the cylinder is not uniform. Finally we decide to use a black cloth as the background and this will make the object's background dark enough to let the object data to be extracted. Also in order to reduce the noises and enhance the signal we use GRAB1 AV 4 in LIPS command language to take the same image 16 times and average them together.

Having taken the cylinder image we find the image data size is too large to be run for BMDP. So we device a program called AVERAGE. FOR (See Appendix) to reduce the size of data. This program will compute the pyramid data structure by forming 2 x 2 averages successively. By using Joystick and the command MENSUR we are able to find the background gray levels. From this we can extract the cylinder data out successfully and the total data count is 607 for averaging two times. Now by relaxing the working size LEN = 22000

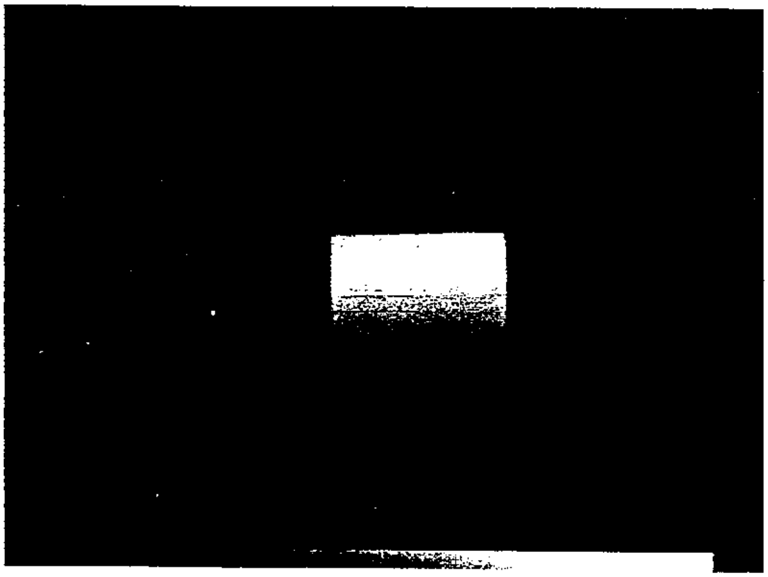
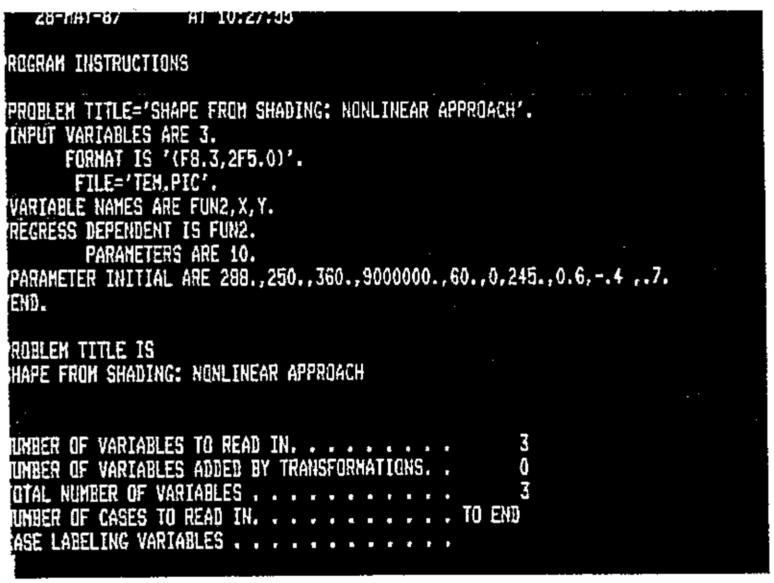


Fig.2. The cylinder image.



```
ROGRAM INSTRUCTIONS

PROBLEM TITLE='SHAPE FROM SHADING: NONLINEAR APPROACH'.
INPUT VARIABLES ARE 3.
      FORMAT IS '(F8.3,2F5.0)'.
       FILE='TEM.PIC'.
VARIABLE NAMES ARE FUN2,X,Y.
REGRESS DEPENDENT IS FUN2.
         PARAMETERS ARE 10.
PARAMETER INITIAL ARE 288.,250.,360.,9000000.,60.,0,245.,0.6,-.4 ,.7.
END.

ROBLEM TITLE IS
HAPE FROM SHADING: NONLINEAR APPROACH


UMBER OF VARIABLES TO READ IN. . . . . . . . .        3
UMBER OF VARIABLES ADDED BY TRANSFORMATIONS. .        0
OTAL NUMBER OF VARIABLES . . . . . . . . . . .        3
UMBER OF CASES TO READ IN. . . . . . . . . . . TO END
ASE LABELING VARIABLES . . . . . . . . . . . .
```

