

# A Secure ID-Based Authenticated Group Key Exchange Protocol Resistant to Insider Attacks\*

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Recently, several identity (ID)-based authenticated group key exchange (IDAGKE) protocols from bilinear pairings were proposed. However, they all suffered from different types of insider (participants) colluding attacks. In this paper, we present a new IDAGKE protocol from bilinear pairings. In the random oracle model and under some security assumptions, we demonstrate that the proposed protocol is a provably secure IDAGKE protocol providing forward secrecy. Meanwhile, it is secure against insider attacks.

**Keywords:** authenticated group key exchange, identity-based, bilinear pairings, insider attacks, malicious participant

## 1. INTRODUCTION

An authenticated group key exchange (AGKE) protocol is an important security mechanism which provides secure group communications for members in cooperative and distributed applications. It allows that group members generate a common key, which can be used to encrypt and authenticate communicating messages in a group. Meanwhile, it also provides entity authentication. In the past, many AGKE protocols based on the traditional certificate-based public-key systems were proposed in [1-8].

In 1984, Shamir [9] first proposed the concept of identity (ID)-based public-key system. With compared to the traditional certificate-based public-key systems, Shamir's system may simplify the certificate management. However, it has a disadvantage that the user's private key must be generated by a single Key Generator Center (KGC). In 2001, Boneh and Franklin [10] presented a practical ID-based encryption system from bilinear pairings defined on elliptic curves. The security of their scheme is based on the discrete logarithm problem. In this case, the user's private key can be generated by several sub-centers using a threshold technique. Subsequently, the ID-based cryptographic schemes based on bilinear pairings have received much attention from cryptographic researchers. Recently, many related ID-based cryptographic schemes and protocols from bilinear pairings have been proposed in [11-24].

In 2004, Choi *et al.* [12] first presented an ID-based authenticated group key exchange (IDAGKE) protocol from bilinear pairings. Unfortunately, their protocol suffered from insider (participants) colluding attacks [23, 24]. Note that insider colluding attack means that some participants can collude to impersonate other participant in this session (or other session) by some ways. In [24], Shim also proposed a modification to overcome

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Received October 19, 2009; revised May 25 & July 27, 2010; accepted September 14, 2010.

Communicated by Wen-Guey Tzeng.

\* This research was partially supported by the National Science Council of Taiwan, R.O.C., under contract No. NSC 97-2221-E-018-010-MY3.

two insider colluding attacks. However, Choi *et al.* [19] proved that Shim's modification is still insecure against the other insider colluding attacks in 2008. They also presented an improvement to resist the mentioned insider colluding attacks. In 2009, we proved that their improvement [19] still suffered from an insider colluding attack in [21]. Meanwhile, these protocols above don't address the problem of malicious participants. Note that, a malicious participant is considered as a legitimate participant, but she/he is fully controlled by an adversary. Fortunately, Katz and Shin [25] presented a security model and a universal composability complier for this issue in authenticated group key exchange (AGKE) protocols. This provides the formal security definition of AGKE protocols in the existence of malicious participants. Adopting the Katz-Shin complier into an AGKE protocol, it can employ the explicit group key confirmation property to prevent malicious participants. However, the complier [25] requires an additional round and  $O(n)$  signature verifications for each participant, where  $n$  is the number of group members.

In this paper, we propose a robust ID-based authenticated group key exchange (ID-AGKE) protocol from bilinear pairings. Particularly, our protocol is based on the Sakai-Kasahara's key construction [26]. In our proposed protocol, we adopt the concept of the Katz-Shin complier to resist malicious participants. For decreasing the computation cost of signature verifying, we use a batch verification technique called the small exponents test [27] into our protocol. In the random oracle model [28] and under some security assumptions, we will prove that our protocol is a provably secure IDAGKE protocol providing forward secrecy and achieves the Katz-Shin's security model. This means that the proposed protocol is secure against insider attacks and malicious participants disturbing. Finally, we make the comparisons between the previously proposed IDAGKE protocols and our proposed protocol.

The remainder of this paper is organized as follows. In section 2, we briefly review the concepts of bilinear pairings, key constructions, and related mathematical assumptions. The security model and notions of authenticated group key exchange is given in section 3. In section 4, we present our IDAGKE protocol. Security analysis of the proposed protocol is given in section 5. In section 6, we make the performance analysis and comparisons. Our conclusions and future work are drawn in section 7.

## 2. PRELIMINARIES

In this section, we depict compendiously the concepts of bilinear pairings, key constructions of ID-based public-key system from bilinear pairings, and the related mathematical assumptions.

### 2.1 Bilinear Pairings

Let  $G_1$  and  $G_2$  be two cyclic groups with a prime order  $q$ , where  $G_1$  is additive and  $G_2$  is multiplicative. Here,  $G_2$  is a subgroup of the multiplicative group over a finite field. A bilinear pairing is defined as a map  $e: G_1 \times G_1 \rightarrow G_2$ . If the map  $e$  satisfies the following three properties, it is called an admissible bilinear map:

- (1) Bilinear:  $e(aP, bQ) = e(P, Q)^{ab}$  for all  $P, Q \in G_1$  and  $a, b \in \mathbb{Z}_q^*$ .
- (2) Non-degenerate: There exists  $P, Q \in G_1$  such that  $e(P, Q) \neq 1$ .

- (3) Computable: For all  $P, Q \in G_1$ , there exists an efficient algorithm to compute the value  $e(P, Q)$ .

For the details of bilinear pairings, we can refer to [10, 16] for full descriptions.

## 2.2 Key Constructions

In general, there are two kinds of key constructions for ID-based public-key systems from bilinear pairings. We describe them as below:

- Construction I: In 2000, Sakai *et al.* presented the key Construction I in [29]. In this construction, there exists a special hash function called a map-to-point hash function  $H_G: \{0, 1\}^* \rightarrow G_1$  [10]. For a user  $U$  with identity  $ID_U$ , the user's private key  $DID_U = H_G(ID_U) \cdot s$ , where  $s$  is the private key of Key Generation Center (KGC). Up to now, many ID-based cryptographic schemes and protocols based on this construction were proposed in [10-13, 16, 17, 19, 20, 22].
- Construction II: In 2003, Sakai and Kasahara proposed the key Construction II in [26]. In this construction, there exists a general hash function  $H: \{0, 1\}^* \rightarrow Z_q^*$ . For a user  $U$  with identity  $ID_U$ , the user's private key  $DID_U = \frac{1}{s + H(ID_U)} \cdot P$ , where  $s$  is the private key of KGC, and  $P$  is a generator of the group  $G_1$ . Until now, some ID-based cryptographic schemes based on this construction were proposed in [14, 15]. Note that the expensive map-to-point hash function is not required in this construction.

## 2.3 Related Mathematical Problems and Assumptions

For convenience to prove the security of our proposed protocol, we summarize some important mathematical hard problems and assumptions for bilinear pairings defined on elliptic curves as follows. For the details of them, we can refer to [15, 16] for full descriptions.

