Parameter Estimation of Chaotic Dynamical Systems
Using HEQPSO

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In this study, a quantum-behaved particle swarm optimization (QPSO) based on hybrid evolution (HEQPSO) approach is proposed to estimate parameters of chaotic dynamic systems, in which the proposed HEQPSO algorithm combines the conceptions of genetic algorithm (GA) and adaptive annealing learning algorithm with the QPSO algorithm. That is, the mutation strategy in GA is used for conquering premature; adaptive decaying learning similar to simulated annealing (SA) is adopted for overcoming stagnation problem in searching optimal solutions. Three examples are illustrated to estimate parameters of chaotic dynamical systems using the proposed HEQPSO approach. From the numerical simulations and comparisons with other extant evolutionary methods in Lorenz system, the validity and superiority of the HEQPSO approach are verified. In addition, the effectiveness and robustness of parameter estimations for Chen and Rossler systems are demonstrated by the proposed HEQPSO approach.

Keywords: quantum-behaved particle swarm optimization, chaotic system, parameter estimation, hybrid evolution, adaptive annealing learning

1. INTRODUCTION

A chaotic system is a nonlinear deterministic system that has some special features of sensitive dependence on initial conditions and unstable bounded trajectories in the phase space. Synchronization of chaotic systems has been investigated intensely in various fields during recent years [1, 2]. Many of the proposed approaches only work under the assumption that the parameters of chaotic systems are known in advance. Nevertheless, the parameters may be difficult to determine due to the complexity of chaotic systems. Therefore, parameter estimation for chaotic systems has become an important issue in the past decade [3-10]. Some studies focused on synchronization-based methods for parameter estimation, recently. A feedback-based synchronization method and an adaptive control method were both introduced to estimate parameters for several chaotic systems in [3, 4]. In [5], an approximate gradient-descent method was adopted to perform the parameter estimation and synchronization of coupled chaotic systems. Wang et al. [6] proposed wavelet de-noising to improve the accuracy of the parameter identification for...
Many researchers paid much attention to evolutionary algorithm (EA) to estimate parameters of chaotic systems from the view of optimization during recent years. Dai et al. [7] used genetic algorithm (GA) to estimate parameters for Lorenz chaotic system, but only one-dimensional parameter estimation was taken into consideration. In [8], differential evolution (DE) algorithms were applied to estimate parameters of Rossler’s, Chen, Lu, and Lorenz chaotic systems without time-delay. He et al. [9] proposed particle swarm optimization (PSO) to estimate the parameters of Lorenz system. In [10], parameter estimation was illustrated using PSO method for semi-supervised support vector machine. Recently, quantum computing based on the principle of quantum mechanics and computing science has been a novel computing technique with some superiority to classical methods. Integrating the quantum computing with evolutionary algorithm, quantum-inspired evolutionary algorithm (QEA) was proposed in [11] and has been an effective optimization technique for many complex optimization problems, such as numerical optimization problems and scheduling problems [11, 12]. As for parameter estimation of nonlinear systems, Wang et al. [13] has shown that QEAs have better performance than classical EAs through numerical simulation results. In [14], Wang and Li have demonstrated the effectiveness and robustness of the hybrid quantum-inspired evolutionary algorithm with DE (HQEDE) estimating the parameters of Lorenz system.

Although the original PSO algorithm possesses the ability of high convergent speed, easily falling in some local optima is its fatal defect. Many researchers have presented revised PSO algorithms [15-17] and obtained good results. Another improvement on traditional PSO algorithm is quantum-behaved particle swarm optimization (QPSO) [18-22]. In the QPSO algorithm, particles move in an environment which likes Delta Potential Well. Because of uncertainty principle, position and velocity of particles cannot be determined simultaneously. Hence, the information of a particle in the QPSO is depicted by probabilities instead of position and velocity. The dynamic behavior of a particle is widely divergent and develops in the time-dependent Schrödinger equation. The QPSO algorithm ensures the congregation of particle swarm without losing the randomness and particles can appear on any position of the whole space which is searched in a certain probability. However, in QPSO, particles fall into local optimal state in multimode optimization problems and cannot find any better state, the QPSO algorithm will take on the premature phenomenon.

