Epistemic Logics for Information Fusion

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Abstract. In this paper, we propose some extensions of epistemic logic for reasoning about information fusion. The fusion operators considered in this paper include majority merging, arbitration, and general merging. Some modalities corresponding to these fusion operators are added to epistemic logics and the Kripke semantics of these extended logics are presented. While most existing approaches treat information fusion operators as meta-level constructs, these operators are directly incorporated into our object logic language. Thus it is possible to reason about not only the merged results but also the fusion process in our logics.
Key Words: Epistemic logic, database merging, belief fusion, majority merging, arbitration, general merging, belief revision, multi-agent systems.

1 Introduction

The philosophical analysis of knowledge and belief has stimulated the development of the so-called epistemic logic[21]. This kind of logic has attracted the attention of researchers from diverse fields such as artificial intelligence(AI), economics, linguistics, and theoretical computer science. Among them, the AI researchers and computer scientists develop some technically sophisticated formalisms and apply them to the analysis of distributed and multi-agent systems[20, 36].

The application of epistemic logic to AI and computer science puts its emphasis on the interaction of agents, so multi-agent epistemic logic is urgently needed. One representative example of such logic is proposed by Fagin et al.[20]. The term "knowledge" is used in a broad sense in [20] to cover cases of belief and information.[1] The most novel feature of their logic is the consideration of common knowledge and distributed knowledge among a group of agents. Distributed knowledge is that which can be deduced by pooling together everyone's knowledge. While it is required that proper knowledge must be true, the belief of an agent may be wrong. Therefore, in general, there will be conflict in the beliefs to be merged. In this case, everything can be deduced from the distributed beliefs due to the notorious omniscience property of epistemic logic, so the merged result will be useless for further reasoning.

[1] More precisely, the logic for belief is called doxastic logic. However, here we will use the three terms knowledge, belief, and information interchangeably, so epistemic logic is assumed to cover all these notions.

Instead of directly putting all beliefs of the agents together, there are other sophisticated techniques for knowledge base merging[12, 15, 25–27, 33–35]. Most of the approaches treat belief fusion operators as meta-level constructs, so given a set of knowledge bases, these fusion operators will return the merged results. More precisely, a fusion operator is used to combine a set of knowledge bases $T_1, T_2, \cdots, T_k$, where each knowledge base is a theory in some logical langauge.

Some of the above-mentioned works present concrete operators that can be used directly in the fusion process, while others stipulate the desirable properties of reasonable belief fusion operators by postulates. However, few of the approaches provide the capability of reasoning about the fusion process. In this paper, we propose that belief fusion operators can be incorporated into the object language of the multi-agent epistemic logic, so we can reason not only with the merged results but also about the fusion process.

1.1 Preliminary

Let $\mathcal{L}$ denote the language of epistemic logic. The alphabet of $\mathcal{L}$ contains the following symbols: a countable set $\Phi_0 = \{p, q, r, \ldots\}$ of atomic propositions; the propositional constants $\bot$ (falsum or falsity constant) and $\top$ (verum or truth constant); the binary Boolean operator $\vee$ (or), and unary Boolean operator $\neg$ (not); a set $Ag = \{1, 2, \ldots, n\}$ of agents; the modal operator-forming symbols "[" and "]"; and the left and right parentheses "(" and ")".

The set of well-formed formulas(wffs)is defined as the smallest set containing $\Phi_0 \cup \{\bot, \top\}$ and closed under Boolean operators and the following rule:

if φ is a wff, then $[G]\varphi$ is a wff for any nonempty $G \subseteq Ag$.

The intuitive meaning of $[G]\varphi$ is "The group of agents G has distributed belief φ"

As usual, other classical Boolean connectives $\wedge$ (and), $\supset$ (implication), and $\equiv$ (equivalence) can be defined as abbreviations. Also, we will write $\langle G\rangle\varphi$ as an abbreviation of $\neg[G]\neg\varphi$. When G is a singleton $\{i\}$, we will write $[i]\varphi$ instead of $[\{i\}]\varphi$, so $[i]\varphi$ means that agent i knows φ.

For the semantics, a possible world model for $\mathcal{L}$ is a structure

$$(W, (\mathcal{R}_i)_{1\leq i\leq n}, V),$$

where

- W is a set of possible worlds,
- $\mathcal{R}_i \subseteq W \times W$ is a serial binary relation[2] over W for $1 \leq i \leq n$,
- $V : \Phi_0 \rightarrow 2^W$ is a truth assignment mapping each atomic proposition to the set of worlds in which it is true.

From the binary relations $\mathcal{R}_i$'s, we can define a derived relation $\mathcal{R}_G$ for each nonempty $G \subseteq Ag$:

$$\mathcal{R}_G = \cap_{i\in G}\mathcal{R}_i.$$

[2] A binary relation $\mathcal{R}$ is serial if $\forall w \exists u.\mathcal{R}(w, u)$.