- Decision Diffie-Hellman problem (DDH) in the group  $G_2$ : Given  $g = e(P, P)$ ,  $g^a, g^b, g^c \in G_2$ , for some  $a, b, c \in_R Z_q^*$ , the DDH problem (in  $G_2$ ) is to distinguish  $(g, g^a, g^b, g^{ab})$  from  $(g, g^a, g^b, g^c)$ .
- DDH assumption: No probabilistic polynomial time (PPT) algorithm  $\mathcal{A}$  can solve the DDH problem (in  $G_2$ ) with a non-negligible advantage.
- $k$ -CAA problem: Let  $k$  be a positive integer and  $s \in_R Z_q^*$ . Given  $c_1, c_2, \dots, c_k \in Z_q^*$ ,  $P \in G_1$ ,  $P_{pub} = s \cdot P$ , and  $\frac{1}{s+c_1}P, \frac{1}{s+c_2}P, \dots, \frac{1}{s+c_k}P$ , the  $k$ -CAA problem is to compute  $\frac{1}{s+c}P$ , where  $c \in Z_q^*$  and  $c \notin \{c_1, c_2, \dots, c_k\}$ .
- $k$ -CAA assumption: No PPT algorithm  $\mathcal{A}$  can solve the  $k$ -CAA problem with a non-negligible advantage.
- Generalized  $k$ -CAA (Gk-CAA) problem: Let  $k = n \cdot m$  be a positive integer and  $s \in_R Z_q^*$ . Given  $c_1, c_2, \dots, c_n, d_1, d_2, \dots, d_m \in Z_q^*$ ,  $P \in G_1$ ,  $P_{pub} = s \cdot P$ ,  $\frac{d_1}{s+c_1}P, \frac{d_2}{s+c_1}P, \dots, \frac{d_m}{s+c_1}P, \frac{d_1}{s+c_2}P, \frac{d_2}{s+c_2}P, \dots, \frac{d_m}{s+c_2}P, \dots$ , and  $\frac{d_1}{s+c_n}P, \frac{d_2}{s+c_n}P, \dots, \frac{d_m}{s+c_n}P$ , the generalized  $k$ -

CAA problem is to compute  $\frac{d}{s+c}P$ , where  $c, d \in \mathbb{Z}_q^*$ ,  $c \notin \{c_1, c_2, \dots, c_n\}$ , and  $d \notin \{d_1, d_2, \dots, d_m\}$ .

The  $k$ -CAA problem and the  $Gk$ -CAA problem in the group  $G_1$  are computationally equivalent [15]. It means that  $Gk$ -CAA problem is a hard problem in the group  $G_1$ .

## 2.4 Notations

The following notations are used throughout this paper:

- $e$ : an admissible bilinear map,  $e: G_1 \times G_1 \rightarrow G_2$ .
- $ID_i$ : the identity of participant  $U_i$ .
- $P$ : a generator of the group  $G_1$ .
- $s$ : the system private key  $s \in \mathbb{Z}_q^*$ .
- $P_{pub}$ : the system public key,  $P_{pub} = s \cdot P$ .
- $g$ : a public value belongs to the group  $G_2$ ,  $g = e(P, P)$ .
- $H()$ : a one-way hash function,  $H: \{0, 1\}^* \rightarrow \mathbb{Z}_q^*$ .
- $H_1()$ : a one-way hash function,  $H_1: \{0, 1\}^* \times G_2 \rightarrow \mathbb{Z}_q^*$ .
- $H_2()$ : a one-way hash function,  $H_2: \{0, 1\}^* \times G_2^2 \rightarrow \mathbb{Z}_q^*$ .

## 3. SECURITY MODEL AND NOTIONS

In this section, we briefly review the security model and notions for authenticated group key exchange protocol. The following definitions are referred to [2, 12, 25, 30].

**Participants and Initialization** We assume that each participant  $U_i$  has an unique identity  $ID_i \in \{0, 1\}^l$ , where  $l$  is a bit length. For simplicity, there is a fixed set  $G = \{U_1, U_2, \dots, U_n\}$  with polynomial-size of potential participants. Here, we allow that each participant  $U_i \in G$  performs the protocol many times with different participants. In other words, each  $U_i$  has the unlimited number of instances to execute the AGKE protocol called oracles  $\Pi_i^t$  with  $t \in \mathbb{N}$ . We denote the  $t$ th instance oracle of participant  $U_i$  as  $\Pi_i^t$ .

An ID-based authenticated group key exchange protocol requires the following algorithms:

- Setup algorithm: The system private key  $s$  and the public parameters are generated by this algorithm.
- Extract algorithm: Each participant  $U_i$  obtains her/his private key  $DID_i$  by this algorithm,  $DID_i \leftarrow \text{Extract}_s(ID_i)$ .

Note that, the public parameters and the set of participants' identities  $ID = \{ID_1, ID_2, \dots, ID_n\}$  are known by all participants (including adversary).

**Session Group key, Session ID, and Partner ID** In each session, a new set  $G$  of  $k$  participating oracles is considered, where  $1 \leq k \leq n$ . By  $g_i$  for  $i \in [1, k]$ , we denote the index of the user related to the  $i$ th oracle involved in  $G$ , where this  $i$ th oracle is defined as  $\Pi(G,$

$i$ ). Thus, for each  $i \in [1, k]$  there exists  $\Pi(G, i) = \Pi_{g_i}^t \in G$  for some  $t \in N$ . Every participating oracle  $\Pi_U^t \in G$  computes the *session group key*  $SK_U^t$ . Each session is identified by a unique *session ID*  $SID(\Pi_U^t)$ . This value is known to all participating oracles in the same session. Similarly, each oracle  $\Pi_U^t \in G$  obtains a *partner ID*  $PID(\Pi_U^t)$  that contains the identities of participants (including  $U$ ), formally  $PID(\Pi_U^t) = \{ID_{U_{g_i}} \mid \Pi(G, i) \in G, \forall i = 1, 2, \dots, k\}$ . Note that the session ID and the partner ID are public information.

**Related Notions** We say that the oracle  $\Pi_i^t$  is *accepted*, when it can compute a valid session key. An oracle may terminate without accepting. In this case, it does not output any session key to all. Whether or no, an oracle has accepted or decided to terminate without accepting is a public information. We say that two oracles  $\Pi_i^t$  and  $\Pi_j^s$  are *partnered* if and only if it satisfies following three conditions: (1) Two oracles have accepted; (2)  $ID_{U_i} \in PID(\Pi_j^s)$  and  $ID_{U_j} \in PID(\Pi_i^t)$ ; (3)  $SID(\Pi_i^t) = SID(\Pi_j^s)$ .

**Adversarial Model** Formally, an adversary  $\mathcal{A}$  is a probabilistic polynomial-time algorithm. We allow  $\mathcal{A}$  to potentially control all communications completely in ID-based authenticated group key exchange (IDAGKE) protocol and may interact with the simulated group participants (*i.e.*, a challenger) by issuing the following oracle queries:

- **Execute query:** When the adversary  $\mathcal{A}$  issues this query on  $ID$  to the oracles, the oracle  $\Pi_i^t$  returns the entire transcripts of an honest execution between the participants belong to  $ID$ . Note that  $\mathcal{A}$  can choose the number of participants.
- **Extract query:** When the adversary  $\mathcal{A}$  issues this query on  $ID_\alpha$  to the oracles, the oracle  $\Pi_i^t$  returns the corresponding private key  $DID_\alpha$ . In particular,  $ID_\alpha$  is a non-target identity,  $ID_\alpha \notin ID$ .
- **Send query:** When the adversary  $\mathcal{A}$  issues this query on messages to the oracles, the oracle  $\Pi_i^t$  returns the response according to the steps of the IDAGKE protocol.
- **Reveal query:** In this query, the adversary  $\mathcal{A}$  can get a group session key from the oracle  $\Pi_i^t$ .
- **Corrupt query:** In this query, the adversary  $\mathcal{A}$  can get a private key  $DID_i$  of  $ID_i$  from the oracle  $\Pi_i^t$ .
- **Test query:** The adversary  $\mathcal{A}$  can issue a single *Test query* to a fresh oracle  $\Pi_i^t$  upon receiving this query, the oracle  $\Pi_i^t$  flips an unbiased coin  $b$ . If  $b = 1$ ,  $\Pi_i^t$  returns the group session key. Otherwise,  $\Pi_i^t$  returns a random string.

