Accuracy Analysis on the Estimation of Camera Parameters for Active Vision Systems

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Abstract

In camera calibration, due to the correlations between certain camera parameters, e.g, the correlation between the image center and the camera orientation, an estimate of a set of camera parameters which minimizes a given criterion does not guarantee that the physical camera parameter estimates are themselves accurate. This problem has not drawn much attention from our computer vision society because most computer vision applications require only accurate 3D measurements and do not care much about the values of the physical parameters as long as their composite effect is satisfactory. However, in calibrating an active vision system where the cameras are motorized such that their parameters can be adapted to the environment, accuracy of the physical parameters is very critical because we need accuracy to establish the relation between the motor positions and the camera parameters (both intrinsic and extrinsic). The contribution of this work is mainly in error analysis of camera calibration, especially in the accuracy of the physical camera parameters themselves, for four different types of calibration problems. The first type is estimation of all the camera parameters simultaneously. The second type is estimation of all the other camera parameters given the image center. The third type is estimation of the extrinsic parameters given the intrinsic parameters. The last one is estimation of the intrinsic parameters given the extrinsic parameters. For each type of calibration problem, we derive (i) the covariance matrices of the estimated camera parameters and (ii) the sensitivity matrices of the estimated parameters with respect to the error of the given parameters. Factors that affect calibration accuracy are found to be the focal length, the area and resolution of the image sensor, the average object distance, the relative object depth, the 2D observation noise and the number of calibration points. Our theoretical analysis has been verified by computer simulations. With our error analysis, the most suitable camera calibration technique and calibration configuration for providing accurate camera parameters can be determined. Also, the accuracy of the estimated physical parameters can be predicted by using our analysis results.

Keywords

Camera Calibration, Error Analysis, Kinematic Calibration, Calibration of Active Vision Systems.

I. Introduction

Camera calibration in the context of computer vision is the process of determining the geometric parameters of a mathematical camera model. In general, camera parameters can be divided into two categories, namely, intrinsic parameters and extrinsic parameters. Intrinsic camera parameters are independent of the position and orientation of the camera. They may include the effective focal length, the width and height of a photo sensor cell, the lens distortion and the image center (i.e, the image coordinates of the intersection of the optical axis and the image sensor plane). Extrinsic camera parameters are essentially the position and orientation of a camera. Hence, they are independent of the intrinsic parameters.

Usually, camera calibration is performed for two major purposes. One purpose is to identify the camera geometry of a 3D computer vision system. Another purpose is to calibrate a robot (either a robot arm or a robot head) by using the extrinsic parameters obtained in the camera calibration process. Due to the strong demands of many computer vision and robotics applications, extensive work has been devoted to the development of accurate and efficient camera calibration techniques, e.g, [1]–[10], [12], [16], [18], [19], [20], [24]–[26], [30]. Among all the existing calibration techniques, the Tsai method [22] may be the most popular one because of its efficiency and accuracy (accurate enough for most vision applications). Also, the source code for the Tsai method is available in the public domain. The Tsai method is a two-stage algorithm, where most of the parameters are solved with a linear method in the first stage, and the remaining parameters are solved in the second stage with a nonlinear optimization method based on the initial values provided by another linear method. Unlike most of the other methods of using non-coplanar calibration points, e.g, [7], [8], [19], [25], [26], and [30], the image center is not estimated in the Tsai method [22]. Instead, the image center and one scale factor are solved in another process described in [16] .

We have noticed that the nonlinear methods which estimate all the camera parameters including the image center can usually obtain more accurate results than can the Tsai method, especially when the given image center is not accurate enough. However, while 3D vision accuracy can be improved by estimating the image center simultaneously with the other parameters, the orientation parameters of the camera become more sensitive to noise than in the original Tsai method. Previous work on the sensitivity problem was done by Kumar and Hanson [13] and Lai [14], where they showed that there was linear dependency on small variations of the image center and the camera orientations and on small variations of the effective focal length and the translation in the optical axis direction. This problem has not drawn much attention from our computer vision society because our emphases have usually concentrated on how accurately we could model

1. the projection of a 3D coordinates in the OCS (Object Coordinate System) to 2D image coordinates in the ICS (Image Coordinate System), and
2. the back-projection of a 2D image point into a 3D ray.

For this reason, little attention has been devoted to the variances of camera parameters in the traditional computer vision literature. However, when applying camera calibration techniques to robot calibration (including hand/eye calibration), e.g, [17], [21], [23], [28] and [31], it is important to have accurate estimates of extrinsic camera parameters. Furthermore, when calibrating an active vision system [15] [27] [30] , the accuracy of both extrinsic and intrinsic parameters is of equal importance. In this case, accurate estimates of *extrinsic* camera parameters are helpful for calibrating the kinematic model of a binocular head, and accurate estimates of *intrinsic* camera parameters can simplify the calibration work for a motorized lens used in an active vision system, as explained below. In an active vision system, the intrinsic parameters are controlled by using motorized lenses. However, the relationships between the motor positions and the intrinsic parameters of a motorized lens are too complex to exactly express as analytic functions [27]. Therefore, in general, look up tables are used for mapping the motor positions to the intrinsic parameters. If the estimated camera parameters are too sensitive to noise such that no systematic behavior can be observed, then a huge comprehensive look up table is required to record the intrinsic parameters with respect to each motor position, which is an impractical approach. On the other hand, if the camera parameters can be estimated accurately, then only several samples are required to describe the correspondence between the camera parameters and the motor position.

When dealing with the above-mentioned problems which require accurate estimates of camera parameters, one may ask:

1. How accurate will the estimated camera parameters be when all the parameters including the image center are estimated simultaneously?
2. Will the accuracy of the estimated extrinsic parameters be improved by using a lens with longer focal length?
3. Which pairs of camera parameters are linearly dependent on each other when the variations are small?
4. What kind of camera calibration configuration will provide the most accurate extrinsic (or intrinsic) parameters?

Our theoretical analysis does reveal something that is not straightforward from intuition. For example, we have found that the answer to the second problem is that accuracy will depend on whether or not the image center is an unknown which needs to be estimated: if the image center is given, the estimation error will be *inversely proportional* to the focal length; otherwise, a longer focal length will *not* promise a more accurate result.

