

Short Paper

$\Theta(N^{5/6})$ Time Complexity of Fault Diagnosis Algorithms in $N \times N$ Dilated Blocking Photonic Switching Networks

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Dilated Optical Multistage Interconnection Networks (DOMINs) based on 2×2 directional coupler photonic switches play an important role in all-optical high-performance networks, especially for the emerging IP over DWDM architectures. The problem of *crosstalk* within photonic switches is underestimated due to the aging of the switching element, control voltage, temperature and polarization, and thus causes undesirable coupling of the signal from one path to the other. Previous works [18] designed an efficient diagnosing *disjoint faults* algorithm in small sized networks, which reduced the number of tests required by overlapping the tests with computations to one half in photonic switching networks. Furthermore, this paper generically derives algorithms and mathematical modules to find the optimal degree of parallelism of faults diagnosis for $N \times N$ dilated blocking networks, as the size of network is larger. Taking advantage of the properties of disjoint faults, diagnosis can be accelerated significantly because the optimal degree of parallel fault diagnosis may grow exponentially. To reduce the diagnosis time, an algorithm is proposed herein to find the maximum number of disjoint faults. Rather than requiring up to $4MN$ tests as a native approach, a two-phase diagnosis algorithm is proposed to reduce the testing requirement to $4N$ tests. This study extends the concept of disjoint faults to reduce the number of tests to the time efficiency of $\Theta(N^{5/6})$ for $N \times N$ DOMINs.

Keywords: dilated photonic switching network, crosstalk fault, degree of parallelism, fault diagnosis algorithm, disjoint-faults

1. INTRODUCTION

The concept of multistage interconnection networks (MINs) was first introduced in the context of circuit switching providing parallel transmission [1-3]. The MINs possess two properties: the unique path for each input-output pair and the digital control routing. Unlike the previous connection-oriented packet switched networks based on the TCP/IP protocol stack, the MINs do not establish connections, but instead the entire packet follows the route selected during set-up process. Because the TCP/IP network bandwidth is

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restricted by the router throughput, router performance benefits when the network scale is increased. A specific architecture is provided by so-called “IP over ATM” switching [4, 5]. In this case, a new packet-passing scenario substitutes for the classic IP routing. After the first IP datagram establishes a new path in the ATM network, the subsequent IP datagrams are forwarded directly through the previously established path. This forwarding technique is termed “cut through” switching [6]. Like the transmission scenario of the IP datagram over ATM networks, electronic-based networks can be promoted to all-optical networks. Furthermore, employing the switching elements as the routing components allows IP to be taken directly over DWDM (Dense Wavelength Division Multiplexing) architecture providing a fully optical high-performance network. This architecture incorporates scalability and cost-effectiveness into recent research [7].

Photonic switching elements are essential in optical networks. Because photonic switching networks provide an all optical transmission, high transmission speed, and high throughput without the conversion between electronic-to-optical and optical-to-electronic [8, 9]. The concept of dilation in the spatial domain for DOMINs [10] to reduce the negative effect of crosstalk is introduced here. The DOMINs, constructed using 2×2 directional couplers (photonic switches) [9, 11-13], can also be classified into three architectures: *dilated blocking network*, *dilated non-blocking network*, and *dilated re-arrangeable network* [10, 14, 15]. A 2×2 switching element, two inlets and two outlets, allows the implementation of self-routing under the unbuffered architecture.

Optical crosstalk is a problem in DOMINs and results from the undesirable coupling between signals from one path to the other within a directional coupler [12, 16], which is called *first-order* crosstalk. It is essential to design different diagnosing crosstalk fault algorithms for DOMINs from the automatic testing generation in the current sophisticated VLSI technology. The diagnosis process for DOMINs [17, 18] can be divided into two basic phases: the fault detection process and the fault location process. In the first phase, all possible suspicious faulty switches are detected from the outputs, while in the second phase, all possible suspicious switches are examined by injecting signals into them and checking if the output signals exceed threshold. Hwang [18] designed the *disjoint faults* algorithm in dilated blocking networks to accelerate diagnosis by putting two first-order crosstalks in parallel. This paper presents the upper bound approach, in which the optimal number of disjoint faults is accommodated in $N \times N$ dilated blocking networks as the size of DOMINs grows. Furthermore, a new diagnosing algorithm is proposed to find other possible pairs of disjoint faults. Because of the property of disjoint faults, the execution time can be reduced during the second phase. This study proves that