To overcome the premature phenomenon in QPSO, a hybrid evolutionary concept algorithm is embedded in the QPSO algorithm, named HEQPSO, will be proposed to perform the parameter estimation of chaotic systems in this study. In HEQPSO, the significant improvement is that the evolutionary algorithm combines the concept of mutation algorithm in GA and adaptive annealing learning similar to SA with QPSO to achieve global search and defeat premature phenomenon in searching optimal solutions. From the illustrated results for three chaotic dynamical systems, the effectiveness and the robustness of the proposed HEQPSO approach are demonstrated.

The remainder of this paper is organized as follows. The parameter estimation of chaotic systems is formulated as a multi-dimensional optimization problem in Section 2. Section 3 presents the PSO and QPSO theory. The proposed HEQPSO algorithm is introduced in Section 4. Numerical simulations and comparisons of three examples are provided in Section 5. Finally, we conclude in Section 6 with a summary of simulation results.
2. PROBLEM FORMULATION

In general, chaotic systems are nonlinear deterministic systems that have some prominent characteristics of sensitive dependency on both initial conditions and parameter variations. To explore the issue, the following $n$-dimensional chaotic system is considered:

$$\dot{X} = F(X, X_0, Q)$$

(1)

where $X = [x_1, x_2, ..., x_n]^T \in \mathbb{R}^n$ denotes the state vector, $X_0$ denotes the initial state and $Q = [q_1, q_2, ..., q_m]^T \in \mathbb{R}^m$ is a set of original parameters. $F : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n$ is a given nonlinear vector function.

A chaotic system can be expressed as Eq. (1). To estimate the unknown parameters, an estimated system can be described as follows:

$$\dot{\hat{X}} = F(X, X_0, \hat{Q})$$

(2)

where $\hat{X} = [\hat{x}_1, \hat{x}_2, ..., \hat{x}_n]^T \in \mathbb{R}^n$ denotes the state vector, and $\hat{Q} = [\hat{q}_1, \hat{q}_2, ..., \hat{q}_m]^T \in \mathbb{R}^m$ is a set of estimated parameters. The problem of parameter estimation can be formulated as an optimization problem. When processing with the optimization operations, a performance criterion or an objective function should be first defined. In general, the heuristic algorithm such as GA and PSO only needs a fitness function to evaluate searching solutions about investigated systems. In this study, the mean squared errors (MSE) between real and estimated responses for a number of given samples is considered as fitness of estimated model parameters. Therefore, the fitness function is defined as follows:

$$J = \frac{1}{M} \sum_{i=1}^{M} \left[ X_i - \hat{X}_i \right]^2$$

(3)

where $M$ denotes the length of data used for parameter estimation, $X_i$ and $\hat{X}_i$ ($k = 1, 2, ..., M$) denote state vectors of the original and the estimated systems at time $k$, respectively.

In Eq. (2), the parameter estimation for chaotic systems is a multi-dimensional continuous optimization problem. The optimization goal is to minimize $J$ in Eq. (3) by searching a suitable vector $\hat{Q}$. Due to the unstable dynamic behavior of chaotic systems, the parameters are not easy to obtain. In addition, there are often multiple variables in the problem and multiple local optima in the landscape of $J$, so traditional optimization methods are easy to trap in local optima and it is difficult to achieve the global optimal parameters. To overcome these drawbacks, a modified quantum-behaved particle swarm optimization approach is adopted to estimate parameters of chaotic dynamic systems.

The scheme of parameter estimation for chaotic systems can be illustrated in Fig. 1.

Fig. 1. The parameter estimation scheme for chaotic systems.
3. PARTICLE SWARM OPTIMIZATION AND QUANTUM BEHAVED PSO

In the PSO algorithm, each particle keeps trajectory of its coordinates in the problem space. The coordinate of each particle is related to its own best position (local best position) and the global best position achieved so far. The trajectories of particles are updated according to the following equations:

\[ v_i(k + 1) = w \cdot v_i(k) + c_1 \cdot r_1 (p^*_i - \varphi_i) + c_2 \cdot r_2 (p^g - \varphi_i), \quad i = 1, 2, \ldots, m, \]  
\[ \varphi_i(k + 1) = \varphi_i(k) + v_i(k + 1), \quad i = 1, 2, \ldots, m, \]