Relatively less work has been devoted to finding the relations among the calibration setup, the 2D measurement noise and the variances of camera parameters. Hui and Ng [11] has developed a method for computing the covariance matrix of the estimated camera parameters. However, they only provided a numerical solution, from which the factors that affected parameter accuracy could not be determined. Kumar and Hanson [13] and Lai [14] showed that there was some linear dependency between some of the extrinsic and intrinsic parameters when the variations are small, but they did not address how seriously the above facts will affect estimation accuracy. In this paper, error analysis on camera parameter estimation is investigated for the following four different types of calibration problems:

- Type 1 calibration problem: to estimate all the camera parameters simultaneously.
- Type 2 calibration problem: to estimate all the other camera parameters given the image center.
- Type 3 calibration problem: to estimate the extrinsic parameters given the intrinsic parameters.
- Type 4 calibration problem: to estimate the intrinsic parameters given the extrinsic

parameters.

Table I lists the major applications related to the four types of calibration problems. In this work, we have also derived the sensitivity matrix of the estimated parameters with respect to the error of the given parameters. The covariance and the sensitivity matrices of the camera parameters were derived as functions of the effective focal length, the size of the CCD sensor area, the size of one photo sensor cell, the average object distance and the *relative object depth* (i.e, the ratio of the object depth to its average distance).

TABLE I

The Relation Between the Applications and the Four Types of Camera Calibration Problems

Applications	Type 1	Type 2	Type 3	Type 4
Stereo Vision	$\surd$	$\surd$		
Robot Calibration	$\surd$	$\surd$	$\surd$	
Calibration of Active Vision System	$\surd$	$\surd$	$\surd$	$\surd$

This paper is organized as follows. Section II describes the camera model and a direct nonlinear calibration technique used for error analysis. Section III addresses the procedures for deriving both the covariance and sensitivity matrix of the camera parameters. Section IV gives the computer simulation results to verify our error analysis. Conclusions are given in Section V.

II. Camera Model and the Direct Nonlinear Calibration Technique

A. The Perspective Projection Camera Model with Radial Lens Distortion

The camera model considered in this paper is the perspective projection model with radial lens distortion, which is commonly used in the field of computer vision. Let p_O be an object point in 3D space, and let (x_O, y_O, z_O) be its coordinates, in millimeters, with respect to a fixed object coordinate system (OCS). Let the projected image coordinates, p_I in pixels, of the object point p_O be (u_I, v_I). The camera model used in this paper requires twelve camera parameters, i.e, u_0, v_0, s_u, s_v, f, κ, ϕ_x, ϕ_y, ϕ_z, t_x, t_y and t_z, where u_0 and

v_0 are the coordinates of the image center, s_u and s_v are the horizontal and vertical pixel spacings, f is the effective focal length, κ is the coefficient of radial lens distortion, ϕ_x , ϕ_y , and ϕ_z are the camera orientation parameterized as the X-Y-Z Euler angles, and t_x , t_y and t_z are the location of the optical center. Equations that relate the 3D and 2D coordinates can be written as follows (refer to [19] or [22]):

$$(1-\kappa\rho^2)(u_I-u_0)s_u = f\frac{x_C}{z_C} \tag{1}$$

$$(1-\kappa\rho^2)(v_I-v_0)s_v = f\frac{y_C}{z_C}, \tag{2}$$

where

$$\rho^2 = (u_I-u_0)^2 s_u^2 + (v_I-v_0)^2 s_v^2,$$

$$\begin{bmatrix} x_C \\ y_C \\ z_C \end{bmatrix} = \mathbf{R}\begin{bmatrix} x_O \\ y_O \\ z_O \end{bmatrix} + \mathbf{t},$$

$\mathbf{R}$ is the 3 by 3 rotation matrix composed by using the X-Y-Z Euler angles ϕ_x, ϕ_y and ϕ_z, and $\mathbf{t} = [\,t_x \quad t_y \quad t_z\,]'$ is the 3 by 1 translation vector, where the notation "$'$" denotes the *transpose* operation. Notice that three of the camera parameters, i.e, the effective focal length, f, the vertical and horizontal pixel spacing, s_u and s_v, can only be solved up to a scale factor. In general, there are two ways to approach this problem. One is to compose f, s_u and s_v into two effective focal length parameters, namely, the horizontal and vertical effective focal length which eliminates the extra degree of freedom (refer to the Weng method [25]). Another way is suitable for a solid state camera and is adopted in the well known Tsai method [22]. This is because the horizontal and vertical pixel spacing of the solid state camera can be directly obtained from the camera supplier. However, the horizontal pixel spacing will be rescaled with an unknown factor after the image is sampled by a frame grabber (refer to [16]). Therefore, only the vertical pixel spacing, s_v, is known and can be used in the calibration process. Nevertheless, if the vertical pixel spacing is unknown, we can simply set s_v to 1, which yields the same representation as in the Weng method [25].

B. Direct Nonlinear Calibration Technique

In general, variances of the camera parameters are relevant to the calibration technique used. In this paper, we chose to use a direct nonlinear calibration technique, because the direct nonlinear calibration technique not only can deals with all four types of calibration problems, but also provides the most accurate calibration results if the given initial value is good enough. The error function for the direct nonlinear calibration technique is defined as follows:

$$J(\beta_p;\beta_g) = \sum_{i=1}^{M}[E_u^2(p_O(i),p_I(i),\beta_p;\beta_g) + E_v^2(p_O(i),p_I(i),\beta_p;\beta_g)], \tag{3}$$

where

$$E_u(p_O,p_I,\beta_p;\beta_g) = (1-\kappa\rho^2)(u_I-u_0)s_u - f\frac{x_C}{z_C} + s_u\epsilon_u, \tag{4}$$

$$E_v(p_O,p_I,\beta_p;\beta_g) = (1-\kappa\rho^2)(v_I-v_0)s_v - f\frac{y_C}{z_C} + s_v\epsilon_v, \tag{5}$$

$$\begin{bmatrix} x_C \\ y_C \\ z_C \end{bmatrix} = \mathbf{R}(\phi_\mathbf{x},\phi_\mathbf{y},\phi_\mathbf{z})\begin{bmatrix} x_O \\ y_O \\ z_O \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix},$$

M is the number of calibration points, $p_O(i)$ and $p_I(i)$ are the 3D and 2D coordinates of the ith calibration point, β_p and β_g are the unknown and given parameter vectors which will be discussed later, and ϵ_u and ϵ_v are the 2D measurement noise (in pixel) in the horizontal and vertical directions, respectively. For instance, when dealing with the Type 1 calibration problem, all the camera parameters are unknown; hence,

$$\beta_p = [\, u_0 \quad v_0 \quad s_u \quad f \quad \kappa \quad \phi_x \quad \phi_y \quad \phi_z \quad t_x \quad t_y \quad t_z \,],$$

and β_g only contains the vertical pixel spacing, s_v. Table II lists the given parameters corresponding to different types of calibration problems.