the number of disjoint faults is $2 \times 2^{\lfloor \frac{\log_2 N - 1}{6} \rfloor}$ and the time efficiency of the new diagnosing algorithm can be enhanced to $\Theta(N^{5/6})$ for $N \times N$ DON as N becomes larger.

This paper is organized as follows. Section 2 introduces $N \times N$ Dilated Omega Network (DON), while section 3 describes the fault model, system assumptions and primitive diagnosing algorithm. Section 4 presents the upper bound approach and shows that the optimal time efficiency of the approach is $\Theta(\log_2 N)$. Section 5 proposes a new diagnosis algorithm for $N \times N$ DON and proves that the time efficiency of the diagnosing algorithm can be reduced to $\Theta(N^{5/6})$. Finally, conclusions are given in section 6.

2. DILATED OMEGA NETWORK

Each of the networks in a class of $N \times N$ dilated blocking networks consists of a set of N input terminals, a set of N output terminals, and $\log_2 N + 1$ stages of logic cells. Some isomorphic dilated blocking topologies have been proven to be topologically equivalent [18], such as the Dilated Omega Network, Dilated Banyan Network, and so on. The $N \times N$ Dilated Omega Network (DON) is an *asymmetrical* architecture recursively constructed from stage i connected to stage $(i + 1)$ by the *shuffle connection*. As in any dilated networks, only half of the input ports and output ports are used, and no single switching can simultaneously carry multiple active paths at a given time [9, 10, 13].

3. PREVIOUS WORKS

This section describes the primitive fault detection and fault location algorithm for disjoint faults, and also defines the crosstalk model and system assumptions.

3.1 Crosstalk Model

The basic component of the photonic switching networks is a 2×2 directional coupler that comprises two single-mode optical waveguides formed by diffusing Titanium into Lithium Niobate (Ti:LiNbO_3) substrate [14]. The voltage controls switching across the electrodes in the directional coupler. By applying a certain control voltage on the electrode, a main signal in the upper (or lower) input is switched to the upper (or lower) output which is equivalent to the BAR state or the CROSS state.

A switch is said to be *faulty*, if it causes its crosstalk is above a given threshold, whose value can be determined according to certain criteria. The crosstalk(s) fault in the network can be isolated and switched to the output switches. Four optical paths exist, and our possibilities for crosstalk faults on these paths in a switching element. $W(0, 0)$ (or Type-1) optical path and $W(1, 1)$ (or Type-2) optical path faults may occur when the switch is in the Bar state, while $W(0, 1)$ (or Type-3) optical path and $W(1, 0)$ (or Type-4) optical path faults may occur when the switch is in the Cross state.

3.2 Fault Diagnosis Method

The diagnosing algorithm contains two phases: fault detection and fault location [17, 18]. The fault detection phase detects possible crosstalk signals from the output terminals, while the fault location phase locates possible fault switches inside the photonic switching chips. Backwards and forwards tracing is necessary to find the input and output terminals, respectively, under test from suspicious fault switches at any stage within the network.

Before investigating the fault detection and fault location methods, the system assumptions are as follows:

- A switch with a crosstalk remains operational.
- Dual access to the edges of the network is allowed and the only accessible parts of the photonic switching network are the input and output terminals.

- A port is called an *even* parity port if its binary representation has an even number of 1s, and an *odd* parity if otherwise.
- Each switch in the network can be referred to by a pair (i, t) , where i denotes the stage number, and t represents the position of the switch at that stage from the top to the bottom.
- Parity code is developed for the input testing signals which guide the testing signals on where to put the set of testing signals into the input terminals and where to measure the crosstalk signals in the output terminals.