Informally, $\mathcal{R}_i(w)$ is the set of worlds that agent i considers possible under w according to his belief, so $\mathcal{R}_G(w)$ is the set of worlds that are considered possible under w according to the direct fusion of agents' beliefs. The informal intuition is reflected in the definition of the satisfaction relation. Let $M = (W, (\mathcal{R}_i)_{1\leq i\leq n}, V)$ be a model and Φ be the set of wffs for $\mathcal{L}$, then the satisfaction relation $\models_M \subseteq W \times \Phi$ is defined by the following inductive rules(we will use the infix notation for the relation and omit the subscript M for convenience):

1. $w \models p$ iff $w \in V(p)$, for each $p \in \Phi_0$,
2. $w \not\models_M \bot$ and $w \models_M \top$,
3. $w \models \neg\varphi$ iff $w \not\models \varphi$,
4. $w \models \varphi \vee \psi$ iff $w \models \varphi$ or $w \models \psi$,
5. $w \models [G]\varphi$ iff for all $u \in \mathcal{R}_G(w)$, $u \models \varphi$.

In the presentation below, we will extensively use the notions of pre-order. Let S be a set, then a pre-order over S is a reflexive and transitive binary relation $\leq$ on S. A pre-order over S is called total (or connected) if for all $x, y \in S$, either $x \leq y$ or $y \leq x$ holds. We will write $x < y$ as the abbreviation of $x \leq y$ and $y \not\leq x$. For a subset S' of S, $\min(S', \leq)$ is defined as the set $\{x \in S' \mid \forall y \in S', y \not< x\}$.

2 Merging by Majority

Majority voting is a method to resolve conflict between agents. For example, if three knowledge bases $T_1 = \{\varphi\}, T_2 = \{\varphi\}$, and $T_3 = \{\neg\varphi\}$ are combined, then the result would be $\{\varphi\}$, since two vote for φ, whereas only one votes against it.

One of the most general merging functions based on majority is defined in [34]. A function $Merge$ is applied to weighted knowledge bases. Let $wt : \{T_1, T_2, \cdots, T_k\} \rightarrow R^+$ be a weight function which assigns a positive real number to each component knowledge base, then a total pre-order over the set of propositional interpretations is defined as:

$$w \preceq_{(\{T_1,T_2,\cdots,T_k\},wt)} w' \text{ iff } \sum_{i=1}^{k} dist(w, T_i) \cdot wt(T_i) \leq \sum_{i=1}^{k} dist(w', T_i) \cdot wt(T_i),$$

where $dist$ is a function denoting the distance between a propositional interpretation and a knowledge base. When the propositional language is finite, the so-called Dalal distance (or Hamming distance) between two interpretations of the language is used[16]. It is defined as the number of atoms whose valuations differs in the two interpretations. Let $dist(w, w')$ denote the Dalal distance between two interpretations w and w', then the distance from w to a theory T, denoted by $dist(w, T)$, is defined as:

$$dist(w, T) = \min\{dist(w, w') \mid w' \models T\}.$$

The merged result $Merge(T_1, T_2, \cdots, T_k, wt)$ is defined as:

$$\{\varphi \mid \forall w \in min(\Omega, \preceq), w \models \varphi\},$$

where Ω is the set of all propositional interpretations and $\preceq$ is $\preceq_{(\{T_1,T_2,\cdots,T_k\},wt)}$.

This kind of weighted merging operator can be incorporated into epistemic logic in the following way. Syntactically, a new class of modal operators $[M(G,wt)]$ for any nonempty $G \subseteq \{1,2,\cdots,n\}$ and weight function $wt : Ag \rightarrow R^+$ is added to our logic language. Then the semantics for the new modal operators is defined by extending a possible world model to $(W,(\mathcal{R}_i)_{1\leq i\leq n},V,\mu)$, where $(W,(\mathcal{R}_i)_{1\leq i\leq n},V)$ is an $\mathcal{L}$ model, whereas $\mu : W \times W \rightarrow R^+ \cup \{0\}$ is a distance metric function between possible worlds satisfying $\mu(w,w) = 0$ and $\mu(w,w') = \mu(w',w)$.