where \( m \) denotes the number of particles in a population; \( \varphi_i(k) = [\varphi_{i1}, \varphi_{i2}, \ldots, \varphi_{in}] \) and \( v_i(k) = [v_{i1}, v_{i2}, \ldots, v_{in}] \) are position and velocity of the \( i \)th particle at generation \( k \) in \( n \)-dimensional search space; \( p^*_i = [p^*_i1, p^*_i2, \ldots, p^*_in] \) and \( p^g = [p^g1, p^g2, \ldots, p^gn] \) are the best position of the \( i \)th particle and the global best position; \( w \) is the inertia weight; \( c_1 \) and \( c_2 \) are cognitive and social constriction coefficients, respectively; \( r_1 \) and \( r_2 \) are random numbers between 0 and 1. The inertia weight \( w \) are introduced to balance the local and global search during the optimization process, a large inertia weight facilitates a global search while a small inertia weight facilitates a local search. From the view of classical dynamics, to avoid explosion and guarantee convergence, particles must be bounded and fly in an attractive potential field. Clerc and Kennedy [23] have proved that if the coefficients, \( i.e., c_1 \cdot r_1 \) and \( c_2 \cdot r_2 \), in Eq. (4) are properly defined, the particle’s position \( \varphi_i \) will converge to the center of potential field, \( p^f_{\text{opt}} = [p^f_{1\text{opt}}, p^f_{2\text{opt}}, \ldots, p^f_{n\text{opt}}] \), and is defined as:

\[ p^f_{\text{opt}} = \frac{(c_1 \cdot r_1 \cdot p^f_i + c_2 \cdot r_2 \cdot p^g)}{(c_1 \cdot r_1 + c_2 \cdot r_2)}, \quad i = 1, 2, \ldots, m. \]

Inspired by the behavior that particles move in a bounded state and preserve the global search ability, Sun et al. [18] proposed the QPSO algorithm. In the QPSO model, particles move in a quantum multi-dimensional space, the state of particles is usually depicted by normalized wave function \( \varphi(r, t) \). That is, a single particle with mass \( m \) is subjected to the influence of a potential field \( V(r, t) \) in quantum space and the wave function of this particle is governed by the Schrödinger equation:

\[ i\hbar \frac{\partial}{\partial t} \varphi(r, t) = -\frac{\hbar^2}{2m} \nabla^2 \varphi(r, t) + V(r, t) \varphi(r, t), \]

where \( \hbar \) is the Planck constant, \( m \) is the mass of particles and \( \nabla^2 \) is the Laplacian operator. Based on the assumption that the attractive potential field is time-independent (called stationary state) and the environment of the Delta Potential Well, the solution of time-independent Schrödinger equation for this system in one dimensional space can be expressed as [22]:

\[ \varphi_i = p^f_{\text{opt}} + \frac{L}{2} \ln \left( \frac{1}{u_i} \right), \]
where \( u \) is a random number uniformly distributed on \([0,1]\) and \( L \) is the characteristic length of Delta Potential Well (or called “Creativity” or “Imagination” of particles) which specifies the search scope of a particle. In order to improve performance, Sun et al. [19] employ a Mainstream Thought Point to evaluate the parameter \( L \). The Mainstream Thought Point and \( L \) can be expressed as the following forms:

\[
m_{best} = \left[ \sum_{i=1}^{m} \frac{\phi_{i,1}}{m}, \sum_{i=1}^{m} \frac{\phi_{i,2}}{m}, \ldots, \sum_{i=1}^{m} \frac{\phi_{i,n}}{m} \right], \quad i = 1, 2, \ldots, m, \quad (9)
\]

\[
L = 2 \cdot \beta |m_{best} - \phi_i|, \quad (10)
\]

where \( \beta \) is a creative coefficient which is used to adjust the convergence speed of an individual particle and the performance of this algorithm.

4. A HYBRID EVOLUTION QPSO ALGORITHM

The QPSO algorithm has possessed many advances better than the traditional PSO algorithm in preventing premature and improving the convergence speed, etc. However, it still exists stagnating phenomenon for searching the global optimal solution in multi-modal problems, meanwhile, it is strongly influenced by the creative coefficient \( \beta \). In order to improve these defects, we propose an improved QPSO algorithm with hybrid evolution (named HEQPSO) which combines the QPSO algorithm with mechanisms of mutation in GA and adaptive annealing learning to achieve global search and conquer premature in optimization process. There are two significant improvements in the HEQPSO algorithm.