3.2.1 Fault detection method

The fault detection procedure has four detection tests to detect the four types of fault, namely the Bar-E, Bar-O, Cross-E, and Cross-O tests. Since the DON is an *asymmetrical* architecture, the possible output faulty signal depends on the type of test (Bar test or Cross test), the parity of the injected signal, and the number of stages in the network. Table 1 lists their relationships among these variables.

Table 1. The relationship of parity between the injected input signal and the faulty output signal will be measured.

Typ of Test	Type of crosstalk	Output signal parity
Bar-Even	no crosstalk	Even
	first-order crosstalk	Odd
Bar-Odd	first-order crosstalk	Even
	no crosstalk	Odd
Cross-Even (No. of stage is odd)	no crosstalk	Odd
	first-order crosstalk	Even
Cross-Even (No. of stage is even)	no crosstalk	Even
	first-order crosstalk	Odd
Cross-Odd (No. of stage is odd)	no crosstalk	Even
	first-order crosstalk	Odd
Cross-Odd (No. of stage is even)	no crosstalk	Odd
	first-order crosstalk	Even

The fault detection procedure is as follows:

1. The whole network is configured to either the Bar or Cross state.
2. A pre-defined set of inputs is illuminated.
3. All four tests are performed, namely the Bar-E test, Bar-O test, Cross-E test, and Cross-O test.
4. Whether the signal at any output specified in step 3 exceeds the threshold value is determined; if so, then the network contains at least one faulty switch.

3.2.2 Algorithm for locating suspicious switches

From the fault detection test, if the crosstalk level in an output terminal exceeds the threshold value, then one suspicious switch in each stage corresponds to that measure-

ment. Tracing back from the output terminal and applying switching function and routing algorithm will locate all possible suspicious switches.

3.2.3 Fault location method

Once all four-fault detection tests are performed, $4N$ location tests ($2N$ Bar tests and $2N$ cross tests) are needed to isolate the faulty switches from among the suspicious ones. One possible way of identifying the suspicious switches is to individually direct a signal to these switches. Should crosstalk exist in any of the suspicious switches, it should be measured at a distinct output that will uniquely identify its location. From the fault detection test, information regarding the state of the switching network and the position of the output at which the signal exceeds the threshold can be used to locate the suspicious switches. The crosstalk for all suspicious switches must be re-routed to different outputs for measurement. The three algorithms are then employed to find the input signal terminal in the first stage, and output terminals from the input signal and the crosstalk signal will be measured in the last stage for the suspicious switch inside the networks [18].

3.2.4 Disjoint faults

The primitive fault detection and the fault location algorithm are the basis of the following proposal for a diagnosing faulty switch algorithm. Notably, the number of unused photonic switches grows in $\Theta(N^2)$ as the size of the network increases for $N \times N$ DON. This phenomenon raises the possibility of finding other possible test patterns that can be generated, or of finding optimal test patterns, which can simultaneously locate faulty switches. First, a diagnosing algorithm which can simultaneously locate two first-order crosstalks called *disjoint faults* is devised. The following lemma is used to define the *disjoint fault*.

Lemma 1 If two output switches are identified as faulty during the last stage, say, $a_{w-1}a_{w-2} \dots a_0$ and $b_{w-1}b_{w-2} \dots b_0$ for a size $N \times N$ DON, $w = \log_2 N + 1$, and if the operation of the *Exclusive-Or* of $a_{w-1} \oplus b_{w-1} = 1$, and $a_0 \oplus b_0 = 1$, and the other bits $a_i = b_i$, $1 \leq i \leq w - 2$; then the two faulty switches are defined as having a pair of disjoint faults, and the suspicious faulty switches can be simultaneously located.