In the above model, there are two types of adversaries according to their queries. A **passive adversary** is allowed to make *Execute*, *Reveal*, *Corrupt*, and *Test queries*; an **active adversary** is additionally to make *Extract* and *Send queries*. Here, we also use the *Execute query* to get more precise analysis, even though this query can be substituted for making the *Send query* repeatedly.

**Freshness** An oracle  $\Pi_i^t$  is called *fresh*, if the following conditions hold: (1)  $\Pi_i^t$  has accepted; (2) Nobody has been asked for a *Corrupt query* before  $\Pi_i^t$  accepts; (3) Neither  $\Pi_i^t$  nor its partners have been issued a *Reveal query*. Here, we assume that all oracles are considered *fresh*.

**Secure AGKE** A secure authenticated group key exchange protocol contains following four parts:

- (1) Freshness: As mentioned above.
- (2) The security of AGKE protocol: The security of ID-based authenticated group key exchange (IDAGKE) protocol is defined in the following game played with an active adversary  $\mathcal{A}$  and an infinite set of oracles  $\Pi_i^t$ , where  $\Pi_i^t$  is the oracle of participant  $U_i \in G$ .
  1. The system private key, the public parameters, and the participants' private keys are generated in the initialization phase.
  2. The adversary  $\mathcal{A}$  can issue some queries and gets back the answers corresponding to oracles.
  3. Finally, the adversary  $\mathcal{A}$  outputs its guess  $b'$  for the coin  $b$  in the *Test* query and terminates.

In the above game, the advantage of  $\mathcal{A}$  is defined by the ability to distinguish the group session key from a random string. Let *Succ* be the event that  $\mathcal{A}$  queries the *Test* oracle and correctly guesses the coin  $b$ , where the coin is used to answer the *Test query* in the *Test* oracle. The advantage of the adversary is defined as  $Adv(k) = |2 \cdot \Pr[Succ] - 1|$ . We say that an IDAGKE protocol is secure, if the advantage  $Adv(k)$  is negligible for any adversary.

- (3) Forward secrecy: We say that an ID-based authenticated group key exchange protocol  $\Psi$  provides *forward secrecy*. It means that the previous establishing session group keys in  $\Psi$  is preserved if an adversary  $\mathcal{A}$  obtains private keys of the protocol participants in later protocol sessions. We define the advantage of  $\mathcal{A}$  to attack  $\Psi$  running in time  $t$  by  $Adv_{\Psi}^{AGKE-fs}(t, q_{ex}, q_s)$ , where  $q_{ex}$  and  $q_s$  are the maximum numbers of issuing the *Execute* and *Send queries*, respectively.
- (4) Authentication: We say that an ID-based AGKE protocol provides the *implicit key authentication*. If participants are assured that no one other than its partners can learn the value of a particular session key. In particular, any adversary should not learn the key. Note that, the property of implicit key authentication does not guarantee that partners have actually obtained the key.

In 2005, Katz and Shin [25] defined the concept of insider attacks for authenticated group key exchange protocol. In the following, we briefly review the definitions of insider attacks in the following.

**Insider Attacks** An ID-based authenticated group key exchange (IDAGKE) protocol is secure against *insider attacks*, if it satisfies following three conditions:

- (1) Secure AGKE: As mentioned above.
- (2) Secure against insider impersonation attack: Firstly, we say that an adversary  $\mathcal{A}$  succeeds in an insider (participant) impersonation attack, if there exist a participant  $U_j$  and an instance  $\Pi_i^t$  such that (1)  $\mathcal{A}$  impersonates the (uncorrupted) participant  $U_j$  to (accepting) the instance  $\Pi_i^t$ ; (2) Neither  $U_i$  nor  $U_j$  are corrupted at the oracle  $\Pi_i^t$  accepts. We say an IDAGKE protocol secure against *insider impersonation attack* if the advantage of any probabilistic polynomial-time adversary succeeds in the above attack is negligible.

- (3) Key agreement: We say that an IDAGKE protocol does not provide *key agreement*, if there exist two partnered instances  $\Pi_i^t$  and  $\Pi_j^s$  satisfying (1) Neither  $U_i$  nor  $U_j$  are corrupted; (2) Two group session keys  $SK_i^t \neq SK_j^s$ , where  $SK_i^t$  and  $SK_j^s$  are generated by two oracles  $\Pi_i^t$  and  $\Pi_j^s$ , respectively.

**Malicious Participant** We say that a participant  $U_m$  is a malicious participant in ID-based authenticated group key exchange protocol, if  $U_m$  is a legitimate participant in the protocol, nevertheless, she/he is fully controlled by an adversary.

**Remark 1:** If an ID-based authenticated group key exchange (IDAGKE) protocol is secure against insider attacks, then the IDAGKE protocol is able to resist malicious participant. Because of the condition “*key agreement*” considers explicit key confirmation property and the condition “*secure against insider impersonation attack*” concerns with mutual authentication property. For a malicious participant, it renders toothless in the protocol.

#### 4. PROPOSED PROTOCOL

In this section, we propose a new ID-based authenticated group key exchange (ID-AGKE) protocol with resisting insider attacks. In other words, the proposed IDAGKE protocol can withstand malicious participants. At first, we present the initialization phase of our protocol. The Key Generation Center (KGC) executes the *Setup algorithm* to generate the system private key  $s$  and the public parameters  $Param = \{e, G_1, G_2, q, P, P_{pub}, g, H, H_1, H_2\}$ , where  $g = e(P, P)$ . When a participant  $U$  with the identity  $ID_U$  wants to obtain her/his private key  $DID_U$ ,  $U$  submits its identity  $ID_U$  to the KGC. Upon receiving the request of the participant  $U$ , the KGC runs the *Extract algorithm* to compute  $DID_U = \frac{1}{s + H(ID_U)} \cdot P$  and returns it to  $U$  via a secure channel.

Let  $\{U_1, U_2, \dots, U_n\}$  be a set of participants who want to establish a group session key  $SK$ . Note that the indices are taken in a cyclic, *i.e.*,  $U_{n+1}$  is  $U_1$  and  $U_0$  is  $U_n$ . Assume that each participant  $U_i$  has a unique identity  $ID_i$ ,  $U_i$ 's public/private key is  $(ID_i, DID_i = \frac{1}{s + H(ID_i)} P)$ ,  $PID$  is the concatenation of the identities of participants taking part in a session, *i.e.*  $PID = ID_1 || ID_2 || \dots || ID_n$ , and  $M \in \{0, 1\}^*$  is a pre-known message by all participants which contains some conference information such as the conference title, date, and location. For authentication, we adopt Cui *et al.*'s ID-based signature scheme [15] into the proposed protocol. In addition, we also use the small exponents test [27] to strengthen the security of batch verification. The details of our protocol are described as follows,

**Round 1** Each participant  $U_i$  picks a random number  $a_i \in Z_q^*$  and computes  $r_i = g^{a_i}$ ,  $h_i = H_1(ID_i, M, PID, r_i)$ , and  $\sigma_i = (a_i + h_i) \cdot DID_i$ . Then, each  $U_i$  broadcasts  $(ID_i, r_i, \sigma_i)$  to all other participants and keeps  $a_i$  as secret.