III. Covariance and Sensitivity Matrices of the Camera Parameters

A. Derivation of the Covariance and Sensitivity Matrices of the Camera Parameters

Before we can derive the covariance and sensitivity matrices of the camera parameters, we have to solve a rather basic problem, i.e, the parameterization stability problem of a rotation matrix when using the X-Y-Z Euler angle representations, where the variations

TABLE II

THE GIVEN PARAMETERS VERSUS THE PROBLEM TYPE

Parameters	Type 1	Type 2	Type 3	Type 4
u_0		$\surd$	$\surd$	
v_0		$\surd$	$\surd$	
s_u			$\surd$	
s_v	$\surd$	$\surd$	$\surd$	$\surd$
f			$\surd$	
κ			$\surd$	
ϕ_x				$\surd$
ϕ_y				$\surd$
ϕ_z				$\surd$
t_x				$\surd$
t_y				$\surd$
t_z				$\surd$

"$\surd$": indicates the parameter is given in this type of problem.

of the Euler angles may be dominated by the representation instability rather than the measurement noise. However, since our goal is not to investigate the estimation error of the camera parameters with respect to the true value of the extrinsic parameters, the orientation and position of the CCS can be arbitrarily assigned. In other words, if the estimated transformation matrix from the OCS to the CCS is

$$ {}^{C}\hat{T}_{O} = \delta^{C}T_{O}{}^{C}T_{O}, \tag{6} $$

then we are interested in $\delta^{C}T_{O}$ rather than ${}^{C}T_{O}$, where $\delta^{C}T_{O}$ and ${}^{C}T_{O}$ are the estimation error and the true value of the transformation matrix, respectively. Therefore, without loss of generality, we may assume that the true values of the extrinsic parameters, i.e, ϕ_x, ϕ_y, ϕ_z, t_x, t_y and t_z, are all *zero*. Notice that this amounts to saying that the true transformation matrix from the OCS to the CCS is identity, and it can be shown that the X-Y-Z Euler angle representation is stable for rotation matrices which are close to identity.

By computing and neglecting the high order terms of the Taylor series expansion of the error function (3) about the true values of the camera parameters and noise free 2D measurements, we have

$$J(\delta\beta_p;\delta\beta_g) \approx \sum_{i=1}^{M} \left(\|a_u(i)\delta\beta_p + b_u(i)\delta\beta_g + s_u\epsilon_u\|^2 + \|a_v(i)\delta\beta_p + b_v(i)\delta\beta_g + s_v\epsilon_v\|^2 \right), \quad (7)$$

where $\delta\beta_p$ and $\delta\beta_g$ are the deviations of the camera parameters from their true values, and

$$a_u(i) = \frac{\partial E_u(p_O(i), p_I(i), \beta_p;\beta_g)}{\partial \beta_p}, \quad (8)$$

$$a_v(i) = \frac{\partial E_v(p_O(i), p_I(i), \beta_p;\beta_g)}{\partial \beta_p}, \quad (9)$$

$$b_u(i) = \frac{\partial E_u(p_O(i), p_I(i), \beta_p;\beta_g)}{\partial \beta_g}, \quad (10)$$

$$b_v(i) = \frac{\partial E_v(p_O(i), p_I(i), \beta_p;\beta_g)}{\partial \beta_g}, \quad (11)$$

are the gradient *row* vectors of the 2D prediction error functions (4) and (5) with respect to the unknown and given camera parameters, i.e, β_p and β_g, respectively. Notice that by ignoring the high order terms of $\delta\beta_p$ and $\delta\beta_g$ when deriving equation (7), we implicitly assume that the given and the estimated camera parameters, i.e, $\hat{\beta}_g$ and $\hat{\beta}_p$, are close enough to their true values. In general, the deviations of camera parameters are due to measurement noise, the error of given parameters and the local minima of the error function. However, in this paper, only the first two factors were considered, which is equivalent to making the following assumption:

Assumption 1: Optimal Solution Assumption — When deriving the covariance matrix of the estimated camera parameters, we assume that the direct nonlinear method will always result in an optimal solution.

Based on Assumption 1, it follows that the estimation error of the camera parameters, $\delta\beta_p$, should satisfy the normal equation:

$$\frac{\partial J(\delta\beta_p;\delta\beta_g)}{\partial \delta\beta_p} = 0. \quad (12)$$

By solving the above equation, we have

$$\delta\beta_p = -(A'A)^{-1}A'\left(B\delta\beta_g + \xi\right), \quad (13)$$

where

$$A = [\, a_u(1)' \quad a_v(1)' \quad \cdots \quad a_u(M)' \quad a_v(M)' \,]' \tag{14}$$

$$B = [\, b_u(1)' \quad b_v(1)' \quad \cdots \quad b_u(M)' \quad b_v(M)' \,]' \tag{15}$$

and

$$\xi = [\, s_u \epsilon_u(1) \quad s_v \epsilon_v(1) \quad \cdots \quad s_u \epsilon_u(M) \quad s_v \epsilon_v(M) \,]' \,. \tag{16}$$

In general, the aspects of both the sensor cell and the image sensor are close to square. In order to simplify the results of the derived covariance matrix, we made the following assumption:

Assumption 2: Square Imager Assumption — Assume that the aspects of both the sensor cell and the total sensing area are square; i.e, we assume that $s_u = s_v$, and that the acquired image is square (e.g, 512 by 512). Henceforth, we will use s to denote the value of the pixel spacing of a square image sensor, and use s_u and s_v to denote, respectively, the parameters of the vertical and horizontal pixel spacing, specifically.

Based on the *square imager assumption* and supposing that the probability distribution functions of the 2D measurement noise, $\epsilon_u(i)$ and $\epsilon_v(i)$, $i = 1, 2, 3, ..., M$, are identically independent Gaussian distributions with zero mean and variance, σ^2, then the covariance matrix of the estimated camera parameters can be derived as follows:

$$Var\,[\delta\beta_p] = s^2 \sigma^2 \left(A'A\right)^{-1}. \tag{17}$$

Also, the sensitivity of the estimated camera parameters with respect to the given parameters can be derived from (13) as follows:

$$E\,[\delta\beta_p] = -\left(A'A\right)^{-1} \left(A'B\right) \delta\beta_g. \tag{18}$$

Notice that A and B are matrix functions of the true camera parameters and the 3D and 2D coordinate pairs of the calibration points. In order to achieve highly accurate camera calibration results, one should use as much as possible calibration points such that the 3D and 2D coordinates of the calibration points are uniformly distributed in the 3D working space and the image plane, respectively. The following assumption states the condition for deriving the camera parameter covariances.