Proof: Recall that the DON is an *asymmetrical* architecture, and possible faulty output signals are measured depending on the type of test (Bar test or Cross test), the parity of the injected signal (even or odd), and the number of stages in the network. Table 2 lists eight situations that must be considered, and [18] provides the detailed proofs for each situation. \square

The methodology called the BCBC and CBCB tests [18] is used to set all switches in the same stage to the same states, with the states alternating between Bar and Cross or Cross and Bar, respectively. The following algorithms are used for locating disjoint faults, namely the BCBC(-CBCB) test and CBCB(-BCBC) test, respectively. The BCBC test and CBCB test are used to locate one possible faulty switch, while the (-CBCB) test and (-BCBC) test are used to locate another faulty switch for disjoint faults.

Table 2. Eight situations are considered in *Lemma 1*.

State	Type of test	The number of stage
Bar	BCBC-CBCB	Even
Bar	BCBC-CBCB	Odd
Bar	CBCB-BCBC	Even
Bar	CBCB-BCBC	Odd
Cross	BCBC-CBCB	Even
Cross	BCBC-CBCB	Odd
Cross	CBCB-BCBC	Even
Cross	CBCB-BCBC	Odd

Lemma 2 $N \times N$ DOMIN can locate one first-order fault switch, provided that N must equal or exceed 2. However, for locating two first-order disjoint faults, the size of DOMIN, N , must equal or exceed 8.

Proof: From the result of *Lemma 1*, it is noted that:

- the number of suspicious faulty switches for one output which is identified as faulty is $\log_2 N + 1$;
- the number of output switches in the last stage passing through after the location algorithm for one output switch is identified as faulty is $2((\log_2 N + 1)/2) = \log_2 N + 1$;
- the number of output switches of the pair of disjoint faults passing through after the location algorithm is $2(\log_2 N + 1)$;
- the number of output switches in the last stage is N ;
- the number of output switches in the last stage must equal or exceed the number of output switches of the disjoint fault passing through after the location algorithm is $N \geq 2(\log_2 N + 1)$, and therefore $N \geq 8$. \square

The results in this section indicate that disjoint faults exists for 8×8 DON. The following two sections can derive the possible maximum pairs of disjoint faults mathematically for the size of $N \times N$ DOMIN, and propose the $\Theta(N^{5/6})$ time complexity of the fault diagnosis algorithm in $N \times N$ DOMIN as N increases larger, thereafter.

4. UPPER BOUND APPROACH

This section presents a mathematical method for predicting the maximum number of disjoint faults for $N \times N$ DOMIN. This approach is based on the relation between the number of switches of $N \times N$ DOMIN and the number of switches passed by the input and crosstalk signals in the last stage for each pair of disjoint faults during the location process. The following lemma proves that the maximum number of disjoint faults for $N \times N$ DOMIN is $2 \cdot \lfloor \frac{N}{2 \log_2 N + 1} \rfloor$.

Lemma 3 A generic equality for the upper bound of disjoint faults for $N \times N$ DOMIN is $2 \cdot \lfloor \frac{N}{2^{\log_2 N + 1}} \rfloor$ and the time efficiency of the approach is $\Theta(\log_2 N)$, where N denotes the number of switches in the last stage and $2^{\log_2 N + 1}$ represents the number of switches that the input signals and crosstalk signals will pass in the last stage during the location process for each pair of disjoint faults.

Proof: During the location process for the Bar test, the status of each stage is set to BCBC to locate possible Bar faulty switches in the even stage of the network, and is set to CBCB to locate possible Bar faulty switches in the odd stage of the network. Like the Cross test, the status of each stage is set to CBCB to locate possible Bar faulty switches in the even stage of the network, and is set to BCBC to locate possible Bar faulty switches in the odd stage of the network. The number of switches that the input signals and crosstalk signals will pass in the last stage depends on the number of stages of $N \times N$ DOMIN, and all the location process for the Bar and Cross tests are summarized as follows:

For the Bar test location processes,

- The number of switches passed by both signal and faulty signals in the last stage is $2^{\log_2 N + 1}$ ($\log_2 N + 1$ during the BCBC location process, and $\log_2 N$ during the CBCB location process).