The distance metric between possible worlds is defined as in the semantics of conditional logic[37, 40]. The distance from a possible world w to the belief state of an agent i in the possible world u is defined by:

$$dist_u(w,i) = \inf\{\mu(w,w') \mid (u,w') \in \mathcal{R}_i\}.$$

Then a total pre-order $\preceq^u_{(G,wt)}$ over the possible worlds is defined for each possible world u and modal operator $[M(G,wt)]$:

$$w \preceq^u_{(G,wt)} w' \text{ iff } \sum_{i\in G} dist_u(w,i)\cdot wt(i) \leq \sum_{i\in G} dist_u(w',i)\cdot wt(i).$$

The most straightforward definition for the satisfaction of the wff $[M(G,wt)]\varphi$ is:

$$u \models [M(G,wt)]\varphi \text{ iff for all } w \in \min(W,\preceq^u_{(G,wt)}), w \models \varphi.$$

However, since for infinite W, the set $\min(W,\preceq^u_{(G,wt)})$ may be empty, the definition may result in $u \models [M(G,wt)]\bot$ in some cases. Alternatively, since $\preceq^u_{(G,wt)}$ is a total pre-order, it is simply a system-of-spheres in the semantics of conditional logic[37], so we can define the satisfaction of the wff $[M(G,wt)]\varphi$ by

$$u \models [M(G,wt)]\varphi \text{ iff there exists } w_0 \text{ such that for all } w \preceq^u_{(G,wt)} w_0, w \models \varphi.$$

Note that the function wt is used only for encoding the reliability of agents. It is tempting to propagate the weights into a group of agents so that we have a weight $wt(G)$ for each group G. This weight may be useful in the belief fusion of two groups of agents. However, we do not really need this because if we want to merge the beliefs of two groups G_1 and G_2, we can simply merge the beliefs of agents in $G_1 \cup G_2$.

3 Arbitration

The notion of distance measure between possible worlds is also used in arbitration, another type of merging operator [32, 38, 39].

A semantic characterization for arbitration is given in [32]. A knowledge base in [32] is identified with a set of propositional models, thus the semantic characterization for this kind of arbitration is given by assigning to each subset of models A a binary relation $\leq_A$ over the set of model sets satisfying the following conditions (the subscript is omitted when it means all binary relations of the form $\leq_A$):

1. transitivity: if $A \leq B$ and $B \leq C$ then $A \leq C$,
2. if $A \subseteq B$ then $B \leq A$,
3. $A \leq A \cup B$ or $B \leq A \cup B$,
4. $B \leq_A C$ for every C iff $A \cap C \neq \emptyset$,
5. $A \leq_{C \cup D} B \Leftrightarrow \begin{cases} C \leq_{A \cup B} D \ \ and \ \ A \leq_C B \ \ or \\ D \leq_{A \cup B} C \ \ and \ \ A \leq_D B. \end{cases}$

By slightly abusing the notation, $\leq_A$ may also denote binary relations between models in the sense that $w \leq_A w'$ iff $\{w\} \leq_A \{w'\}$. The arbitration between two sets of models A and B is then defined as:

$$A \triangle B = \min(A, \leq_B) \cup \min(B, \leq_A). \tag{1}$$

To incorporate the arbitration operator of [32] into epistemic logic, we first note that according to (1), the arbitration is commutative but not necessarily associative. Thus, the arbitration operator should be a binary operator between two agents. We can add a class of modal operators for arbitration into our logic just as in the case of majority merging. However, to be more expressive, we will also consider the interaction between arbitration and other epistemic operators, so we define the set of *arbitration expressions* over Ag recursively as the smallest set containing Ag and closed under the binary operators $+, \cdot$, and $\triangle$. Here $+$ and $\cdot$ correspond respectively to the distributed belief and the so-called "everybody knows" operators in multi-agent epistemic logic[20]. Then the operator $[G]$ in epistemic logic can be replaced with a new class of modal operators $[a]$ where a is an arbitration expression.

For the semantics, a model is extended to $(W, (\mathcal{R}_i)_{1 \leq i \leq n}, V, \leq)$, where $\leq$ is a function assigning to each subset of possible worlds A a binary relation $\leq_A \subseteq 2^W \times 2^W$ satisfying the above-mentioned five conditions. Note that the first two conditions imply that $\leq_A$ is a pre-order over 2^W. Then for each arbitration expression, we can define the binary relations $\mathcal{R}_{a \triangle b}, \mathcal{R}_{a \cdot b}$ and $\mathcal{R}_{a+b}$ over W recursively by:

$$\mathcal{R}_{a \triangle b}(w) = \min(\mathcal{R}_a(w), \leq_{\mathcal{R}_b(w)}) \cup \min(\mathcal{R}_b(w), \leq_{\mathcal{R}_a(w)})$$

$$\mathcal{R}_{a+b} = \mathcal{R}_a \cap \mathcal{R}_b$$

$$\mathcal{R}_{a \cdot b} = \mathcal{R}_a \cup \mathcal{R}_b$$

Thus the satisfaction for the wff $[a]\varphi$ is defined as:

$$u \models [a]\varphi \ \text{ iff for all } \ w \in \mathcal{R}_a(u), w \models \varphi.$$

Note that the original distributed belief operator $[G]$ is equivalent to $[i_1 + (i_2 + \cdots (i_{k-1} + i_k))]$ if $G = \{i_1, i_2, \cdots, i_k\}$. Furthermore, it has been shown that the only associative arbitration satisfying postulates 7 and 8 of [32] is $A \triangle B = A \cup B$, so if $\triangle$ is an associative arbitration satisfying those postulates, then $[a \triangle b]\varphi$ is reduced to $[a \cdot b]\varphi$, which is in turn equivalent to $[a]\varphi \wedge [b]\varphi$.