Assumption 3: Uniform Distribution Assumption — Assume that the calibration points are uniformly distributed within the depth range $[Z_{min}, Z_{max}]$ in the z-axis direction, and that their corresponding horizontal and vertical image coordinates of the calibration points are uniformly distributed within the region $[0, I_{max}]$ (see Fig. 1). In general, if the number of calibration points is large enough, then this assumption can be well approximated; otherwise, the derived covariance matrix is simply a lower bound of the real one since we use an integral to approximate the summation operation in the following derivation.

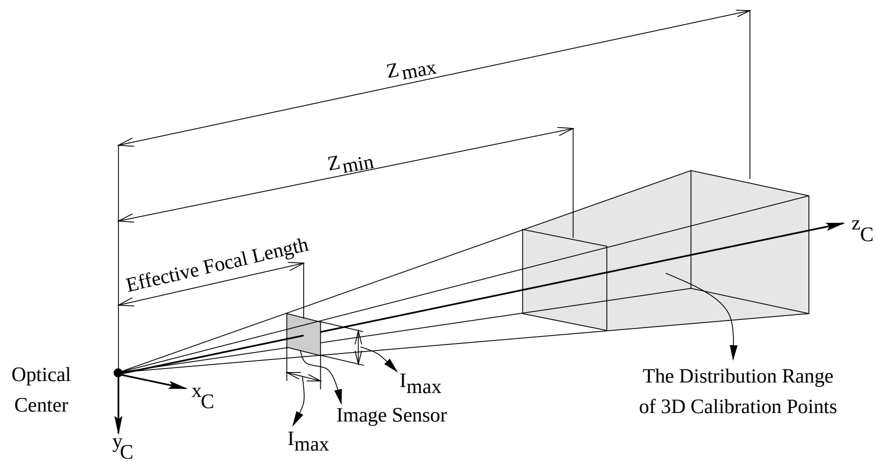

Fig. 1. The distribution range of the calibration points.

Based on Assumption 3, we have

$$A'A \approx \frac{M \int_0^{I_{max}} \int_0^{I_{max}} \int_{Z_{min}}^{Z_{max}} \left[a_u' a_u + a_v' a_v\right] dz_C du_I dv_I}{I_{max}^2 \left(Z_{max} - Z_{min}\right)}, \tag{19}$$

$$A'B \approx \frac{M \int_0^{I_{max}} \int_0^{I_{max}} \int_{Z_{min}}^{Z_{max}} \left[a_u' b_u + a_v' b_v\right] dz_C du_I dv_I}{I_{max}^2 \left(Z_{max} - Z_{min}\right)}, \tag{20}$$

subject to

$$x_C = \frac{z_C}{f} \left[\left(1 - \kappa\rho^2\right)\left(u_I - u_0\right) s_u\right], \tag{21}$$

and

$$y_C = \frac{z_C}{f} \left[\left(1 - \kappa\rho^2\right)\left(v_I - v_0\right) s_v\right]. \tag{22}$$

By using Mathematica, we are able to compute equations (19) and (20) and even the matrix inverse of equation (19). However, the results are too complex to analyze. Therefore, we concentrate on the case of calibrating a camera with a non-wide angle and low distortion lens, and make the following two assumptions, *Assumption 4: Low Lens Distortion Assumption* — Assume that the amount of lens distortion is less than 1% at the four edges of the image, i.e,

$$\left|\left(\frac{sI_{max}}{2}\right)^2 \kappa\right| < 0.01.$$

Assumption 5: Non-wide Angle Lens Assumption — Assume that the effective focal length is larger than the dimension of the image sensor. Or, more specifically, suppose that

$$f > 1.3sI_{max},$$

which is equivalent to having a view angle of less than 42°.

Based on the above two assumptions, the covariance matrix of the estimated camera parameters, $s^2\sigma^2\left(A'A\right)^{-1}$, and their sensitivity matrix with respect to given parameters, $-\left(A'A\right)^{-1}\left(A'B\right)$, can be derived and easily simplified. The criterion for simplifying the derived results is that for every two terms, both from the denominator or the numerator of an expression, the GCD (greatest common divider) is first computed, and that if we can determine that one of the remainders is at least ten times larger than another, then the smaller one is eliminated; otherwise, both of them are reserved. The notations used in the derived results are summarized in Table III. The variances of the estimated camera parameters with respect to the four types of calibration problems are listed in Table IV. The normalized correlation values of some camera parameters are listed in Table V, where the normalized correlation value of two zero-mean random variables, say δx and δy, are defined as follows:

$$\frac{E\left[\delta x \delta y\right]}{\sqrt{E\left[\delta x^2\right]E\left[\delta y^2\right]}} = \frac{\sigma_{\delta x \delta y}}{\sigma_{\delta x}\sigma_{\delta y}}. \tag{23}$$

Furthermore, the sensitivity values of some estimated camera parameters with respect to the given parameters are listed in Table VI for the four types of calibration problems.

B. Notes for Robot Kinematic Calibration

When using the results of camera calibration for robot kinematic calibration, it is very important to choose a calibration technique which provids accurate extrinsic parameters. Based on the derived variances of the estimated parameters as listed in Table IV, we find that the techniques for solving the Type 1 calibration problem are not suitable for

TABLE III

NOTATION TABLE

σ^2 — The variance of the 2D measurement noise.
δx — A small variation of the parameter x.
$\sigma^2_{\delta x}$ — The variance of δx.
$\sigma_{\delta x \delta y}$ — The covariance of δx and δy.
Z_{avg} — The average object distance.
η — The relative object depth, i.e, $\frac{Z_{max}-Z_{min}}{Z_{avg}}$.
(u_0, v_0) — The coordinates of the image center.
s_u — The horizontal pixel spacing.
s_v — The vertical pixel spacing.
f — The effective focal length.
κ — The coefficient of the radial lens distortion.
(ϕ_x, ϕ_y, ϕ_z) — The X-Y-Z Euler angles.
(t_x, t_y, t_z) — The translation vector of a camera.
I_{max} — The size of the digitized image, e.g, $I_{max} = 512$ for a 512 by 512 image.
s — The horizontal and vertical pixel spacing of a square image sensor.
a — The area of a square image sensor, i.e, $I^2_{max}s^2$.
M — The number of calibration points.