For the Cross test location processes,

- The number of switches passed by both signal and faulty signals in the last stage is $2^{\log_2 N + 1}$ ($\log_2 N$ during the BCBC location process, and $\log_2 N + 1$ during the CBCB location process).

Recall that the BCBC(-CBCB) and CBCB(-BCBC) tests for Bar-E test involve the BCBC and CBCB tests locating one possible faulty switch, while the (-CBCB) and (-BCBC) tests involving locating another faulty switch for disjoint faults. The number of switches in the last stage of the BCBC(-CBCB) test, which are passed by both signal and faulty signals, is $2^{\log_2 N + 1}$ ($\log_2 N + 1$ during the BCBC location process, and $\log_2 N$ during the (-CBCB) location process). Furthermore, the number of switches which both signal and faulty signals pass in the last stage of the CBCB(-BCBC) test is $2^{\log_2 N + 1}$ ($\log_2 N$ during the CBCB location process, and $\log_2 N + 1$ during the (-BCBC) location process). Similarly, the same results are found for the Bar-O test and Cross test (Cross-E and Cross-O tests), showing that both the Bar test and Cross test are independent of the size of the $N \times N$ DOMIN. Therefore, the upper bound of disjoint faults for $N \times N$ DOMIN is $2 \cdot \lfloor \frac{N}{2^{\log_2 N + 1}} \rfloor$. The total number of tests required by the two phases diagnosis algorithm is $4N$, and the number of tests is $\frac{4N}{2 \cdot \lfloor \frac{N}{2^{\log_2 N + 1}} \rfloor}$. The time efficiency of the

upper bound approach when N is infinity is $\Theta(\frac{N}{\log_2 N}) = \Theta(\log_2 N)$. □

The upper bound of the disjoint faults in $N \times N$ DOMIN is illustrated in Table 3.

Table 3. Upper bound of the disjoint faults for $N \times N$ DOMIN.

Size of DOMIN, N	Upper bound of disjoint faults
8	2
16	2
32	4
64	8
128	16
256	30
512	52

5. $\Theta(N^{5/6})$ TIME EFFICIENCY OF THE NEW DIAGNOSING CROSSTALK FAULT ALGORITHM

The previous section showed that the upper bound approach can predict the optimal number of disjoint faults without considering possible merging into the same switches between pairs of disjoint faults with growing DOMIN. In fact, it is possible that the test and crosstalk signals will merge into the same switches before the last stage. Essentially, the number of tests exceeds that with the upper bound approach. This section shows the $\Theta(N^{5/6})$ time efficiency of the diagnosing algorithm. The following *Propositions* are presented to aid the proof of *Lemma 4*.

Proposition 1: The “*Hamming distance*” is defined as the number of bit differences between two signals (input signals and/or crosstalk signals).

Proposition 2: The *switching function* of the *cross* state within the switch can be expressed as follows:

$S(C, b_{w-1}b_{w-2} \dots b_1b_0) = b_{w-1}b_{w-2} \dots b_1\bar{b}_0$, where $b_{w-1}b_{w-2} \dots b_1b_0$ denote the input signal and $b_{w-1}b_{w-2} \dots b_1\bar{b}_0$ denotes the output signal. This is referred to a *Type-1 change*. Meanwhile, a *Type-2 change* is defined as a one bit complement of the faulty signal compared with the input signal.

Proposition 3: The “*stage distance*” is defined as the difference of the stages between two possible suspicious switches located.

Proposition 4: Any two signals (input signals and/or faulty signals) will not route to the same switch if they can keep more than a 2-bit hamming distances in the same stage.

Proof: Assume two signals, $b_{w-1}b_{w-2} \dots b_0$ at stage i , and $a_{w-1}a_{w-2} \dots a_0$ at stage j . When these two signals route to the same stage k , the output signals at stage k are $b'_{w-1}b'_{w-2} \dots b'_0$, and $a'_{w-1}a'_{w-2} \dots a'_0$, respectively. If two signals merge into the same switch, then $b_{w-1} = a'_{w-1}$, $b'_{w-2} = a'_{w-2}$, ..., $b'_1 = a'_1$. So, either one ($b'_0 \neq a'_0$) or no hamming distance ($b'_0 = a'_0$) exists.