By this kind of modal operators, the postulates 2-8 of [32] can be translated into the following axioms:

1. $[a\triangle b]\varphi \equiv [b\triangle a]\varphi$,
2. $[a\triangle b]\varphi \supset [a+b]\varphi$,
3. $\neg[a+b]\bot \supset ([a+b]\varphi \supset [a\triangle b]\varphi)$,
4. $[a\triangle b]\bot \supset [a]\bot \wedge [b]\bot$,
5. $([a\triangle(b\cdot c)]\varphi \equiv [a\triangle b]\varphi) \vee ([a\triangle(b\cdot c)]\varphi \equiv [a\triangle c]\varphi) \vee ([a\triangle(b\cdot c)]\varphi \equiv [(a\triangle b)\cdot (a\triangle c)]\varphi)$,
6. $[a]\varphi \wedge [b]\varphi \supset [a\triangle b]\varphi$,
7. $\neg[a]\bot \supset \neg[a+(a\triangle b)]\bot$.

However, since the set of possible worlds W may be infinite in our logic, the minimal models in (3) may not exist, so the axioms 4 and 7 are not sound with respect to the semantics. To make them sound, we must add the following limit assumption[2] to the binary relations $\leq_A$ for any $A \subseteq W$:

$$\text{for any nonempty } U \subseteq W,\ \min(U, \leq_A) \text{ is nonempty.}$$

4 General Merging

In [26], an axiomatic framework unifying the majority merging and arbitration operators is presented. A set of postulates common to majority and arbitration operators is first proposed to characterize the general merging operators and then additional postulates for differentiating them are considered respectively. In that framework, a knowledge base is also a finite set of propositional sentences. The general merging operator is defined as a mapping from a multi-set[3] of knowledge base, called a *knowledge set*, to a knowledge base. Therefore, the arbitration operator defined via this approach can merge more than two knowledge bases, whereas the definition of arbitration operator in [32] is limited to two knowledge bases. The merging operator is denoted by $\triangle$, so for each knowledge set E, $\triangle(E)$ is a knowledge base. Two equivalent semantic characterizations are also given for the merging operators. One is based on the so-called *syncretic assignment*. A syncretic assignment maps each knowledge set E to a pre-order $\leq_E$ over interpretations such that some conditions reflecting the postulated properties of the merging operators must be satisfied. Then $\triangle(E)$ is the knowledge base whose models are the minimal interpretations according to $\leq_E$.

This logical framework is further extended to dealing with integrity constraints in [27]. Let E be a knowledge set and φ be a propositional sentence denoting the integrity constraints, then the merging of knowledge bases in E with integrity constraint φ, $\triangle_\varphi(E)$, is a knowledge base which implies φ. The models of $\triangle_\varphi(E)$ are characterized by $\min(Mod(\varphi), \leq_E)$, i.e., the minimal models of φ with respect to the ordering $\leq_E$. $\triangle_\varphi(E)$ is called an IC merging operator. According to the semantics, it is obvious that $\triangle(E)$ is a special case of IC merging operator $\triangle_\top(E)$. It is also shown that when E contains exactly one knowledge base, the operator is reduced to the AGM revision operator proposed in [1].

[3] A multi-set, also called a bag, is a collection of elements over some domain which allows multiple occurrences of elements.

Therefore, IC merging is general enough to cover majority merging, arbitration, and AGM revision operator.

To incorporate IC merging operators into epistemic logic, we will extend its syntax with the following formation rule:

– if φ and ψ are wffs, then for any nonempty $G \subseteq \{1, 2, \dots, n\}$, $[\triangle_\varphi(G)]\psi$ is also a wff.