**Round 2** Upon receiving  $(ID_{i-1}, r_{i-1}, \sigma_{i-1})$  and  $(ID_{i+1}, r_{i+1}, \sigma_{i+1})$ , each participant  $U_i$  verifies them independently by the following equation. For  $j \in \{i-1, i+1\}$ ,

$$g^{h_j} \cdot r_j \stackrel{?}{=} e(P_{pub} + H(ID_j) \cdot P, \sigma_j),$$

where  $h_j = H_1(ID_j, M, PID, r_j)$ . If the two verifications hold, each  $U_i$  uses  $r_{i-1}$  and  $r_{i+1}$  to compute  $D_i = (r_{i+1}/r_{i-1})^{a_i}$ . Note that the indices are taken in a cyclic, i.e.  $U_{n+1}$  is  $U_1$  and  $U_0$  is  $U_n$ . Thus,  $D_1 = (r_2/r_n)^{a_1}$  and  $D_n = (r_1/r_{n-1})^{a_n}$ .

Then, each  $U_i$  chooses a random number  $b_i \in Z_q^*$  and computes  $t_i = g^{b_i}$ ,  $k_i = H_2(ID_i, PID, R_1, D_i, t_i)$ , and  $\rho_i = (b_i + k_i)DID_i$ , where  $R_1 = r_1 \parallel r_2 \parallel \dots \parallel r_n$ . Finally,  $U_i$  broadcasts  $(ID_i, D_i, t_i, \rho_i)$  to all other participants.

**Round 3** Upon receiving all  $(ID_j, D_j, t_j, \rho_j)$  for  $j = 1, 2, \dots, n$  and  $j \neq i$ , each participant  $U_i$  first selects a random vector  $(v_1, v_2, \dots, v_{i-1}, v_{i+1}, \dots, v_n)$  with the security level  $l_b = 160$  bits elements from  $Z_q$  and verifies them by the following equation:

$$g^{\sum_{j=1, j \neq i}^n v_j \cdot k_j} \cdot \prod_{j=1, j \neq i}^n t_j^{v_j} \stackrel{?}{=} e(P_{pub}, \sum_{j=1, j \neq i}^n (v_j \cdot \rho_j)) \cdot e(P, \sum_{j=1, j \neq i}^n (v_j \cdot H(ID_j) \cdot \rho_j)),$$

where  $k_j = H_2(ID_j, PID, R_1, D_j, t_j)$  and  $R_1 = r_1 \parallel r_2 \parallel \dots \parallel r_n$ . If the batch verification holds, each  $U_i$  computes the group session key

$$SK_i = (r_{i-1})^{na_i} \cdot D_i^{n-1} \cdot D_{i+1}^{n-2} \cdot \dots \cdot D_n^{i-1} \cdot D_1^{i-2} \cdot \dots \cdot D_{i-2}.$$

Then, each  $U_i$  selects a random number  $c_i \in Z_q^*$  and computes  $u_i = g^{c_i}$ ,  $w_i = H_2(ID_i, PID, R_2, SK_i, u_i)$ , and  $\tau_i = (c_i + w_i) \cdot DID_i$ , where  $R_2 = D_1 \parallel D_2 \parallel \dots \parallel D_n$ . Finally,  $U_i$  broadcasts  $(ID_i, u_i, \tau_i)$  to all other participants.

**Key Confirmation** Upon receiving all  $(ID_j, u_j, \tau_j)$  for  $j = 1, 2, \dots, n$  and  $j \neq i$ , each participant  $U_i$  first selects a random vector  $(v_1, v_2, \dots, v_{i-1}, v_{i+1}, \dots, v_n)$  with the security level  $l_b = 160$  bits elements from  $Z_q$  and verifies them by the following equation:

$$g^{\sum_{j=1, j \neq i}^n v_j \cdot w_j} \cdot \prod_{j=1, j \neq i}^n u_j^{v_j} \stackrel{?}{=} e(P_{pub}, \sum_{j=1, j \neq i}^n (v_j \cdot \tau_j)) \cdot e(P, \sum_{j=1, j \neq i}^n (v_j \cdot H(ID_j) \cdot \tau_j)),$$

where  $w_j = H_2(ID_j, PID, R_2, SK_j, u_j)$  and  $R_2 = D_1 \parallel D_2 \parallel \dots \parallel D_n$ . If the batch verification holds, each  $U_i$  can confirm that her/his  $SK_i$  is equal to all  $SK_j$  for  $j = 1, 2, \dots, n$  and  $j \neq i$ . In this case, no malicious participant exists in this group key establishment. All participants can use the group session key  $SK_i$  to communicate securely. Otherwise, there exists at least one malicious participant in the group.

**Remark 2:** After running the proposed protocol, if no malicious participants exist in the group, all participants can compute the same group session key. Here, we demonstrate that all the computed keys  $SK_i = (r_{i-1})^{na_i} \cdot D_i^{n-1} \cdot D_{i+1}^{n-2} \cdot \dots \cdot D_{i-2}$  are identical for  $i = 1, 2, \dots, n$ . Set  $A_{i-1} = (r_{i-1})^{a_i} = g^{a_{i-1}a_i}$ ,  $A_i = (r_{i-1})^{a_i} \cdot D_i = g^{a_{i-1}a_i}$ ,  $A_{i+1} = (r_{i-1})^{a_i} \cdot D_i \cdot D_{i+1} = g^{a_{i-1}a_{i+2}}$ , ..., etc., where  $r_i = g^{a_i}$ ,  $D_i = (r_{i+1}/r_{i-1})^{a_i}$ ,  $g = e(P, P)$ . Thus, we can obtain  $SK_i = A_{i-1} \cdot A_i \cdot A_{i+1} \cdot \dots \cdot A_{i-2}$ . Indeed, all participants can compute the same group session key  $SK = e(P, P)^{a_1 a_2 + a_2 a_3 + \dots + a_n a_1}$ .



## 5. SECURITY ANALYSIS

In this section, we prove that proposed protocol can achieve the security requirements defined in section 3 in the random oracle model [28] and under some security assumptions defined in section 2.3.

### 5.1 Insider Impersonation Attack

Here, we will prove that the proposed protocol is secure against insider (participants) impersonate attack. Firstly, we want to show that the adopted batch verification in Round 3 and Key confirmation is secure in the following lemma. Note that for the security proof of Lemma 1 we use the similar way in [27].

**Lemma 1** For the security level  $l_b = 160$  bits, the batch verification of our proposed protocol in Round 3 and Key confirmation is secure. Precisely, the probability of accepting an invalid batch signature is only  $2^{-l_b} = 2^{-160}$ .