kinematic calibration. Notice that, for a *distortion-free* camera, as the image center drifts to a new position, this is equivalent to a change in the direction of the optical axis such that the piercing point of the optical axis with repsect to the image plane is right at the new image center. However, after doing so, the optical axis is no longer perpendicular to the image plane, which will cause a kind of distortion known as thin prism distortion (refer to Weng [25]). The amount of distortion caused by a slightly tilt of optical axis is large when the effective focal length is small. In contrast, if the effective focal length is large, then the amount of thin prism distortion is small (see Fig. 2). Hence, the estimated image center and the direction of the optical axis (i.e, the camera orientation) can drift farther than it can for a camera with a short effective focal length. However, when the

TABLE IV

VARIANCES OF THE CAMERA PARAMETERS FOR THE FOUR TYPES OF CALIBRATION PROBLEMS

	Type 1	Type 2	Type 3	Type 4
$\sigma^2_{\delta u_0}$	$\frac{\left(108a^2+720\eta^2 f^4\right)\sigma^2}{P(f^2\kappa)a^2\eta^2 M}$	–	–	$\frac{\sigma^2}{M}$
$\sigma^2_{\delta v_0}$	$\frac{\left(108a^2+720\eta^2 f^4\right)\sigma^2}{P(f^2\kappa)a^2\eta^2 M}$	–	–	$\frac{\sigma^2}{M}$
$\sigma^2_{\delta s_u}$	$\frac{24s^4\sigma^2}{aM}$	$\frac{24s^4\sigma^2}{aM}$	–	$\frac{24s^2\sigma^2}{aM}$
$\sigma^2_{\delta s_v}$	–	–	–	–
$\sigma^2_{\delta f}$	$\frac{72f^2s^2\sigma^2}{a\eta^2 M}$	$\frac{72f^2s^2\sigma^2}{a\eta^2 M}$	–	$\frac{45.2f^2s^2\sigma^2}{aM}$
$\sigma^2_{\delta\kappa}$	$\frac{609.68s^2\sigma^2}{a^3M}$	$\frac{609.68s^2\sigma^2}{a^3M}$	–	$\frac{609.68s^2\sigma^2}{a^3M}$
$\sigma^2_{\delta\phi_x}$	$\frac{720f^2s^2\sigma^2}{P(f^2\kappa)a^2M}$	$\frac{240f^2s^2\sigma^2}{(3a^2+20f^4\eta^2)M}$	$\frac{240f^2s^2\sigma^2}{(3a^2+20f^4\eta^2)M}$	–
$\sigma^2_{\delta\phi_y}$	$\frac{720f^2s^2\sigma^2}{P(f^2\kappa)a^2M}$	$\frac{240f^2s^2\sigma^2}{(3a^2+20f^4\eta^2)M}$	$\frac{240f^2s^2\sigma^2}{(3a^2+20f^4\eta^2)M}$	–
$\sigma^2_{\delta\phi_z}$	$\frac{6s^2\sigma^2}{aM}$	$\frac{6s^2\sigma^2}{aM}$	$\frac{6s^2\sigma^2}{aM}$	–
$\sigma^2_{\delta t_x}$	$\frac{12Z^2_{avg}s^2\sigma^2}{\eta^2 f^2 M}$	$\frac{240f^2Z^2_{avg}s^2\sigma^2}{(3a^2+20\eta^2 f^4)M}$	$\frac{240f^2Z^2_{avg}s^2\sigma^2}{(3a^2+20\eta^2 f^4)M}$	–
$\sigma^2_{\delta t_y}$	$\frac{12Z^2_{avg}s^2\sigma^2}{\eta^2 f^2 M}$	$\frac{240f^2Z^2_{avg}s^2\sigma^2}{(3a^2+20\eta^2 f^4)M}$	$\frac{240f^2Z^2_{avg}s^2\sigma^2}{(3a^2+20\eta^2 f^4)M}$	–
$\sigma^2_{\delta t_z}$	$\frac{72Z^2_{avg}s^2\sigma^2}{\eta^2 aM}$	$\frac{72Z^2_{avg}s^2\sigma^2}{\eta^2 aM}$	$\frac{6Z^2_{avg}s^2\sigma^2}{aM}$	–

Note: $P\left(f^2\kappa\right) = 60\left(f^2\kappa + \frac{11}{30}\right)^2 + \frac{14}{15}$

lens distortion is not negligible, misalignment of the image center will cause another kind of residual error. Therefore, variances of both the estimated image center and camera orientation will not be simply proportional to the the focal length when considering the effects of lens distortion. Notice that the center of the radial lens distortion (i.e, the image point where the amount of radial lens distortion is exactly zero) and the piercing point (i.e, the image center) are aligned in the camera model. As a result, the radial lens distortion will help us to locate the image center, (u_0, v_0). The above qualitative analysis of the effects of the effective focal length and the radial lens distortion is consistent with the derived theoretical results. Notice that one of the common denominators of the variances of the estimated image center and camera orientation is the following bivariable polynomial (refer to Table IV):

$$P\left(f^2\kappa\right) = 60\left(f^2\kappa + \frac{11}{30}\right)^2 + \frac{14}{15}. \tag{24}$$

TABLE V

NORMALIZED CORRELATIONS OF CAMERA PARAMETERS

Parameters	Type 1	Type 2	Type 3	Type 4
δu_0 v.s. $\delta\phi_y$	$\frac{60\eta f^2}{\sqrt{180(3a^2+20\eta^2 f^4)}}$	-	-	-
δv_0 v.s. $\delta\phi_x$	$\frac{60\eta f^2}{\sqrt{180(3a^2+20\eta^2 f^4)}}$	-	-	-
δu_0 v.s. δt_x	$-\frac{\left(9+22f^2\kappa\right)a}{\sqrt{3P(f^2\kappa)(3a^2+20\eta^2 f^4)}}$	-	-	-
δv_0 v.s. δt_y	$-\frac{\left(9+22f^2\kappa\right)a}{\sqrt{3P(f^2\kappa)(3a^2+20\eta^2 f^4)}}$	-	-	-
δs_u v.s. δf	0.29η	0.29η	-	0.36
δs_u v.s. $\delta\kappa$	$-0.2\kappa a$	$-0.2\kappa a$	-	$-0.2\kappa a$
δf v.s. $\delta\kappa$	-0.68η	-0.68η	-	-0.86
δf v.s. δt_z	1	1	-	-
$\delta\phi_x$ v.s. δt_y	$\frac{0.26\kappa a\left(11+30f^2\kappa\right)}{\eta\sqrt{P(f^2\kappa)}}$	1	1	-
$\delta\phi_y$ v.s. δt_x	$\frac{0.26\kappa a\left(11+30f^2\kappa\right)}{\eta\sqrt{P(f^2\kappa)}}$	1	1	-