For the convenience of naming, we will call a subset of possible worlds a belief state. Let $\mathcal{U} = \{U_1, U_2, \dots, U_k\}$ denote a multi-set of belief states, then $\bigcap \mathcal{U} = U_1 \cap \cdots U_k$. For the semantics, a possible world model is extended to $(W, (\mathcal{R}_i)_{1 \leq i \leq n}, V, \leq)$, where $\leq$ is an assignment mapping each multi-set of belief states $\mathcal{U}$ to a total pre-order $\leq_\mathcal{U}$ over W satisfying the following conditions:

1. If $w, w' \in \bigcap \mathcal{U}$, then $w \leq_\mathcal{U} w'$,
2. If $w \in \bigcap \mathcal{U}$ and $w' \notin \bigcap \mathcal{U}$ then $w <_\mathcal{U} w'$,
3. For any $w \in U_1$, there exists $w' \in U_2$, such that $w' \leq_{\{U_1, U_2\}} w$, where U_1 and U_2 are two belief states,
4. If $w \leq_{\mathcal{U}_1} w'$ and $w \leq_{\mathcal{U}_2} w'$, then $w \leq_{\mathcal{U}_1 \sqcup \mathcal{U}_2} w'$, where $\sqcup$ denotes the union of two multi-sets,
5. If $w <_{\mathcal{U}_1} w'$ and $w \leq_{\mathcal{U}_2} w'$, then $w <_{\mathcal{U}_1 \sqcup \mathcal{U}_2} w'$.

These conditions are model-theoretic correspondences of those for syncretic assignments in [26, 27]. Condition 1 says that possible worlds appearing in the belief states of all agents are equally plausible. Condition 2 asserts that a possible worlds appearing in the belief states of all agents is more plausible than those not. Condition 3 requires that all agents are treated fairly. Therefore, if agent 1 considers w possible, then w is not more plausible than all worlds in the belief state of agent 2. Conditions 4 and 5 essentially require that if two groups of agents agree on the ordering between w and w', then the united group of these two groups does not reverse the ordering.

For a group of agents G and a possible world u, let us define a total pre-order $\leq_G^u$ over W as follows:

$$w \leq_G^u w' \text{ iff } w \leq_{\{\mathcal{R}_i(u) | i \in G\}} w'.$$

The truth condition of $[\triangle_\varphi(G)]\psi$ is defined as that for conditional logic[10, 9]. Formally, $u \models [\triangle_\varphi(G)]\psi$ iff

(i) there are no possible worlds in W satisfying φ, or
(ii) there exists $w_0 \in W$ such that $w_0 \models \varphi$ and for any $w \leq_G^u w_0$, $w \models \varphi \supset \psi$.

Note that in IC merging, a knowledge set consists of a multi-set of objective sentences, whereas for the modal operator $[\triangle_\varphi(G)]$, G is a set of agents whose beliefs may contain subjective sentences or beliefs of other agents. Also, an integrity constraint in [27] must be an objective sentence, whereas φ may be arbitrary complex wffs of our extended language. Furthermore, instead of selecting minimal models of φ, since the set of possible worlds may be infinite in our case, we adopt the system-of-spheres semantics as in section 2 for the epistemic operator $[\triangle_\varphi(G)]$.

5 Belief Change

Unlike knowledge merging, where the component knowledge bases are equally important, belief change is a kind of asymmetry operator, where new information always outweighs the old. The main belief change operators are belief revision and update. They are characterized by different postulates[1, 23, 24]. In [23], a uniform model-theoretic framework is provided for the semantic characterization of the revision and update operators. In that context, a knowledge base is a finite set of propositional sentences, so it can also be represented by a single sentence(i.e., the conjunction of all sentences in the knowledge base).

For the revision operator, it is assumed that there is a total pre-order $\leq_\psi$ over the propositional interpretations for each knowledge base ψ. The revision operators satisfying the AGM postulates in [1] are exactly those that select from the models of the new information φ the minimal ones with respect to the ordering $\leq_\psi$. More precisely, let ψ be a knowledge base and φ denote the new information, then the result of revising ψ by φ, denoted by $\psi \circ \varphi$, will have the set of models

$$Mod(\psi \circ \varphi) = \min(Mod(\varphi), \leq_\psi).$$

As for the update operator, assume for each propositional interpretation w, there exists some partial pre-order $\leq_w$ over the interpretations for closeness to w, then update operators select for each model w in $Mod(\psi)$ the set of models from $Mod(\varphi)$ that are closest to w. The updated theory is characterized by the union of all such models. That is,

$$Mod(\psi \diamond \varphi) = \bigcup_{w \in Mod(\psi)} \min(Mod(\varphi), \leq_w),$$

where $\psi \diamond \varphi$ is the result of updating the knowledge base ψ by φ.

Both belief revision and update may occur in the observation of new information φ. For belief revision, it is assumed that the world is static, so if the new information is incompatible with the agent's original beliefs, then the agent may have an incorrect belief about the world. Thus he will try to accommodate the new information by minimally changing his original beliefs. However, for the belief update, it is assumed that the observation may be due to dynamic changes of the outside world, so the agent's belief may be out-of-date, though it may be totally correct for the original world. Thus the agent will assume the possible worlds are those resulting from the minimal change of the original world. In [11], a generalized update model is proposed which combines aspects of both revision and update. It is shown that a belief update model will be inadequate without modelling the dynamic aspect (i.e. the events causing the update) in the same time. Since the dynamic change of the external worlds does not play a role in the belief fusion process, we will not model belief update in our logic. Therefore, in what follows, we will concentrate on the belief revision operator.