**Proof:** At first we prove that the batch verification in Round 3 is secure. It is easy to see

that if  $g^{k_j} \cdot t_j = e(P_{pub}, \rho_j) \cdot e(P, H(ID_j) \cdot \rho_j)$  holds for all  $j \in [1, n] \setminus i$ , then  $g^{\sum_{j=1, j \neq i}^n v_j \cdot k_j} \cdot \prod_{j=1, j \neq i}^n t_j^{v_j} = e(P_{pub}, \sum_{j=1, j \neq i}^n (v_j \cdot \rho_j)) \cdot e(P, \sum_{j=1, j \neq i}^n (v_j \cdot H(ID_j) \cdot \rho_j))$  holds for any vector  $(v_1, v_2, \dots, v_{i-1}, v_{i+1}, \dots, v_n) \in Z_q$ . Without of loss generality, we let  $j = 1, 2, \dots, n-1$ . Now, we must consider the other direction that if the batch verification holds, then  $g^{k_j} \cdot t_j = e(P_{pub}, \rho_j) \cdot e(P, H(ID_j) \cdot \rho)$  holds for all  $j \in [1, n-1]$  except with probability at most  $2^{-l_b}$ .

Set  $X_j = g^{k_j} \cdot t_j$  and  $Y_j = e(P_{pub}, \rho_j) \cdot e(P, H(ID_j) \cdot \rho_j)$ . Since  $X_j$  and  $Y_j$  are in the group  $G_2$ , we have  $X_j = e(P, P)^{x_j}$  and  $Y_j = e(P, P)^{y_j}$  for some  $x_j, y_j \in Z_q$ . Thus, the batch verification can be written as  $\prod_{j=1}^{n-1} e(P, P)^{x_j} = \prod_{j=1}^{n-1} e(P, P)^{y_j}$ . It implies  $e(P, P)^{\sum_{j=1}^{n-1} z_j} = 1$ , where  $z_j = x_j - y_j$ . Because the batch verification holds, the following equation must be true  $\sum_{j=1}^{n-1} v_j \cdot z_j \equiv 0 \pmod q$  for the vector  $(v_1, v_2, \dots, v_{n-1}) \in Z_q$ .

Assume that at least one individual equation does not hold. Without of loss generality, we let the first equation does not hold, *i.e.*,  $j = 1$ . This means that the value  $z_1 \neq 0$ . Since  $q$  is a prime, we can find an inverse  $\alpha_1$  of  $z_1$  such that  $z_1 \cdot \alpha_1 \equiv 1 \pmod q$ . Hence, we have  $v_1 \equiv -\alpha_1 \cdot \sum_{j=2}^{n-1} v_j \cdot z_j \pmod q$ . Let  $E$  be the event that  $X_1 \neq Y_1$  but the batch verification holds. Then, we can get a result that given a fixed vector  $(v_2, \dots, v_{n-1})$ , there exists exactly one value  $v_1$  such that the event  $E$  occurs. Therefore, the probability of  $E$  with  $v_1$  is  $\Pr[E | \Delta'] = 2^{-l_b}$ , where  $\Delta' = (v_2, \dots, v_{n-1})$ . If we select  $v_1$  at random and the sum of all possible values of  $(v_2, \dots, v_{n-1})$ , we obtain  $\Pr[E] \leq \sum_{j=1}^{|\Delta'|} (\Pr[E | \Delta'] \cdot \Pr[\Delta'])$ , where  $|\Delta'|$  be the number of all possible values for  $\Delta'$ . That is  $\Pr[E] \leq \sum_{j=1}^{2^{b(n-2)}} (2^{-l_b} \cdot 2^{-l_b(n-2)}) = 2^{-l_b}$ , where  $l_b = 160$  bits. Following the similar method, the batch verification in Key confirmation is also secure.  $\square$

Then, combining Lemma 1 and the security of Cui *et al.*'s single signature [15] we

prove that the proposed protocol is secure against insider (participants) impersonation attack in Theorem 2.

**Theorem 2** Under the generalized  $k$ -CAA assumption and the security level  $l_b$ , the proposed ID-based authenticated group key exchange (IDAGKE) protocol is secure against insider impersonation attack.

**Proof:** Firstly, we have known that Cui *et al.*'s single signature is secure against forgery attack under the generalized  $k$ -CAA assumption [15]. Since we only verify the single signature in Round 2, we can claim that Round 2 of our protocol is secure against insider impersonation attack. In Round 3 and Key confirmation, we use the batch verification technique to reduce the computational cost. By Lemma 1, we have proven that the batch verification of our protocol in Round 3 and Key confirmation is secure against forgery attack under the security level  $l_b = 160$  bits. The probability of accepting an invalid signature is  $2^{-l_b} = 2^{-160}$ . In other words, even if  $(n - 1)$  insider participants collude to impersonate other participant in the group key establishment, the generated batch signature tuple is still unable to pass the batch verification equation in Round 3 and Key confirmation. Therefore, the proposed IDAGKE protocol is secure against insider impersonation attack.  $\square$

## 5.2 Secure AGKE Providing Forward Secrecy

Now, we want to prove that the proposed ID-based authenticated group key exchange (IDAGKE) protocol is a secure AGKE protocol providing forward secrecy. Firstly, we focus on the following theorem. Note that we use the similar way in [12, 31] to prove Theorem 3.

**Theorem 3** Assume that the hash functions  $H$ ,  $H_1$ , and  $H_2$  are random oracles. Then, the proposed IDAGKE protocol  $\Psi$  is a secure AGKE providing forward secrecy under the  $k$ -CAA and decision Diffie-Hellman (DDH) in the group  $G_2$  assumptions. Precisely,

$$Adv_{\Psi}^{AGKE-fs}(t, q_{ex}, q_s) \leq 2nq_{ex} \cdot Adv_{G_2}^{DDH}(t) + Adv_{\Phi}^{forge}(t),$$

where  $Adv_{\Phi}^{forge}(t)$  is the advantage of any forger attacking the adopted signature scheme  $\Phi$  and  $Adv_{G_2}^{DDH}(t)$  is the advantage of solving the  $DDH$  problem in the group  $G_2$ .

**Proof:** Assume that  $\mathcal{A}_2$  is an active adversary in attacking the proposed protocol  $\Psi$  with a non-negligible advantage. Here, we consider two possible attacking cases. One is that  $\mathcal{A}_2$  can impersonate a participant (forging authentication transcripts). The other is that  $\mathcal{A}_2$  breaks the protocol  $\Psi$  without modifying any transcripts with the advantage.

**Case 1:** We assume that  $\mathcal{A}_2$  has an adaptive impersonation ability to break  $\Psi$ . Using the adversary  $\mathcal{A}_2$ , we can construct a forger  $\mathcal{F}$  who generates a valid signature pair  $(ID, r, \sigma)$  with respect to the adopted signature scheme  $\Phi$  in the following. The forger  $\mathcal{F}$  generates all other public and private keys for the system.  $\mathcal{F}$  simulates the oracle queries of  $\mathcal{A}_2$ . This simulation is perfect indistinguishable from  $\mathcal{A}_2$ 's oracle queries unless  $\mathcal{A}_2$  makes the *Corrupt*( $ID$ ) query. If it occurs, then the forger  $\mathcal{F}$  terminates. Otherwise, if  $\mathcal{A}_2$  outputs a valid

signature pair  $(ID, r, \sigma)$ , then  $\mathcal{F}$  generates the  $\mathcal{A}_2$ 's pair  $(ID, r, \sigma)$ . Let *Forge* be the event that  $\mathcal{A}_2$  can output a valid signature pair and  $\Pr[\textit{Forge}]$  be the corresponding probability of this event. Then, the probability of  $\mathcal{F}$  which successfully generates a valid signature pair satisfies  $\Pr[\textit{Forge}] \leq \textit{Adv}_{\mathcal{F}, \Phi}^{\textit{forge}}(t) \leq \textit{Adv}_{\Phi}^{\textit{forge}}(t)$ .