Firstly, in order to achieve global searching, \( \beta \) should be set to a large number at the beginning. Then accompanying the reduction of fitness value, the parameter \( \beta \) is adjusted decreasingly. The decreasing rate of \( \beta \) can be linear, but nonlinear revision according to the convergence of optimization process is more reasonable. The creative coefficient \( \beta \) with adaptive annealing learning mechanism according to the change rate of optimal estimation has the form:

\[
\beta = \beta_{inc} - \Delta \beta \cdot (1 + \exp(-\Delta fit))^{-\gamma}, \quad (11)
\]

\[
\Delta fit = |p^g - p^i|, \quad (12)
\]

where \( \Delta \beta \) is step length of \( \beta \), \( \Delta fit \) is the change rate of optimal estimation so far. The mechanism of adaptive annealing learning can overcome the stagnation problem to accelerate the convergent speed. Another improvement of the HEQPSO is elitist reproduction. The mutation mechanism is usually used for keeping diversity and avoiding premature. Inspired by these two mechanisms, an index of stagnation (named INST) is used for monitoring the convergence of optimization procedure in the HEQPSO algorithm.

In the HEQPSO algorithm, each \( p^g \) during optimization procedure is preserved. Meanwhile, the INST is set to be zero whenever \( p^g \) is updated according to their fitness values. Of course, the INST is increased when \( p^g \) is unchanged. Before ending this itera-
tion, the HEQPSO algorithm makes a judgment whether \( \text{INST} \) exceeds the specific criteria. If it is true, a certain rate (20\%) of new population is randomly generated and to replace the worse 20\% particles. The concepts of random populations come from the mutation strategy in GA that can obtain the optimal solution. Hence, these random populations can conquer premature problem to achieve the optimal solution. Finally, the proposed HEQPSO algorithm is described as follows:

**HEQPSO Algorithm:**

**Step 1:** Randomly initialize the particles and evaluate their fitness values.

**Step 2:** Initialize \( p^i \) and \( p^f \).

**Step 3:** Evaluate the fitness values of initial particles.

**Step 4:** Sort particles according to their fitness values.

**Step 5:** Calculate \( pf^{\text{cnt}} \), \( mbest \), and \( L \), in Eqs. (6), (9), and (10), respectively.

**Step 6:** Select the Eq. (13) or Eq. (14) with randomly probability to update \( \varphi_i \).

\[
\begin{align*}
\varphi_i(k+1) &= pf^{\text{cnt}} - \beta \cdot \text{norm}_2(mbest - \varphi_i(k)) \cdot \ln(1/\text{rand}), & \text{rand} \geq 0.5, \\
\varphi_i(k+1) &= pf^{\text{cnt}} + \beta \cdot \text{norm}_2(mbest - \varphi_i(k)) \cdot \ln(1/\text{rand}), & \text{rand} < 0.5,
\end{align*}
\]

where \( \text{norm}_2(ps_1 - ps_2) \) denotes the distance between \( ps_1 \) and \( ps_2 \). \( \beta \) is a nonlinearly decaying with Eq. (11).

**Step 7:** Evaluate the fitness values of new particles.

**Step 8:** Check \( p^i \) and \( p^f \) should be updated? If \( p^f \) is updated, sets \( \text{INST} \) to be 0 and go to Step 4. If \( p^f \) is unchanged, increase \( \text{INST} \) by 1.

**Step 9:** Check whether \( \text{INST} \) reaches 10? If yes, go to next Step, else go to Step 11.

**Step 10:** Rank \( \varphi(t+1) \) according to fitness values calculated in Step 7. Sort \( \varphi(t+1) \) and give up the worse 20\% particles. The new population is randomly generated and to replace the worse 20\% particles.

**Step 11:** Check whether the maximum iteration is reached or the termination criterion is satisfied? If yes, go to next Step, else perform Step 4.

**Step 12:** Check whether \( p^i \) and \( p^f \) should be updated and output results.

### 5. SIMULATION RESULTS

To verify the effectiveness and feasibility of the proposed HEQPSO algorithm, three examples are given to estimate the parameters of chaotic dynamical systems including Lorenz system, Chen system, and Rossler system. The fourth Runge-Kutta method is adopted to solve differential equations of chaotic systems in numerical simulations. In the simulations, the chaotic systems firstly evolve freely from a random initial state. The sampling time is equal to 0.005 and the number of states for calculating the fitness \( J \) in Eq. (3) is set as 300 \((M = 300)\) for three simulated examples. In HEQPSO, the maximum generation number, the population size, \( \text{INST} \), and rate of randomly generate particles in Step 3 are set to 100, 30, 10 and 20\%, respectively. Meanwhile, the values of parameters \( \beta_{ini} \) and \( \Delta \beta \) in Eq. (11) are given as 0.9 and 0.5, respectively. All simulations are implemented in the Matlab environment and conducted on Intel Core 2 Duo CPU P8400, 4GB Ram capacity PC.
Example 1: A typical chaotic system, Lorenz system, is considered as an example described as follows [7, 9, 14]:

\[
\begin{align*}
  \dot{x}_1 &= q_1(x_2 - x_1) \\
  \dot{x}_2 &= q_2 x_1 - x_2 - x_1 x_3 \\
  \dot{x}_3 &= x_1 x_2 - q_3 x_3
\end{align*}
\]  

(15)

where \( q_1 = 10, \ q_2 = 28, \ q_3 = 8/3 \) are the original parameters. In this simulation, the searching ranges for parameters are set as \( 5 \leq q_1 \leq 15, \ 20 \leq q_2 \leq 30, \ \text{and} \ 2 \leq q_3 \leq 5. \) In the Lorenz system (15), three-dimensional parameters are unknown and need to be estimated.

Problem 1-1: To verify the effectiveness of the creative coefficient \( \beta \), firstly, in HEQPSO, the decreasing rate of \( \beta \) with variant visions is adopted to estimate the parameters of Lorenz system. The simulated results are shown in Table 1. From the results, we can assert that the nonlinear revision with adaptive annealing learning has promising solutions.

<table>
<thead>
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<th>( \beta )</th>
<th>Best</th>
<th>Average</th>
<th>Worst</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
<tr>
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<td>3.4887e-14</td>
<td>1.9914e-13</td>
</tr>
</tbody>
</table>

Note: Linear with \( \beta = \beta_0 \cdot \frac{\Delta \beta}{\text{generation number}} \cdot \text{iter} \).

Problem 1-2: To show the validity and superiority of the proposed HEQPSO. Some comparisons of HEQPSO, GA [7], PSO [9] and HQEDE [14] are shown in Table 2, in which each algorithm is run for 20 times independently. From the results, it is clear that the best, the average, and the worst results obtained by HEQPSO with less population size are better than those obtained by GA [7], PSO [9] and HQEDE [14] whose parameter settings are shown in Table 3.

The history of creative coefficient \( \beta \) with respect to the change rate of optimal estimation \( \Delta J/\Delta t \) is shown in Fig. 2. Furthermore, the evolving process of the average fitness values \( J \) (MSE) in Eq. (3) by the proposed HEQPSO is illustrated in Fig. 3, which shows that the MSE converges to zero efficiently. The evolutionary process of estimated parameters in each generation is shown in Fig. 4. It can be seen that the estimated parameters converge to true values very fast. Further, two prominent chaotic systems are considered for parameter estimation to verify the effectiveness and feasibility of the proposed HEQPSO method, in which three-dimensional parameters in Rossler and Chen chaotic systems are unknown and need to be estimated.
Fig. 2. The history of creative coefficient $\beta$ with respect to the change rate of optimal estimation $\Delta fit$ for Lorenz system.

Fig. 3. The average fitness value $J$ with MSE in Eq. (3) obtained by HEQPSO when estimating parameters of Lorenz system.

Fig. 4. Tuning trajectories of estimated parameters $q_1$, $q_2$, and $q_3$ for Lorenz system.

Table 2. Statistical results of three-dimensional parameter estimation for Lorenz system using various methods, the average values are obtained through 20 times independently; meanwhile, the best values and the worst values are determined.

<table>
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Table 3. The parameter settings of the benchmark algorithms [7, 9, 14] and HEQPSO.

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<td>Population size</td>
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<td>Generation number</td>
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<td>100</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

Example 2: Rossler chaotic system described by [24] is considered.

\[
\begin{align*}
\dot{x}_1 &= -x_2 - x_3 \\
\dot{x}_2 &= x_1 + q_1 x_2 \\
\dot{x}_3 &= (x_1 - q_3)x_3 + q_2
\end{align*}
\]  

(16)

where \(q_1 = 0.2, q_2 = 0.2, q_3 = 5.7\) are the original parameters. The searching ranges are set as \(0 \leq q_1 \leq 1, 0 \leq q_2 \leq 1,\) and \(0 \leq q_3 \leq 10.\)

Problem 2-1: Firstly, the effective of the creative coefficient \(\beta\) is tested. Different visions of the decreasing rate \(\beta\) in HEQPSO are adopted to estimate the parameters of Rossler chaotic system. Simulations are shown in Table 4. From the results, we can assert that the nonlinear revision with adaptive annealing learning has promising solutions.