Let us now consider the possibility of incorporating the belief revision operator into epistemic logic. In addition to the original meaning of revising a knowledge base ψ by new information φ, there is an alternative reading for the revision operator. That is, we can consider $\circ$ as a prioritized belief fusion operator that gives priority to its second argument[22]. In the context of knowledge base revision, these two interpretations are essentially equivalent. However, from the perspective of our logic in multi-agents systems, they may be quite different. Roughly speaking, $i \circ \varphi$ will denote the result of revising the beliefs of agent i by new information φ, whereas $i \circ j$ is the result of merging the beliefs of agents i and j by giving priority to j. More formally, a *revision expression* will be defined inductively as follows:

- If $1 \leq i, j \leq n$ and φ is a wff, then $i \circ j$ and $i \circ \varphi$ are revision expressions.
- If r is a revision expression, $1 \leq i \leq n$ and φ is a wff, then $r \circ i$ and $r \circ \varphi$ are revision expressions.

The syntactic rule is extended to include the modal operators $[r]$ for any revision expression r, so $[r]\varphi$ would be a wff if φ is. Note that a revision expression allows us to represent a revision sequence, which is directly related to iterated revision in [8, 17].

To interpret the modal operator in our semantic framework, a possible world model is extended to $(W, (\mathcal{R}_i)_{1 \leq i \leq n}, V, \leq)$, where $\leq$ is an assignment mapping each belief state (i.e. subset of possible worlds) U to a total pre-order $\leq_U$ over W such that (i) if $w, w' \in U$, then $w \leq_U w'$ and (ii) if $w \in U$ and $w' \notin U$, then $w <_U w'$. Let $S{\cdot}U$ denote the sequence $(U_1, U_2, \cdots, U_k, U)$ if $S = (U_1, U_2, \cdots, U_k)$ is a sequence of belief state, then the assignment $\leq$ is extended to sequences of belief states in the following way (we assume $\leq_{(U)} = \leq_U$):

1. $w <_{S \cdot U} w'$ if $w \in U$ and $w' \notin U$,
2. $w \leq_{S \cdot U} w'$ iff $w \leq_S w'$ when both $w, w' \in U$ or both $w, w' \notin U$.

For each wff φ, let the truth set of φ, denoted by $|\varphi|$, be defined as $\{w \in W \mid w \models \varphi\}$. For each possible world u, define a function mapping any agent i and revision expression r into a sequence of belief states $u(i)$ and $u(r)$ as follows:

1. $u(i) = (\mathcal{R}_i(u))$,
2. $u(r \circ i) = u(r) \cdot \mathcal{R}_i(u)$,
3. $u(r \circ \varphi) = u(r) \cdot |\varphi|$.

Then the truth condition for the wff $[r \circ \varphi]\psi$ is $u \models [r \circ \varphi]\psi$ iff

(i) there are no possible worlds in W satisfying φ, or
(ii) there exists $w_0 \in W$ such that $w_0 \models \varphi$ and for any $w \leq_{u(r)} w_0$, $w \models \varphi \supset \psi$.

Analogously, the truth condition for the wff $[r \circ i]\psi$ is

$u \models [r \circ i]\psi$ iff there exists $w_0 \in \mathcal{R}_i(u)$ such that for any $w \leq_{u(r)} w_0$, if $w \in \mathcal{R}_i(u)$, then $w \models \psi$.

It can be seen that $[i \circ \varphi]\psi$ is equivalent to $[\triangle_\varphi(\{i\})]\psi$ in section 4 according to the semantics.

6 Concluding Remarks

In preceding sections, we assume an agent's belief states are represented as a subset of possible worlds, i.e. $\mathcal{R}_i(w)$ is the belief state of agent i in world w. However, some more fine-grained representations have been also proposed, such as total pre-orders over the set of possible worlds [8, 17, 28, 41], ordinal conditional functions [11, 43, 44], possibility distributions[3, 18, 19], belief functions[42] and pedigreed belief states[22]. Further development of logical systems that incorporate fusion operators based on more fine-grained representations of belief states should be a very interesting research direction.