**Case 2:** We assume that the adversary  $\mathcal{A}_2$  breaks the proposed protocol  $\Psi$  without modifying any transcripts. Let  $n$  be the number of participants chosen by  $\mathcal{A}_2$ . Considering the case, the adversary  $\mathcal{A}_2$  makes the *Execute* $(ID_1, ID_2, \dots, ID_n)$  query once. The real execution of  $\Psi$  is given by

$$\textit{Param} = \left\{ \begin{array}{l} (G_1, G_2, e) \leftarrow \text{KGC}; P \leftarrow G_1; s \leftarrow Z_q^*; P_{pub} = s \cdot P; g = e(P, P); \\ DID_1 = \frac{1}{s + H(ID_1)} P, \dots, DID_n = \frac{1}{s + H(ID_n)} P; (G_1, G_2, e, P, P_{pub}, g) \end{array} \right\}$$

and

$$\textit{Real} = \left\{ \begin{array}{l} a_1, \dots, a_n, h_1, \dots, h_n, b_1, \dots, b_n, k_1, \dots, k_n, c_1, \dots, c_n, w_1, \dots, w_n \leftarrow Z_q^*; \\ r_1 = g^{a_1}, \dots, r_n = g^{a_n}; \sigma_1 = (a_1 + h_1)DID_1, \dots, \sigma_n = (a_n + h_n)DID_n; \\ D_1 = (r_2/r_n)^{a_1}, D_2 = (r_3/r_1)^{a_2}, \dots, D_n = (r_1/r_{n-1})^{a_n}; \\ t_1 = g^{b_1}, \dots, t_n = g^{b_n}; \rho_1 = (b_1 + k_1)DID_1, \dots, \rho_n = (b_n + k_n)DID_n; \\ u_1 = g^{c_1}, \dots, u_n = g^{c_n}; \tau_1 = (c_1 + w_1)DID_1, \dots, \tau_n = (c_n + w_n)DID_n; \\ T = (r_1, \dots, r_n, \sigma_1, \dots, \sigma_n, D_1, \dots, D_n, t_1, \dots, t_n, \rho_1, \dots, \rho_n, u_1, \dots, u_n, \tau_1, \dots, \tau_n); \\ SK = (r_n)^{na_1} \cdot D_1^{n-1} \cdot D_2^{n-2} \cdot \dots \cdot D_{n-1} : (T, SK) \end{array} \right\},$$

where  $T$  denotes the transcript and  $SK$  is the group session key.

Note that  $D_i = (r_{i+1}/r_{i-1})^{a_i} = \frac{e(a_i a_{i+1} P, P)}{e(a_{i-1} a_i P, P)}$ . Here, we use a value  $d_{1,2} \in Z_q^*$  to substitute the value  $a_1 a_2$  and define a distribution *Fake*<sub>1</sub> as follows,

$$\textit{Fake}_1 = \left\{ \begin{array}{l} d_{1,2}, a_1, \dots, a_n, h_1, \dots, h_n, b_1, \dots, b_n, k_1, \dots, k_n, c_1, \dots, c_n, w_1, \dots, w_n \leftarrow Z_q^*; \\ r_1 = g^{a_1}, \dots, r_n = g^{a_n}; \sigma_1 = (a_1 + h_1)DID_1, \dots, \sigma_n = (a_n + h_n)DID_n; \\ D_1 = \frac{e(d_{1,2} P, P)}{e(a_n a_1 P, P)}, D_2 = \frac{e(a_2 a_3 P, P)}{e(d_{1,2} P, P)}, \dots, D_n = \frac{e(a_n a_1 P, P)}{e(a_{n-1} a_n P, P)}; \\ t_1 = g^{b_1}, \dots, t_n = g^{b_n}; \rho_1 = (b_1 + k_1)DID_1, \dots, \rho_n = (b_n + k_n)DID_n; \\ u_1 = g^{c_1}, \dots, u_n = g^{c_n}; \tau_1 = (c_1 + w_1)DID_1, \dots, \tau_n = (c_n + w_n)DID_n; \\ T = (r_1, \dots, r_n, \sigma_1, \dots, \sigma_n, D_1, \dots, D_n, t_1, \dots, t_n, \rho_1, \dots, \rho_n, u_1, \dots, u_n, \tau_1, \dots, \tau_n); \\ SK = (r_n)^{na_1} \cdot D_1^{n-1} \cdot D_2^{n-2} \cdot \dots \cdot D_{n-1} : (T, SK) \end{array} \right\}.$$

Since  $\mathcal{A}_2$  can obtain all private keys  $DID_i$  and hash values  $h_i, k_i,$  and  $w_i$  by making the *Corrupt* and *Hash* queries,  $\mathcal{A}_2$  can compute all  $a_i DID_i = \sigma_i - h_i DID_i$ ,  $b_i DID_i = \rho_i - k_i DID_i$ , and  $c_i DID_i = \tau_i - w_i DID_i$  for  $i = 1, 2, \dots, n$ . By the modified  $k$ -CAA problem, we know that

these values offer no information about  $a_i, b_i,$  and  $c_i$  for  $i = 1, 2, \dots, n.$

In the following Claim, we want to prove that the problem of distinguishing two distributions *Real* from *Fake*<sub>1</sub> can be reduced to solve the decision Diffie-Hellman (DDH) problem in the group  $G_2.$  Here, we let  $\varepsilon(t) = Adv_{G_2}^{DDH}(t).$

**Claim** For any algorithm  $\mathcal{A}_2$  running in time  $t,$  we have

$$|\Pr[(T, SK) \leftarrow Real: \mathcal{A}_2(T, SK) = 1] - \Pr[(T, SK) \leftarrow Fake_1: \mathcal{A}_2(T, SK) = 1]| \leq \varepsilon(t).$$

**Proof:** Note that  $D_i = (r_{i+1}/r_{i-1})^{a_i} = \frac{e(a_i a_{i+1} P, P)}{e(a_{i-1} a_i P, P)} = \frac{e(P, P)^{a_i a_{i+1}}}{e(P, P)^{a_{i-1} a_i}}.$  We use the symbol  $\Gamma_{i,i+1}$  to substitute  $e(P, P)^{a_i a_{i+1}}.$  Hence, each  $D_i$  can be rewritten into  $\frac{\Gamma_{i,i+1}}{\Gamma_{i-1,i}}$  and the group session key  $SK = (r_n)^{na_1} \cdot D_1^{n-1} \cdot D_2^{n-2} \cdot \dots \cdot D_{n-1}$  can be transformed into  $(\Gamma_{n,1})^n \cdot D_1^{n-1} \cdot D_2^{n-2} \cdot \dots \cdot D_{n-1},$  where  $(r_n)^{na_1} = e(P, P)^{na_1} = (\Gamma_{n,1})^n.$