Table 4. Statistical results of three-dimensional parameter estimation for Rossler system using variant revisions of \(\beta\), the average values of fitness \(J\) with MSE are obtained through 20 times independently; meanwhile, the best values and the worst values are determined.

<table>
<thead>
<tr>
<th>(\beta)</th>
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</tbody>
</table>

Problem 2-2: To show the validity and superiority of the proposed HEQPSO. Some results obtained by the proposed HEQPSO and extant methods, GA [7], PSO [9] and HQEDE [14] are compared as shown as Table 5, in which each algorithm is run for 20 times independently. From the results, it is clear that the average results obtained by HEQPSO with same population size with 30 and generation number with 100 are better than those obtained by GA [7], PSO [9] and HQEDE [14].

The history of creative coefficient \(\beta\) with respect to the change rate of optimal estimation \(\Delta fit\) is shown in Fig. 5. Furthermore, the results of evolving process are illustrated in Figs. 6 and 7. Figs. 6 and 7 show that the MSE converges to zero efficiently and the estimated parameters efficiently converge to true values.
Table 5. Statistical results of three-dimensional parameter estimation for Rossler system using various methods, the average values are obtained through 20 times independently; meanwhile, the best values and the worst values are determined.

<table>
<thead>
<tr>
<th></th>
<th></th>
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<tbody>
<tr>
<td>Best</td>
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<tr>
<td>$q_1$</td>
<td>0.2000001002</td>
<td>0.2000000312</td>
<td>0.200000012</td>
<td>0.200000002</td>
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<tr>
<td>$q_2$</td>
<td>0.200000569</td>
<td>0.200000325</td>
<td>0.200000101</td>
<td>0.200000097</td>
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<tr>
<td>$q_3$</td>
<td>5.700004911</td>
<td>5.700003857</td>
<td>5.700003934</td>
<td>5.700002818</td>
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<td>$J$</td>
<td>6.1273e-10</td>
<td>4.1273e-16</td>
<td>3.5631e-18</td>
<td>1.1899e-19</td>
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<td>Average</td>
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</tr>
<tr>
<td>$q_1$</td>
<td>0.2000012953</td>
<td>0.2000001031</td>
<td>0.2000000304</td>
<td>0.2000000143</td>
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<tr>
<td>$q_2$</td>
<td>0.2000050694</td>
<td>0.2000040130</td>
<td>0.2000031205</td>
<td>0.2000023348</td>
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<td>$q_3$</td>
<td>5.7000093567</td>
<td>5.7000081423</td>
<td>5.7000714351</td>
<td>5.7000703971</td>
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<tr>
<td>$J$</td>
<td>7.3352e-06</td>
<td>5.1132e-10</td>
<td>2.7315e-12</td>
<td>1.2475e-13</td>
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<td>$q_1$</td>
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<td>0.2000006920</td>
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<td>$q_2$</td>
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<td>0.2000406416</td>
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<td>$q_3$</td>
<td>5.8012117360</td>
<td>5.8010003102</td>
<td>5.701210258</td>
<td>5.7012258987</td>
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<tr>
<td>$J$</td>
<td>7.2419e-03</td>
<td>3.2419e-08</td>
<td>5.6136e-11</td>
<td>2.2545e-12</td>
</tr>
</tbody>
</table>

Fig. 5. The history of creative coefficient $\beta$ with respect to the change rate of optimal estimation $\Delta fit$ for Rossler system.

Fig. 6. The average fitness value $J$ with MSE in Eq. (3) obtained by HEQPSO when estimating parameters of Rossler system.

Fig. 7. Tuning trajectories of estimated parameters $q_1$, $q_2$, and $q_3$ for Rossler system.
Example 3: Chen system is a typical chaos anti-control model with three state differential equations described as follows [22]:

\[
\begin{align*}
\dot{x}_1 &= q_1 (x_2 - x_1) \\
\dot{x}_2 &= (q_2 - q_1) x_1 + q_2 x_2 - x_1 x_3 \\
\dot{x}_3 &= x_1 x_2 - q_3 x_3
\end{align*}
\]  

(17)

System Eq. (17) has a chaotic dynamical behavior for \( q_1 = 35, q_2 = 28, q_3 = 3 \). The searching ranges are set as \( 30 \leq q_1 \leq 40, 25 \leq q_1 \leq 30, \) and \( 2 \leq c \leq 5 \).