We mainly present the semantics of epistemic logics for information fusion in this paper. However, to do practical reasoning, we must develop proof methods for these logics. There have been some previous works on the development of axiomatic or Gentzen-style calculi for information fusion. For example, in [4–7], logics for information fusion based on possibility theory are proposed. The Hilbert-style or Gentzen-style proof systems of those logics are also presented. In particular, the logic $\mathbf{PL}_n^{\otimes}$ in [4] is an extension of QML in [29–31] with distributed belief operator, so the fusion operator in $\mathbf{PL}_n^{\otimes}$ is different than the merging operators used in this paper. The axiomatic system and theorem prover for a majority fusion logic MF have also been developed in [13, 14]. The belief bases in MF are sets of literals, so it does not allow nested modalities as in our logics. In spite of these differences, the further development of proof theory for logics proposed in this paper could take these previous works as good starting points.

References

1. C.E. Alchourrón, Gärdenfors, and D. Makinson. "On the logic of theory change: Partial meet contraction and revision functions". *Journal of Symbolic Logic*, 50:510–530, 1985.
2. H.L. Arlo-Costa and S.J. Shapiro. "Maps between nonmonotonic and conditional logic". In B. Nebel, C. Rich, and W. Swartout, editors, *Proceedings of the 3rd International Conference on Principle of Knowledge Representation and Reasoning*, pages 553–564. Morgan Kaufmann, 1992.
3. S. Benferhat, D. Dubois, and H. Prade. "From semantic to syntactic approaches to information combination in possibilistic logic". In B. Bouchon-Meunier, editor, *Aggregation and Fusion of Imperfect Information*, pages 141–161. Physica-Verlag, 1997.
4. L. Boldrin and A. Saffiotti. "A modal logic for merging partial belief of multiple reasoners". *Journal of Logic and Computation*, 9(1):81–103, 1999.
5. L. Boldrin and C. Sossai. "An algebraic semantics for possibilistic logic". In *Proceedings of the 11th International Conference on Uncertainty in ArtiOcial Intelligence*, pages 24–37, 1995.
6. L. Boldrin and C. Sossai. "Partial information and necessity-valued logic". In *Proceedings of the 6th International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems*, pages 941–946, 1996.
7. L. Boldrin and C. Sossai. "Local possibilistic logic". *Journal of Applied Non-Classical Logic*, 7(3):309–333, 1997.

8. C. Boutilier. "Revision sequences and nested conditionals". In *Proceedings of the 13th International Joint Conference on Artificial Intelligence*, pages 519–525, 1993.
9. C. Boutilier. "Conditional logics of normality : a modal approach". *Artificial Intelligence*, 68:87–154, 1994.
10. C. Boutilier. "Toward a logic for qualitative decision theory". In *Proceedings of the Fourth International Conference on Principles of Knowledge Representation and Reasoning*, pages 75–86, 1994.
11. C. Boutilier. "Generalized update: Belief change in dynamic settings". In *Proceedings of the 14th International Joint Conference on Artificial Intelligence*, pages 1550–1556, 1995.
12. L. Cholvy. "A logiccal approach to multi-souces reasoning". In M. Masuch and L. Pólos, editors, *Knowledge Representation and Reasoning under Uncertainty*, LNCS 808, pages 183–196. Springer-Verlag, 1994.
13. L. Cholvy and Ch. Garion. "A logic to reason with contradictory beliefs with a majority approach". In *Proceedings of the IJCAI Workshop on Inconsistency in Data and Knowledge*, 2001.
14. L. Cholvy and Ch. Garion. "Answering queries addressed to several databases: a query evaluator which implements a majority merging approach". In M.-S. Hacid, Z.W. Ra, and D.A. Zighed andY. Kodratoff, editors, *Proc of 13th International Symposium on Methodologies for Intelligent Systems*, LNAI 2366, pages 131–139. Springer-Verlag, 2002.
15. L. Cholvy and A. Hunter. "Information fusion in logic: A brief overview". In *Qualitative and Quantitative Practical Reasoning(ECSQARU'97/FAPR'97)*, LNAI 1244, pages 86–95. Springer-Verlag, 1997.
16. M. Dalal. "Investigations into a theory of knowledge base revision: Preliminary report". In *Proceedings of the 7th National Conference on Artificial Intelligence*, pages 475–479. AAAI Press, 1988.
17. A. Darwiche and J. Pearl. "On the logic of iterated belief revision". *Artificial Intelligence*, 89(1):1–29, 1997.
18. D. Dubois and H. Prade. "Belief change and possibility theory". In P. Gärdenfors, editor, *Belief Revision*, pages 142–182. Cambridge University Press, 1992.
19. D. Dubois and H. Prade. "Possibility theory in information fusion". In *Proc. of the Third International Conference on Information Fusion*, pages TuA–1, 2000.
20. R. Fagin, J.Y. Halpern, Y. Moses, and M.Y. Vardi. *Reasoning about Knowledge*. MIT Press, 1996.
21. J. Hintikka. *Knowledge and Belief*. Cornell University Press, 1962.
22. P. Maynard-Reid II and Y. Shoham. "Belief fusion: Aggregating pedigreed belief states". *Journal of Logic, Language and Information*, 10(2):183–209, 2001.
23. H. Katsuno and A. Medelzon. "On the difference between updating a knowledge base and revising it". In *Proceedings of the Second International Conference on Principles of Knowledge Representation and Reasoning (KR'91)*, pages 387–394. Morgan Kaufmann Publisher, 1991.
24. H. Katsuno and A. Medelzon. "Propositional knowledge base revision and minimal change". *Artificial Intelligence*, 52:263–294, 1991.
25. S. Konieczny. "On the difference between merging knowledge bases and combining them". In *Proceedings of the Seventh International Conference on Principles of Knowledge Representation and Reasoning (KR'00).*, pages 135–144. Morgan Kaufmann Publisher, 2000.
26. S. Konieczny and R. Pino Pérez. "On the logic of merging". In *Proceedings of the Sixth International Conference on Principles of Knowledge Representation and Reasoning (KR'98)*, pages 488–498. Morgan Kaufmann Publisher, 1998.