Here, we use a technique to dispose related parameters, which are used to solve the DDH problem in  $G_2.$  Giving the adversary  $\mathcal{A}_2$  and considering the following algorithm  $\mathcal{D}$  which takes  $r_a = g^a = e(P, P)^a, r_b = g^b = e(P, P)^b, r_c = g^c = e(P, P)^c \in G_2$  as input for some  $a, b, c \in Z_q^*.$   $\mathcal{D}$  generates  $(T, SK)$  according to the distribution  $Dist^1$  and outputs whatever  $\mathcal{A}_2$  outputs after running  $\mathcal{A}_2(T, SK).$  The distribution  $Dist^1$  is defined as follows,

$$Dist^1 = \left\{ \begin{array}{l} \alpha_1, \dots, \alpha_{n-2}, a_1, \dots, a_n, h_1, \dots, h_n, b_1, \dots, b_n, k_1, \dots, k_n, c_1, \dots, c_n, w_1, \dots, w_n \leftarrow Z_q^*; \\ r_1 = g^{a_1}, \dots, r_n = g^{a_n}; \sigma_1 = (a_1 + h_1)DID_1, \dots, \sigma_n = (a_n + h_n)DID_n; \\ \Gamma_{1,2} = g^{ab} \in G_2, \Gamma_{2,3} = (r_b)^{\alpha_1} \text{ for } j = 3 \text{ to } n - 1: \\ \Gamma_{j,j+1} = e(P, P)^{\alpha_{j-2}\alpha_{j-1}}, \Gamma_{n,1} = (r_a)^{\alpha_{n-2}}; D_1 = \frac{\Gamma_{1,2}}{\Gamma_{n,1}}, D_2 = \frac{\Gamma_{2,3}}{\Gamma_{1,2}}, \dots, D_n = \frac{\Gamma_{n,1}}{\Gamma_{n-1,n}}; \\ t_1 = g^{b_1}, \dots, t_n = g^{b_n}; \rho_1 = (b_1 + k_1)DID_1, \dots, \rho_n = (b_n + k_n)DID_n; \\ u_1 = g^{c_1}, \dots, u_n = g^{c_n}; \tau_1 = (c_1 + w_1)DID_1, \dots, \tau_n = (c_n + w_n)DID_n; \\ T = (r_1, \dots, r_n, \sigma_1, \dots, \sigma_n, D_1, \dots, D_n, t_1, \dots, t_n, \rho_1, \dots, \rho_n, u_1, \dots, u_n, \tau_1, \dots, \tau_n); \\ SK = (\Gamma_{n,1})^n \cdot D_1^{n-1} \cdot D_2^{n-2} \cdot \dots \cdot D_{n-1} : (T, SK) \end{array} \right\}.$$

By the above distribution, let  $r_a = e(P, P)^a, r_b = e(P, P)^b,$  and  $\Gamma_{1,2} = e(P, P)^{ab},$  we refer to the resulting distribution as  $Dist^1_{DDH}.$  It is obvious that the distribution  $Dist^1_{DDH}$  is identical to *Real* as follows,

$$\begin{aligned} SK &= (\Gamma_{n,1})(\Gamma_{1,2})(\Gamma_{2,3}) \cdots (\Gamma_{n-2,n-1})(\Gamma_{n-1,n}) \\ &= e(P, P)^{\alpha_{n-2}a} \cdot e(P, P)^{ab} \cdot e(P, P)^{b\alpha_1} \cdot \dots \cdot e(P, P)^{\alpha_{n-4}\alpha_{n-3}} \cdot e(P, P)^{\alpha_{n-3}\alpha_{n-2}} \\ &= e(P, P)^{\alpha_{n-2}a + ab + b\alpha_1 + \dots + \alpha_{n-4}\alpha_{n-3} + \alpha_{n-3}\alpha_{n-2}}. \end{aligned}$$

Next, we again examine the distribution  $Dist^1.$  Let  $r_c = e(P, P)^c = \Gamma_{1,2}$  for some  $c \in Z_q^* \setminus \{ab\}.$  We refer to the resulting distribution as  $Dist^1_{\text{random}}.$  It is obvious that the distribution  $Dist^1_{\text{random}}$  is identical to *Fake*<sub>1</sub> as follows,

$$\begin{aligned}
SK &= (\Gamma_{n,1})(\Gamma_{1,2})(\Gamma_{2,3}) \cdots (\Gamma_{n-2,n-1})(\Gamma_{n-1,n}) \\
&= e(aP, P)^{\alpha_{n-2}a} \cdot e(P, P)^c \cdot e(P, P)^{b\alpha_1} \cdot \dots \cdot e(P, P)^{\alpha_{n-4}\alpha_{n-3}} \cdot e(P, P)^{\alpha_{n-3}\alpha_{n-2}} \\
&= e(P, P)^{\alpha_{n-2}a+c+b\alpha_1+\dots+\alpha_{n-4}\alpha_{n-3}+\alpha_{n-3}\alpha_{n-2}}.
\end{aligned}$$

Therefore, we have

$$|\Pr[(T, SK) \leftarrow Real: \mathcal{A}_2(T, SK) = 1] - \Pr[(T, SK) \leftarrow Fake_1: \mathcal{A}_2(T, SK) = 1]| \leq \varepsilon(t). \quad \square$$

By the same approach, other distributions  $Fake_i$  can be defined for  $i = 2, 3, \dots, n$ . Using the same construction of  $Dist^1$ , for any adversary  $\mathcal{A}_2$  running in time  $t$ , we can get the following equations:

$$\begin{aligned}
&|\Pr[(T, SK) \leftarrow Fake_1: \mathcal{A}_2(T, SK) = 1] - \Pr[(T, SK) \leftarrow Fake_2: \mathcal{A}_2(T, SK) = 1]| \leq \varepsilon(t), \\
&\vdots \\
&|\Pr[(T, SK) \leftarrow Fake_{n-1}: \mathcal{A}_2(T, SK) = 1] - \Pr[(T, SK) \leftarrow Fake_n: \mathcal{A}_2(T, SK) = 1]| \leq \varepsilon(t).
\end{aligned}$$

This implies

$$|\Pr[(T, SK) \leftarrow Real: \mathcal{A}_2(T, SK) = 1] - \Pr[(T, SK) \leftarrow Fake_n: \mathcal{A}_2(T, SK) = 1]| \leq n\varepsilon(t).$$

In the distribution  $Fake_n$ , the values  $d_{1,2}, d_{2,3}, \dots, d_{n,1}$  are constrained by  $T$  according to the following  $n$  equations:  $\log_g D_1 = d_{1,2} - d_{n,1}$ ,  $\log_g D_2 = d_{2,3} - d_{1,2}$ ,  $\dots$ ,  $\log_g D_n = d_{n,1} - d_{n-1,n}$ , where  $g = e(P, P)$ . Only  $n - 1$  of these equations are linear independent. Due to  $SK = e(P, P)^{d_{1,2}+d_{2,3}+\dots+d_{n,1}}$ , we have  $\log_g SK = d_{1,2} + d_{2,3} + \dots + d_{n,1}$ . Since this final equation is linear independent from the set of equations above, the value of  $SK$  is independent of transcript  $T$ . This implies that even for a computationally-unbounded adversary  $\mathcal{A}_2$ :  $\Pr[(T, SK_0) \leftarrow Fake_n; SK_1 \leftarrow G_2; b \leftarrow \{0, 1\} | \mathcal{A}_2(T, SK_b) = 1] = 1/2$ .