Problem 3-1: Similarly, the effective of the creative coefficient \( \beta \) is tested. Different visions of the decreasing rate \( \beta \) in HEQPSO are adopted to estimate the parameters of Chen system. Simulations are shown in Table 6. From the results, the validity of the non-linear revision with adaptive annealing learning has been verified.

Table 6. Statistical results of three-dimensional parameter estimation for Chen system using variant revisions of \( \beta \), the average values of fitness \( J \) with MSE are obtained through 20 times independently; meanwhile, the best values and the worst values are determined.

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>Best</th>
<th>Average</th>
<th>Worst</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>3.0887e-04</td>
<td>1.4072e-02</td>
<td>1.2285e-01</td>
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<tr>
<td>0.8</td>
<td>3.0823e-06</td>
<td>1.1184e-02</td>
<td>1.7154e-01</td>
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<td>2.3219e-02</td>
<td>3.7239e-01</td>
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<tr>
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<td>7.5254e-01</td>
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<td>5.8331e-10</td>
<td>1.4738e-01</td>
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<td>0.4</td>
<td>1.4337e-06</td>
<td>4.7596e-01</td>
<td>2.7351</td>
</tr>
<tr>
<td>Linear</td>
<td>3.3960e-06</td>
<td>4.5022e-01</td>
<td>3.5064</td>
</tr>
<tr>
<td>Nonlinear with Eq. (11)</td>
<td>3.6706e-05</td>
<td>9.6466e-03</td>
<td>5.3970e-02</td>
</tr>
</tbody>
</table>

Problem 3-2: To show the validity and superiority of the proposed HEQPSO. Some results obtained by the proposed HEQPSO and extant methods, GA [7], PSO [9] and HQEDE [14] are compared as shown as Table 7, where each algorithm is run for 20 times independently. From the comparisons, the superiority of by the proposed HEQPSO with same population size with 30 and generation number with 100 are certified.

The history of creative coefficient \( \beta \) with respect to the change rate of optimal estimation \( \Delta \text{fit} \) is shown in Fig. 8. The results of evolving process are illustrated in Figs. 9 and 10. From Figs. 9 and 10, the estimation performance of the proposed HEQPSO method is confirmed. From the above simulation results, it is clear that the values of estimated parameters obtained by the proposed HEQPSO method are still very close to the true values of original parameters of Rossler and Chen systems. Meanwhile, the efficiency of convergence is prominent. Then, we concluded that the proposed HEQPSO method has efficiency and robustness for estimating parameters of chaotic systems.
Table 7. Statistical results of three-dimensional parameter estimation for Chen system using various methods, the average values are obtained through 20 times independently; meanwhile, the best values and the worst values are determined.

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
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<td>Best</td>
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<td>$q_1$</td>
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<td>35.0050073261</td>
<td>35.0020015246</td>
<td>35.0019017388</td>
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<td>35.0596324561</td>
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<td>9.2335e-01</td>
<td>7.1135e-01</td>
<td>5.3970e-02</td>
</tr>
</tbody>
</table>

Fig. 8. The history of creative coefficient $\beta$ with respect to the change rate of optimal estimation $\Delta fit$ for Chen system.

Fig. 9. The average fitness value $J$ with MSE in Eq. (3) obtained by HEQPSO when estimating parameters of Chen system.

Fig. 10. Tuning trajectories of estimated parameters $q_1$, $q_2$ and $q_3$ for Chen system.
6. CONCLUSION

From the viewpoint of optimization, parameter estimation for chaotic systems is formulated as a multi-dimensional optimization problem. In this paper, the proposed HEQPSO algorithm was proposed to solve such an issue, in which we efficiently combine mechanisms of mutation and adaptive annealing learning into the QPSO algorithm. From the results and comparisons with the extant literature based on Lorenz system, the feasibility and superiority of the proposed method were verified. Furthermore, two chaotic systems, Rossler system and Chen system, have been tested to demonstrate the efficiency and robustness of the proposed HEQPSO algorithm from some comparisons. The future work is to apply HEQPSO for investigating more complex chaotic systems.

REFERENCES

12. B. B. Li and L. Wang, “A hybrid quantum-inspired genetic algorithm for multiobjec-


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