Note: $P\left(f^2\kappa\right) = 60\left(f^2\kappa + \frac{11}{30}\right)^2 + \frac{14}{15}$

Because the coefficient of radial lens distortion can be either positive or negative, equation (24) has a minimum at

$$f^2 = -\frac{11}{30\kappa}. \tag{25}$$

Therefore, around the above minimum in (25), there exist a maximum of the estimation variance. For those cameras having an effective focal length and a coefficient of radial lens distortion which approximately satisfy equation (25), the techniques for solving the Type 1 calibration problem will cause large deviation when estimating the image center and the camera orientation.

Furthermore, there is yet another pair of camera parameters that are linearly dependent when the variations are small, i.e, the effective focal length and the Z-component of the translation vector (refer to Table V). Therefore, to improve the accuracy of the estimated extrinsic parameters, the three intrinsic parameters, u_0, v_0 and f, should be determined in one process, and the remanding parameters should then be estimated in another in-

TABLE VI

SENSITIVITY OF THE ESTIMATED CAMERA PARAMETERS WITH RESPECT TO THE ERROR OF THE GIVEN PARAMETERS

	Type 1	Type 2	Type 3	Type 4
$\frac{\partial u_0}{\partial \phi_y}$	–	–	–	$\frac{f}{s}$
$\frac{\partial v_0}{\partial \phi_x}$	–	–	–	$\frac{f}{s}$
$\frac{\partial u_0}{\partial t_x}$	–	–	–	$-\frac{f}{sZ_{avg}}$
$\frac{\partial v_0}{\partial t_y}$	–	–	–	$-\frac{f}{sZ_{avg}}$
$\frac{\partial s_u}{\partial s_v}$	1	1	–	1
$\frac{\partial f}{\partial s_v}$	$\frac{f}{s}$	$\frac{f}{s}$	–	$\frac{f}{s}$
$\frac{\partial f}{\partial t_z}$	–	–	–	$\frac{f}{Z_{avg}}$
$\frac{\partial \kappa}{\partial s_v}$	$\frac{2\kappa}{s}$	$\frac{2\kappa}{s}$	–	$\frac{2\kappa}{s}$
$\frac{\partial \phi_x}{\partial v_0}$	–	$\frac{20\eta^2 f^3 s}{3a^2+20\eta^2 f^4}$	$\frac{20\eta^2 f^3 s}{3a^2+20\eta^2 f^4}$	–
$\frac{\partial \phi_y}{\partial u_0}$	–	$\frac{20\eta^2 f^3 s}{3a^2+20\eta^2 f^4}$	$\frac{20\eta^2 f^3 s}{3a^2+20\eta^2 f^4}$	–
$\frac{\partial t_x}{\partial u_0}$	–	$-\frac{sa^2 Z_{avg}\left(9+22f^2\kappa\right)}{3f(3a^2+20\eta^2 f^4)}$	$-\frac{sa^2 Z_{avg}\left(9+22f^2\kappa\right)}{3f(3a^2+20\eta^2 f^4)}$	–
$\frac{\partial t_y}{\partial v_0}$	–	$-\frac{sa^2 Z_{avg}\left(9+22f^2\kappa\right)}{3f(3a^2+20\eta^2 f^4)}$	$-\frac{sa^2 Z_{avg}\left(9+22f^2\kappa\right)}{3f(3a^2+20\eta^2 f^4)}$	–
$\frac{\partial t_z}{\partial s_u}$	–	–	$-\frac{Z_{avg}}{2s}$	–
$\frac{\partial t_z}{\partial s_v}$	–	–	$-\frac{Z_{avg}}{2s}$	–
$\frac{\partial t_z}{\partial f}$	–	–	$\frac{Z_{avg}}{f}$	–
$\frac{\partial t_z}{\partial \kappa}$	–	–	$\frac{7aZ_{avg}}{30}$	–

dependent process. However, so far as we know, there is no calibration method which performs exactly in this way. On the other hand, there is a simple, inexpensive and very accurate method for calibrating the image center, (u_0, v_0), namely, the auto-collimated laser technique, which is one of the methods frequently used to determine the optical axis of lenses in the field of optics (refer to [27] for a description of this method). Since the auto-collimated laser technique is a kind of direct optical method which can be performed independently of the extrinsic parameters, we strongly recommend that this be done first; then the techniques for solving either the Type 2 or Type 3 calibration problems can be

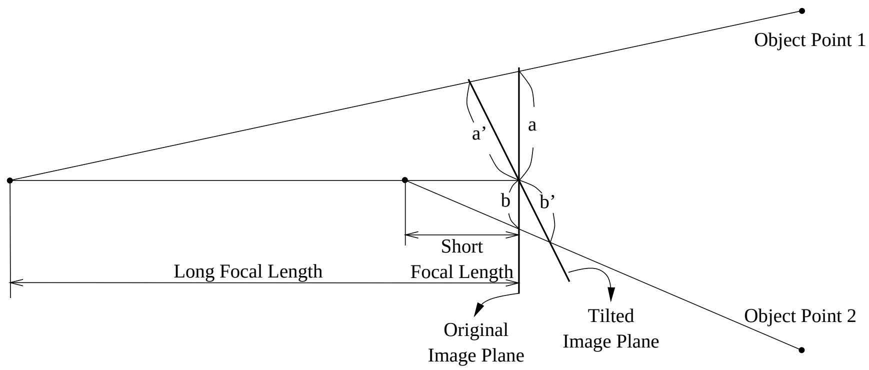

Fig. 2. The amount of thin prism distortion versus two effective focal lengths when lens distortion is negligible. Notice that a' is approximately equal to a, but that b' obviously deviates from b.

applied to obtain accurate calibration results. Moreover, the accuracy of the estimated parameters obtained by using the Type 2 technique is approximately equal to that obtained using the Type 3 technique. To summarize, the conditions for achieving highly accurate extrinsic calibration results are listed in the following:

- The image center should be calibrated using another independent process such as the auto-collimated laser technique or the Lenz and Tsai method [16]. Otherwise, the estimated camera orientation will be very sensitive to noise.
- The object distance should be made as small as possible because the estimation error of the camera is proportional to the object distance.
- Calibration points should be uniformly distributed in the 3D space, and the relative depth should be made as large as possible.
- The Number of calibration points should be selected to be as large as possible, and the measurement noise should be made small.