Since  $\varepsilon(t) = Adv_{G_2}^{DDH}(t)$ , we have the result that the advantage of  $\mathcal{A}_2$  on the event  $\neg Forge$  is bounded by  $2n \cdot Adv_{G_2}^{DDH}(t)$ . Hence, we have  $Adv_{\Psi}^{AGKE-fs}(t, 1, q_s) \leq 2n \cdot Adv_{G_2}^{DDH}(t) + Adv_{\Phi}^{forge}(t)$ . For the case of  $q_{ex} > 1$ , a standard hybrid argument immediately shows that  $Adv_{\Psi}^{AGKE-fs}(t, q_{ex}, q_s) \leq 2nq_{ex} \cdot Adv_{G_2}^{DDH}(t) + Adv_{\Phi}^{forge}(t)$ .  $\square$

According to Cui *et al.*'s single signature [15] and Lemma 1, we obtain that the advantage of forging the adopted signature scheme  $Adv_{\Phi}^{forge}(t)$  is negligible. Under the decision Diffie-Hellman (DDH) assumption in the group  $G_2$ , the advantage  $Adv_{G_2}^{DDH}(t)$  also is negligible. Thus, the proposed protocol  $\Psi$  is a secure authenticated group key exchange protocol providing forward secrecy.

## 5.4 Insider Attacks

Here, we prove that the proposed protocol is secure against insider attacks in the following Theorem 4. In other words, our protocol can resist malicious participants.

**Theorem 4** In the random oracle model, the proposed ID-based authenticated group key exchange (IDAGKE) protocol is secure against insider attacks under the security level  $l_b$ , the  $k$ -CAA assumption, and the decision Diffie-Hellman (DDH) assumption in the group.

**Proof:** As mentioned in section 3, we say that an IDAGKE protocol is secure against insider attacks, if the protocol satisfies following three conditions: (1) it is a secure AGKE; (2) it is secure against insider impersonation attack; (3) it provides key agreement. By Theorems 2 and 3, we have proven that the proposed protocol can withstand insider impersonation attack and is a secure authenticated group key exchange protocol providing forward secrecy. Thus, the conditions (1) and (2) have satisfied.

In the key confirmation phase, each  $U_i$  can confirm that her/his group key  $SK_i$  is equal to all  $SK_j$  for  $j = 1, 2, \dots, n$  and  $j \neq i$  by the batch verifying process. By Lemma 1, we have shown that the adopted batch verification technique is secure under the security level  $l_b$ . Thus, each participant can confirm that all computed group session keys from other participants are identical. This means that key agreement is achieved. Therefore, our proposed protocol is secure against insider attacks.  $\square$

## 6. PERFORMANCE ANALYSIS AND COMPARISONS

In this section, we want to analyze the computational cost for each participant in our proposed ID-based authenticated group key exchange (IDAGKE) protocol. Furthermore, we present the comparisons between our protocol and the previously proposed IDAGKE protocols in terms of key construction, the number of rounds, performance, and security property.

For convenience to evaluate the computational cost of our protocol, we only consider some time-consuming operations and define the following notations:

- $TG_e$ : The time of executing a bilinear map operation  $e: G_1 \times G_1 \rightarrow G_2$ .
- $TG_{mul}$ : The time of executing a scalar multiplication operation of point in  $G_1$ .
- $TG_H$ : The time of executing a map-to-point hash function  $H_G: \{0, 1\}^* \rightarrow G_1$ .
- $T_{exp}$ : The time of executing a modular exponentiation operation in  $Z_q$ .

In Round 1,  $TG_{mul} + T_{exp}$  is required for computing  $(r_i, \sigma_i)$ . In Round 2, each participant  $U_i$  requires  $2TG_e + 3TG_{mul} + 4T_{exp}$  to verify  $(r_j, \sigma_j)$  for  $j \in \{i-1, i+1\}$  and to compute  $(D_i, t_i, \rho_i)$ . In Round 3, it requires  $2TG_e + (2n-1)TG_{mul} + (n+2)T_{exp}$  for verifying all  $(t_j, \rho_j)$  and computing  $(SK_i, u_i, \tau_i)$ . Note that computing the key  $SK_i = (r_{i-1})^{a_i} \cdot D_i^{n-1} \cdot D_{i+1}^{n-2} \cdot \dots \cdot D_{i-2}$  requires  $T_{exp}$  due to  $A_{i-1} = (r_{i-1})^{a_i}$ ,  $A_i = A_{i-1} \cdot D_i$ ,  $A_{i+1} = A_i \cdot D_{i+1}$ ,  $\dots$ , etc. so that  $SK_i = A_{i-1} \cdot A_i \cdot A_{i+1} \cdot \dots \cdot A_{i-2}$ . In the key confirmation phase,  $2TG_e + (2n-2)TG_{mul} + nT_{exp}$  is required to verify all  $(u_j, \tau_j)$  and to confirm all  $SK_j$ . As a result, it requires  $6TG_e + (4n+1)TG_{mul} + (2n+7)T_{exp}$  for each participant  $U_i$ .

As described in section 1, we know that Choi *et al.*'s protocol [12] suffered from insider (participants) colluding attacks in [23, 24]. Although Shim also proposed a modification to overcome the insider colluding attacks in [24]. Actually, Shim's modification is still insecure against other insider colluding attacks in [19]. Recently, Choi *et al.* [19] also presented an improvement to resist the mentioned insider colluding attacks. However, we proved that their improvement still suffered from an insider colluding attack in [21]. In Table 1, we make the comparisons between our proposed protocol, Shim's modification [24], and Choi *et al.*'s improvement [19] in terms of key construction, number of rounds, performance, and security property. In section 5, we have demonstrated that our proposed

protocol is a provably secure ID-based authenticated group key exchange protocol with resisting malicious participants.

**Table 1. Comparisons between our protocol and the previously proposed IDAGKE protocols.**

	Shim's modification [24]	Choi <i>et al.</i> 's improvement [19]	Our protocol
Key construction	I	I	II
Rounds	2	2	3
Computational cost for each participant	$6TG_e + (2n + 6)TG_{mul} + (2n - 2)TG_H$	$6TG_e + (n + 11)TG_{mul} + (n + 3)TG_H$	$6TG_e + (4n + 1)TG_{mul} + (2n + 7)T_{exp}$
Security	Existing attacks [19]	Existing attacks [21]	Provably secure
Resistant to malicious participants	No	No	Yes

## 7. CONCLUSIONS AND FUTURE WORK

As compared to the certificate-based public-key systems, ID-based cryptographic protocols may simplify the certificate management. In this paper, we have proposed a secure ID-based authenticated group key exchange (IDAGKE) protocol based on key construction II. To reduce the cost of signature verifications, we adopt the batch verification technique into our protocol. In the random oracle model, we have proven that the proposed protocol satisfies the Katz-Shin's defined security model under the security level  $l_b$ , the  $k$ -CAA and decision Diffie-Hellman (DDH) in the group  $G_2$  assumptions. Thus, our proposed protocol is a provably secure IDAGKE protocol with resistant to insider attacks. This means that our protocol can withstand malicious participants disturbing.

Recently, the certificateless cryptography [32, 33] was proposed to solve the key escrow issue in the ID-based public-key system. The key escrow means that the Key Generation Center (KGC) knows all users' private keys. In the certificateless public-key system, the KGC issues a partial key to user with respect to her/his identity and the user can generate an additional public/private key pair. When both the user's partial key and private key are known, the user can execute some cryptographic operations. Thus, the KGC cannot do any cryptographic operations on behalf of users. In the future, we will try to propose a secure certificateless authenticated group key exchange protocol.

## ACKNOWLEDGMENTS

The authors would like to thank the anonymous referees for their valuable comments and constructive suggestions.

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