According to the apagoge, it can be proved that any two signals will not route to the same switch provided that the two signals (input signals and/or faulty signals) can keep more than 2-bit hamming distances.

Lemma 4 For $N \times N$ DON (when the size of the network, N , is increased beyond 128), the output detected faulty signals of the pair of disjoint faults defined in *Lemma 1* are $b_{w-1} \dots b_6 b_5 b_4 b_3 b_2 b_1 b_0$ and $\overline{b_{w-1} \dots b_6 b_5 b_4 b_3 b_2 b_1 b_0}$, where $w = \log_2 N + 1$. Whenever the number of internal bits (excluding b_{w-1} and b_0) increases in every other six successive bits with 6-bit Hamming distances based on the previous pair of disjoint faults, a new pair of disjoint faults can be generated, for example, when the new pair of disjoint faults are $b_{w-1} \dots \overline{b_6 b_5 b_4 b_3 b_2 b_1 b_0}$ (pair of $b_{w-1} \dots b_6 b_5 b_4 b_3 b_2 b_1 b_0$) and $\overline{b_{w-1} \dots \overline{b_6 b_5 b_4 b_3 b_2 b_1 b_0}}$ (pair of $b_{w-1} \dots b_6 b_5 b_4 b_3 b_2 b_1 b_0$).

Proof: Recall that in *Proposition 4*, any two disjoint signals in the same stage during routing must keep more than 2-bit Hamming distances. According to the *Lemma 1*, the first pair of the disjoint faults is $b_3 b_2 b_1 b_0$, $\overline{b_3 b_2 b_1 b_0}$ for 8×8 DON. It is proved that when the number of internal bits (excluding b_{w-1} and b_0) increases in every other six successive bits with 6-bit Hamming distances based on the previous disjoint faults, a new pair of disjoint faults can be generated. The proof is divided into two parts:

Initially, if the number of successive bits is less than 6, for example 5, then because the setting for the status of the stage is BCBC... or CBCB... for the Bar test, there are two or three of the five successive different bits that will be Type-1 change, and another bit will be Type-2 change if a faulty switch occurs. If two of the five successive different bits are Type-1 change, the hamming distance is at least 2, while if three of the five successive different bits are Type-1 change, then the hamming distance is at least 1, which violates *proposition 4*. The same argument for the Cross test, and another pair of the disjoint faults can be found if six or more different successive bits exist.

When the number of successive bits is 6, a maximum of three will be Type-1 change and another will be Type-2 change if a faulty switch occurs. Because the Type-1 change occurs between two suspicious switches from stage i to stage $i + 5$, a maximum of four changes occur for any two signals when the stage distance is equal or greater than 6. Consequently, it is only proved that the stage distance between any two crosstalk faulty signals in either a BCBC... or CBCB... setting will be ≥ 6 .

If the stage distance equals 6 or greater, then four possibilities exist for the Bar test:

First, the suspicious signals may both be non-faulty. When these two suspicious signals route to the same stage, they will be 3-bit complemented by Type-1 changes, and will be 0 bit complemented by Type-2 changes. Consequently, any two signals in the same stage will be separated by at least 3 bits Hamming distance of.

Second, the left suspicious signal may be non-faulty and the other may be faulty. When the two suspicious signals route to the same stage, they will be 3-bit complemented by the Type-1 changes, and will be 1 bit complemented by the Type-2 change. Then, any two signals in the same stage will be separated by contain at least 2 bits Hamming distance.

Third, the left suspicious signal may be faulty and the other may be non-faulty. When these two suspicious signals route to the same stage, they will be 3-bit complemented by Type-1 changes, and will be 0 bit complemented by the Type-2 change. Con-

sequently, any two signals in the same stage will be separated by at least 2 bits of Hamming distance.