C. Notes for Calibration of an Active Vision System

Calibration of an active vision system consists of two major steps: one is extrinsic calibration and another is intrinsic calibration. The purpose of extrinsic calibration of an active vision system is to obtain the kinematic model of the vision system, which is exactly a robot calibration problem. Some important issues for accurately calibrating a robot based on camera calibration techniques were discussed in the previous subsection. The intrinsic calibration problem is trivial if the active vision system contains no motorized lens. However, the intrinsic calibration problem for a motorized lens is more complicated. As can be seen from Table IV and Table V, the linear dependency of small variations of some parameters will cause the estimated intrinsic parameters to be very sensitive to noise. Fortunately, there are two approaches for solving this problem. One is to use the auto-collimated laser technique to estimate the image center with respect to each lens setting so as to eliminate the most sensitive parameters (refer to Willson[27]). Another approach is to make full use of the extra small degrees of freedom, i.e, the linear dependency between some parameters for small variations. For instance, we can assume that the camera orientation is independent of the lens setting and use the image center to compensate for the error induced by the incorrect orientation parameters. Also, from the sensitivity analysis results listed in Table VI, we find that the intrinsic parameters are less sensitive to the error of the z-component of the translation vector. Additionally, in our experience, the drifting of the x- and y-components of the translation vector is negligible when the lens setting is changed. Therefore, if the variation of the focal length is not very large (e.g, the focus setting only is changed), then we may assume that the position of the camera is independent of the focus setting when solving the Type 4 calibration problem.

IV. Experiments

Two computer simulations were performed to verify our theoretical analysis. In the computer simulations, the 3D and their corresponding 2D calibration data were generated according to a set of given camera parameters. Then, Gaussian random noise was added to the 2D calibration data to simulate the measurement noise. The initial estimate of the camera parameters required by the direct nonlinear optimization method was simply assigned to be the true values of the camera parameters in the computer simulations. For each set of given camera parameters, the above process were repeated several times, and the sample mean and variance of the estimated camera parameters were computed to verify our theoretical analysis.

In the first experiment, we set $\sigma = 0.1$ pixel, $M = 100$, $s = 0.01mm$, $\kappa = -0.0002mm^{-2}$, $I_{max} = 512$ (i.e, $a = 26.2mm^2$), $\eta = 0.2$, $Z_{avg} = 1200mm$, and $f = 10, 20, 30, \cdots, 100$, and set all the extrinsic parameters to zero. For each effective focal length, the corresponding mean and variance of the estimated parameters were computed from 100 random trials.

Fig. 3 shows some of the results of the first experiment, where the theoretical prediction results and the sample variances of the camera orientation estimation error are plotted. Both the theoretical and experimental results show that the Type 2 and Type 3 calibration techniques can provide much better orientation estimation than can the Type 1 technique. Notice that the theoretical prediction results deviate slightly from the simulation results. We believe that this is partially because of the effects of eliminating the high order terms and partially because of the local minima of the nonlinear error functions. Nevertheless, the theoretical analysis results are accurate enough to serve as a guideline for selecting calibration techniques to fit different kinds of requirements.

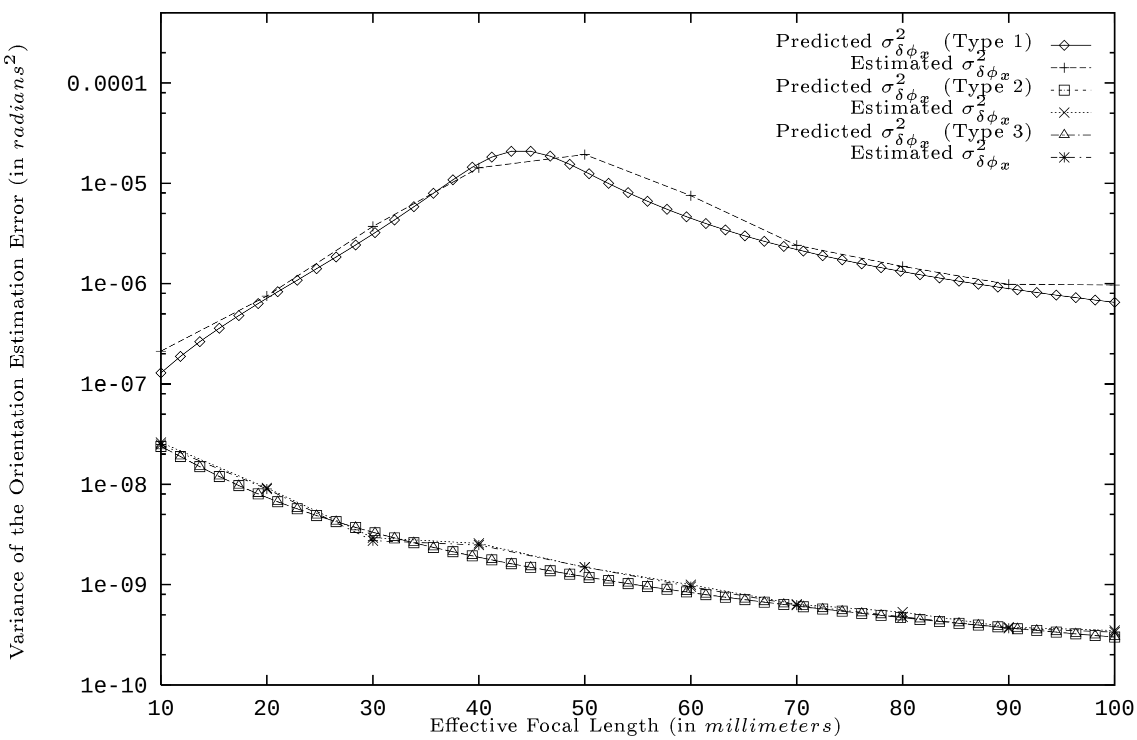

Fig. 3. The orientation estimation error versus the effective focal length for the first three types of calibration techniques.