27. S. Konieczny and R. Pino Pérez. "Merging with integrity constraints". In A. Hunter and S. Parsons, editors, *Proccedings of the Fifth European Conference on Symbolic and Quantitative Approaches to Reasoning with Uncertainty (ECSQARU'99)*, LNAI 1638, pages 233–244. Springer-Verlag, 1999.
28. D. Lehmann. "Belief revision, revised". In *Proceedings of the 14th International Joint Conference on Artificial Intelligence*, pages 1534–1540, 1995.
29. C.J. Liau and I.P. Lin. "Quantitative modal logic and possibilistic reasoning". In B. Neumann, editor, *Proceedings of the 10th ECAI*, pages 43–47. John Wiley & Sons. Ltd, 1992.
30. C.J. Liau and I.P. Lin. "Proof methods for reasoning about possibility and necessity". *International Journal of Approximate Reasoning*, 9(4):327–364, 1993.
31. C.J. Liau and I.P. Lin. "Possibilistic Reasoning—A Mini-survey and Uniform Semantics". *Artificial Intelligence*, 88:163–193, 1996.
32. P. Liberatore and M. Schaerf. "Arbitration: A commutative operator for belief revision". In *Proceedings of the Second World Conference on the Fundamentals of Artificial Intelligence (WOCFAI '95)*, pages 217–228, 1995.
33. J. Lin. "Information sharing and knowledge merging in cooperative information systems". In *Proceedings of the Fourth Workshop on Information Technologies and Systems*, pages 58–66, 1994.
34. J. Lin. "Integration of weighted knowledge bases". *Artificial Intelligence*, 83(2):363–378, 1996.
35. J. Lin and A.O. Mendelzon. "Knowledge base merging by majority". In R. Pareschi and B. Fronhoefer, editors, *Dynamic Worlds: From the Frame Problem to Knowledge Management*. Kluwer Academic Publisher, 1999.
36. J.-J. Ch. Meyer and W. van der Hoek. *Epistemic Logic for AI and Computer Science*. Cambridge University Press, 1995.
37. D. Nute. "Conditional logic". In D.M. Gabbay and F. Guenthner, editors, *Handbook of Philosophical Logic, Vol II: Extensions of Classical Logic*, pages 387–439. D. Reidel Publishing Company, 1984.
38. P. Z. Revesz. "On the semantics of theory change: Arbitration between old and new information". In *Proceedings of the Twelfth ACM SIGACT-SIGMOD-SIGART Symposium on Principles of Database Systems*, pages 71–82, 1993.
39. P.Z. Revesz. "On the semantics of arbitration". *International Journal of Algebra and Computation*, 7(2):133–160, 1997.
40. K. Schlechta and D. Makinson. "Local and global metrics for the semantics of counterfactual conditionals". *Journal of Applied Non-Classical Logics*, 4(2):129–140, 1994.
41. K. Segerberg. "Belief revision from the point of view of doxastic logic". *Bull. of the IGPL*, 3(4):535–553, 1995.
42. P. Smets. " Data fusion in the transferable belief model". In *Proc. of the Third International Conference on Information Fusion*, pages WeA–1, 2000.
43. W. Spohn. "Ordinal conditional functions: a dynamic theory of epistemic states". In W.L. Harper and B. Skyrms, editors, *Causation in Decision, Belief Change, and Statistics, II*, pages 105–134. Kluwer Academic Publishers, 1988.
44. M.A. Williams. "Transmutations of knowledge systems". In J. Doyle, E. Sandewall, and P. Torasso, editors, *Proceedings of the 4th International Conference on Principle of Knowledge Representation and Reasoning*, pages 619–629. Morgan Kaufmann Publishers, 1994.