Fig.3. BMDP instructions and the parameter initial values.

we finally are able to run BMDP — PAR. By properly choosing the initial values of unknown parameters, shown in Fig.3, we get the parameter estimated values shown in Fig.4.

The parameters are

$P_1 \longleftrightarrow x_0 = 288$

$P_2 \longleftrightarrow y_0 = 250$

$P_3 \longleftrightarrow a = 360$

$P_4 \longleftrightarrow b = 9000000$

$P_5 \longleftrightarrow c = 60$

$P_6 \longleftrightarrow \theta = 0$

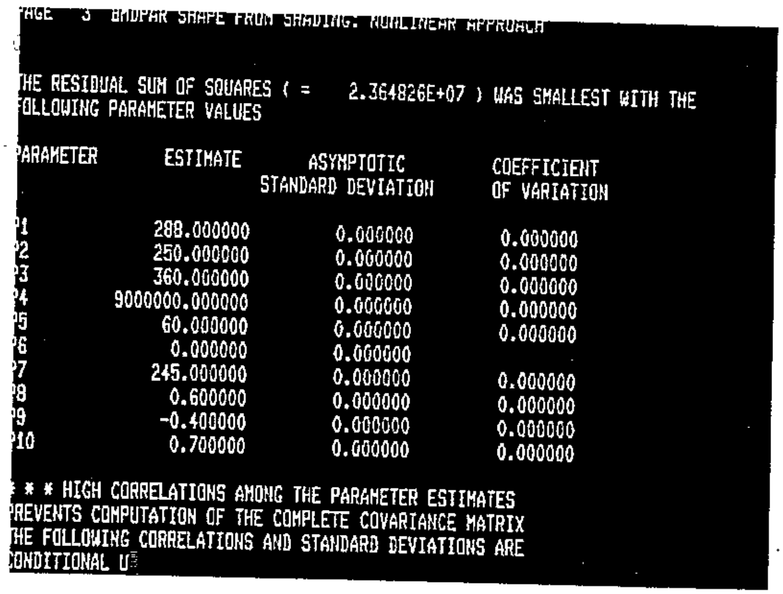
$P_7 \longleftrightarrow \rho\lambda = 245$

$P_8 \longleftrightarrow l_1 = 0.6$

$P_9 \longleftrightarrow l_2 = -0.4$

$P_{10} \longleftrightarrow l_3 = 0.7$.

The second image to be analyzed is a stone shown in Fig. 5. With properly chosen initial values the final estimated parameter values, shown in Fig.6, are 256, 120, 100, 120, 100, 0, 245, 0, 0, 1. The image data is stored in file TEMS. PIC and the BMDP instruction set is stored in file INSTRUC.DAT. The assumed Lambertian model is stored in file FUN2.FOR.



AGE 3 GHDPAR SHAPE FROM SHADING: NONLINEAR APPROACH

HE RESIDUAL SUM OF SQUARES (= 2.364826E+07) WAS SMALLEST WITH THE
OLLOWING PARAMETER VALUES

ARAMETER	ESTIMATE	ASYMPTOTIC STANDARD DEVIATION	COEFFICIENT OF VARIATION
1	288.000000	0.000000	0.000000
2	250.000000	0.000000	0.000000
3	360.000000	0.000000	0.000000
4	9000000.000000	0.000000	0.000000
5	60.000000	0.000000	0.000000
6	0.000000	0.000000	
7	245.000000	0.000000	0.000000
8	0.600000	0.000000	0.000000
9	-0.400000	0.000000	0.000000
10	0.700000	0.000000	0.000000

* * HIGH CORRELATIONS AMONG THE PARAMETER ESTIMATES
REVENTS COMPUTATION OF THE COMPLETE COVARIANCE MATRIX
HE FOLLOWING CORRELATIONS AND STANDARD DEVIATIONS ARE
ONDITIONAL U

Fig.4. The estimated parameter values.