In the second experiment, we randomly generated 100 sets of camera parameters, where the true value of the image center and the extrinsic parameters were set to zero, and the camera parameters were generated uniformly from the ranges, $s_u \in [0.005, 0.02]$,

$s_v = s_u$, $f \in [8, 100]$, $\kappa \in [-.0008, .0008]$, $50 \leq M \leq 200$, $\sigma \in [0, 0.5]$, $\eta \in [0.01, 0.5]$, and $Z_{avg} \in [100, 2000]$. For each set of camera parameters, 100 random trials were performed to compute the mean and variance of the estimated parameters. The computed sample variances were then normalized by the theoretical variances. The mean, minimum, maximum and standard deviation of the 100 normalized variances are listed in Tables VII–VIII. Notice that the means of the normalized variances are close to unity, and that their standard deviations are small, which means that the derived theoretical variances are very accurate.

TABLE VII

Some Statistics of the Normalized Variances of the Estimated Camera Parameters for the Type 1 and Type 2 Calibration Problems

	Type 1				Type 2			
	min	mean	max	std.dev.	min	mean	max	std.dev.
$\tilde{\sigma}^2_{\delta u_0}$	0.6	1.5	10	1.4	-	-	-	-
$\tilde{\sigma}^2_{\delta v_0}$	0.7	1.4	6	0.9	-	-	-	-
$\tilde{\sigma}^2_{\delta s_u}$	0.7	1.2	7.1	0.9	0.7	1	1.3	0.1
$\tilde{\sigma}^2_{\delta s_v}$	-	-	-	-	-	-	-	-
$\tilde{\sigma}^2_{\delta f}$	0.8	1.2	1.8	0.2	0.8	1.1	1.8	0.2
$\tilde{\sigma}^2_{\delta \kappa}$	0.8	1.2	1.8	0.2	0.7	1.1	1.8	0.2
$\tilde{\sigma}^2_{\delta \phi_x}$	0.7	1.6	10	1.3	0.7	1.1	1.5	0.2
$\tilde{\sigma}^2_{\delta \phi_y}$	0.7	1.7	10	1.5	0.7	1	1.3	0.1
$\tilde{\sigma}^2_{\delta \phi_z}$	0.7	1.1	4.7	0.4	0.7	1	1.3	0.1
$\tilde{\sigma}^2_{\delta t_x}$	0.5	1	2	0.2	0.7	1	1.4	0.1
$\tilde{\sigma}^2_{\delta t_y}$	0.7	1.1	4.7	0.4	0.7	1	1.4	0.2
$\tilde{\sigma}^2_{\delta t_z}$	0.8	1.1	1.6	0.2	0.7	1.1	1.6	0.2

V. Conclusion

In camera calibration, due to the correlation between certain camera parameters, e.g, the correlation between the image center and the camera orientation, an estimate of a set of

TABLE VIII

Some Statistics of the Normalized Variances of the Estimated Camera Parameters for the Type 3 and Type 4 Calibration Problems

	Type 3				Type 4			
	min	mean	max	std.dev.	min	mean	max	std.dev.
$\tilde{\sigma}^2_{\delta u_0}$	-	-	-	-	0.8	1	1.4	0.1
$\tilde{\sigma}^2_{\delta v_0}$	-	-	-	-	0.7	1	1.4	0.1
$\tilde{\sigma}^2_{\delta s_u}$	-	-	-	-	0.7	1	1.3	0.1
$\tilde{\sigma}^2_{\delta s_v}$	-	-	-	-	-	-	-	-
$\tilde{\sigma}^2_{\delta f}$	-	-	-	-	0.7	1	1.4	0.2
$\tilde{\sigma}^2_{\delta \kappa}$	-	-	-	-	0.7	1	1.5	0.2
$\tilde{\sigma}^2_{\delta \phi_x}$	0.7	1	1.4	0.2	-	-	-	-
$\tilde{\sigma}^2_{\delta \phi_y}$	0.6	1	1.3	0.1	-	-	-	-
$\tilde{\sigma}^2_{\delta \phi_z}$	0.7	1	1.3	0.1	-	-	-	-
$\tilde{\sigma}^2_{\delta t_x}$	0.6	0.9	1.3	0.1	-	-	-	-
$\tilde{\sigma}^2_{\delta t_y}$	0.7	1	1.4	0.2	-	-	-	-
$\tilde{\sigma}^2_{\delta t_z}$	0.7	1	1.5	0.2	-	-	-	-

camera parameters which minimizes a given criterion does not guarantee that the physical camera parameter estimates are themselves accurate. This problem has not drawn much attention from our computer vision society because most computer vision applications require only accurate 3D measurements and do not care much about the values of the physical parameters as long as their composite effect is satisfactory. However, in calibrating an active vision system where the cameras are motorized such that their parameters can be adapted to the environment, accuracy of the physical parameters is very critical because we need accuracy to establish the relation between the motor positions and the camera parameters (both intrinsic and extrinsic). The contribution of this work has mainly been in error analysis of camera calibration, especially with regard to the accuracy of the physical camera parameters themselves, for four different types of calibration problems.

In this work, we have successfully derived the formulas for the covariance matrix of the estimated camera parameters and for the sensitivity matrix of the estimated camera parameters with respect to the error of the given parameters. From the derived results, we conclude that the most accurate estimation technique among the four types of calibration techniques is to split the camera calibration process into two independent processes; the extrinsic parameters are calibrated in one process, i.e, the Type 3 technique, and the intrinsic camera parameters are calibrated in another process, i.e, the Type 4 technique. However, since there is no easy way to estimate accurate intrinsic parameters independently of the extrinsic parameters, or vice versa, a second best choice is to use the Type 2 calibration technique where the image center can be estimated by using the auto-collimated laser technique. If one does not want to use the auto-collimated laser technique yet wants to estimate the extrinsic parameters with high accuracy, we recommand use of the Type 1 calibration technique first and then use of the estimated intrinsic parameters as the given parameters for the Type 3 technique in subsequent calibration tasks. In this way, the Type 3 technique will usually provide more accurate results. Notice that when using the Type 1 or Type 2 technique, one should follow the guidelines listed below to achieve more accurate estimation results:

- The relative object depth, η, should be made as large as possible.
- For the Type 2 technique, the focal length of the camera lens should be selected to be as large as possible.
- The object distance, Z_{avg}, should be made as small as possible.
- The number of calibration points, M, should be made as large as possible.
- The standard deviation of the 2D measurement noise, σ, should be reduced to be as small as possible.

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