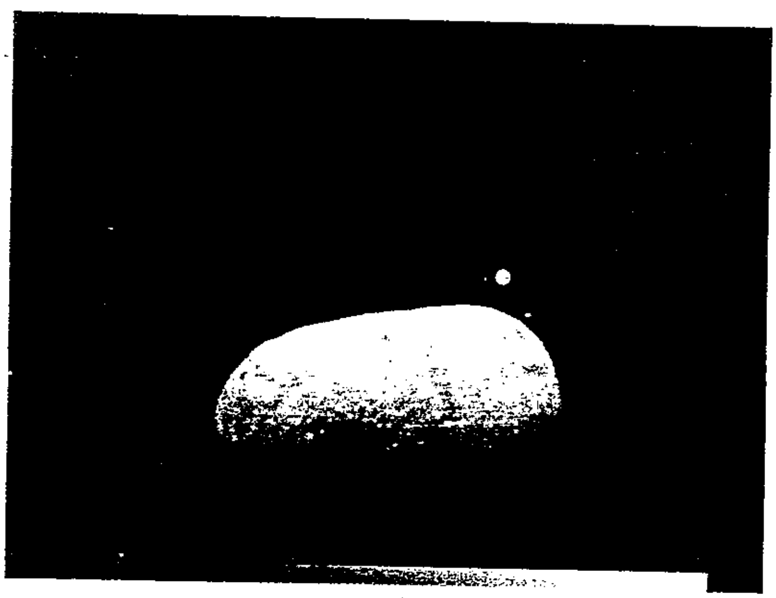


Fig.5. The stone image.

AGE 3 BNDPAR SHAPE FROM SHADING: NONLINEAR APPROACH

HE RESIDUAL SUM OF SQUARES (= 1.888920E+07) WAS SMALLEST WITH THE
OLLOWING PARAMETER VALUES

ARAMETER	ESTIMATE	ASYMPTOTIC STANDARD DEVIATION	COEFFICIENT OF VARIATION
1	256.000000	0.000000	0.000000
2	120.000000	0.000000	0.000000
3	100.000000	0.000000	0.000000
4	120.000000	0.000000	0.000000
5	100.000000	0.000000	0.000000
6	0.000000	0.000000	
7	245.000000	0.000000	0.000000
8	0.000000	0.000000	
8	0.000000	0.000000	
10	1.000000	0.000000	0.000000

* * HIGH CORRELATIONS AMONG THE PARAMETER ESTIMATES
REVENTS COMPUTATION OF THE COMPLETE COVARIANCE MATRIX
HE FOLLOWING CORRELATIONS AND STANDARD DEVIATIONS ARE
ONDITIONAL UPON SOME OF THE PARAMETERS BEING HELD CONST

Fig.6. The estimated parameter values.

5. DISCUSSION

The nonlinear regression model used for solving the shape from shading problem is clear and workable. However in general we do not have "good" initial values to run; these initial values are very important to the success of the computer running. A general way of choosing initial values is by randomly selecting a definite number of initial values in some predefined region. Then choose the one with the smallest sum of squares of errors. In our experience with BMDP — PAR, this program does not workwell. The future research of this topic will be a delicated program that can be adjusted for many parameters such as the error tolerance and the number of iterations, etc. A better way for writing this program is by APL language. It may take some time to buy such language.

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Appendix. BMDP input files and related programs

```
/PROBLEM TITLE='SHAPE FROM SHADING: NONLINEAR APPROACH'.
/INPUT VARIABLES ARE 3.
       FORMAT IS '(F8.3,2F5.0)'.
        FILE='TEMS.PIC'.
/VARIABLE NAMES ARE FUN2,X,Y.
/REGRESS DEPENDENT IS FUN2.
         PARAMETERS ARE 10.
/PARAMETER INITIAL ARE 256.,120.,100.,120.,100.,0,245.,0.,0. ,1..
/END.
```

```
      FUNCTION FUN2(X,Y,P)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION P(10)
      P1=1./P(3)
      P2=1./P(4)
      P34=P1**2+P2**2
      P34M=P1**2-P2**2
      S1=(X-P(1))**2*(-DCOS(P(6))**2*P1-DSIN(P(6))**2*P2+P(5)**2*.5*(
    $    DCOS(2*P(6))*P34M+P34))
      S2=(Y-P(2))**2*(P(5)**2*.5*(P34-DCOS(2*P(6))*P34M)-DSIN(P(6))**2*P1
    $   -DCOS(P(6))**2*P2)
      S3=(X-P(1))*(Y-P(2))*DSIN(2*P(6))*(P(5)**2*P34M-P1+P2)+1.
      S=S1+S2+S3
      T=1-((X-P(1))*DCOS(P(6))+(Y-P(2))*DSIN(P(6)))**2*P1
    $   -(-(X-P(1))*DSIN(P(6))+(Y-P(2))*DCOS(P(6)))**2*P2
      S1=P(8)*P(5)*((X-P(1))*(DCOS(P(6))**2*P1+DSIN(P(6))**2*P2)
    $   +           (Y-P(2))*DSIN(2*P(6))*.5*(P1-P2))
      S2=P(9)*P(5)*((X-P(1))*DSIN(2*P(6))*.5*(P1-P2)+(Y-P(2))*(DSIN(P(6))**
2
    $    *P1+DCOS(P(6))**2*P2))
      IF(T.GE.0)GOTO100
      TYPE *,'T < 0 ERROR'
      FUN2=1.E10
      RETURN
100   DSQT=DSQRT(T)
      SS=S1+S2+P(10)*DSQT
      FUN2=P(7)*DSQT*SS/S
      RETURN
      END
```

AVERAGE.FOR

```
      BYTE IMA(512,480)
      REAL TEM(512,480),TEMP(512,480),KK,ISUM
      LOGICAL*1 OFL(20)
      TYPE *,'---INPUT THE SOURCE IMAGE---'
      ACCEPT 1,(OFL(I),I=1,19)
1     FORMAT(20A1)
      OPEN(UNIT=1,FILE=OFL,STATUS='OLD',ACCESS='DIRECT',READONLY
     1        ,FORM='UNFORMATTED',RECL=128,ASSOCIATEVARIABLE=NREC)
      NREC=32
      DO 2 J=480,1,-1
        READ(1'NREC)(IMA(I,J),I=1,512)
2     CONTINUE
      TYPE *,'---INPUT THE TIMES OF AVERAGE---'
      ACCEPT *,N
      IX=512
      IY=480
      DO 12 I=1,IX
      DO 12 J=1,IY
      TEM(I,J)=IZEXT(IMA(I,J))
      IF(TEM(I,J).LT.30.    )TEM(I,J)=0
12    CONTINUE
      DO 10 K=1,N
        DO 11 I=1,IX
        DO 11 J=1,IY
          TEMP(I,J)=0
11      CONTINUE
        IX=IX/2
        IY=IY/2
        DO 20 I=1,IX
        DO 20 J=1,IY
          ISUM=0
          DO 30 II=2*I-1,2*I
          DO 30 JJ=2*J-1,2*J
            ISUM=ISUM+TEM(II,JJ)
30        CONTINUE
              TEMP(I,J)=ISUM/4.
20      CONTINUE
        DO 40 I=1,512
        DO 40 J=1,480
          TEM(I,J)=TEMP(I,J)
40      CONTINUE
10    CONTINUE
      TYPE *,'---INPUT THE OUTPUT IMAGE---'
      ACCEPT 1,(OFL(I),I=1,19)
      OPEN(UNIT=2,FILE=OFL,STATUS='NEW',ACCESS='SEQUENTIAL'
     1        ,FORM='FORMATTED',RECL=18)
      II=2**N
      J1=0
      DO 50 J=50,480
      I1=0
      J1=II+J1
      DO 50 I=1,512
      IF(TEM(I,J).LT.15.    )GOTO50
      I1=II+I1
        WRITE(2,51)(TEM(I,J),I1,J1)
51    FORMAT(F8.3,2I5)
50    CONTINUE
      CALL CLOSE(2)
      STOP
      END
```