Fourth, both of the suspicious signals may be both faulty: When these two suspicious signals route to the same stage, they will be 3 bit complemented by the Type-1 changes, and will be 0 bit complemented by the Type-2 change, so any two signals in the same stage will be separated by at least 3 bits of Hamming distance. \square

In summary, Table 4 lists the fault location cluster relation for various sizes of DOMIN.

Table 4. Fault location cluster relation for different sizes of $N \times N$ DON.

Size of N	Fault Location Cluster
8	$(b_3b_2b_1b_0, \bar{b}_3b_2b_1\bar{b}_0)$
16	$(b_4b_3b_2b_1b_0, \bar{b}_4b_3b_2b_1\bar{b}_0)$
32	$(b_5b_4b_3b_2b_1b_0, \bar{b}_5, b_4b_3b_2b_1\bar{b}_0)$
64	$(b_6b_5b_4b_3b_2b_1b_0, \bar{b}_6b_5b_4b_3b_2b_1\bar{b}_0)$
128	$(b_7b_6b_5b_4b_3b_2b_1b_0, \bar{b}_7b_6b_5b_4b_3b_2b_1\bar{b}_0, b_7\bar{b}_6\bar{b}_5\bar{b}_4\bar{b}_3\bar{b}_2\bar{b}_1b_0, \bar{b}_7\bar{b}_6\bar{b}_5\bar{b}_4\bar{b}_3\bar{b}_2\bar{b}_1\bar{b}_0)$
256	$(b_8b_7b_6b_5b_4b_3b_2b_1b_0, \bar{b}_8b_7b_6b_5b_4b_3b_2b_1\bar{b}_0, b_8b_7\bar{b}_6\bar{b}_5\bar{b}_4\bar{b}_3\bar{b}_2\bar{b}_1b_0, \bar{b}_8b_7\bar{b}_6\bar{b}_5\bar{b}_4\bar{b}_3\bar{b}_2\bar{b}_1\bar{b}_0)$
512	$(b_9b_8b_7b_6b_5b_4b_3b_2b_1b_0, \bar{b}_9b_8b_7b_6b_5b_4b_3b_2b_1\bar{b}_0, b_9b_8b_7\bar{b}_6\bar{b}_5\bar{b}_4\bar{b}_3\bar{b}_2\bar{b}_1b_0, \bar{b}_9b_8b_7\bar{b}_6\bar{b}_5\bar{b}_4\bar{b}_3\bar{b}_2\bar{b}_1\bar{b}_0)$
1024	$(b_{10}b_9b_8b_7b_6b_5b_4b_3b_2b_1b_0, \bar{b}_{10}b_9b_8b_7b_6b_5b_4b_3b_2b_1\bar{b}_0, b_{10}b_9b_8b_7\bar{b}_6\bar{b}_5\bar{b}_4\bar{b}_3\bar{b}_2\bar{b}_1b_0, \bar{b}_{10}b_9b_8b_7\bar{b}_6\bar{b}_5\bar{b}_4\bar{b}_3\bar{b}_2\bar{b}_1\bar{b}_0)$
2048	$(b_{11}b_{10}b_9b_8b_7b_6b_5b_4b_3b_2b_1b_0, \bar{b}_{11}b_{10}b_9b_8b_7b_6b_5b_4b_3b_2b_1\bar{b}_0, b_{11}b_{10}b_9b_8b_7\bar{b}_6\bar{b}_5\bar{b}_4\bar{b}_3\bar{b}_2\bar{b}_1b_0, \bar{b}_{11}b_{10}b_9b_8b_7\bar{b}_6\bar{b}_5\bar{b}_4\bar{b}_3\bar{b}_2\bar{b}_1\bar{b}_0)$
4096	$(b_{12}b_{11}b_{10}b_9b_8b_7b_6b_5b_4b_3b_2b_1b_0, \bar{b}_{12}b_{11}b_{10}b_9b_8b_7b_6b_5b_4b_3b_2b_1\bar{b}_0, b_{12}b_{11}b_{10}b_9b_8b_7\bar{b}_6\bar{b}_5\bar{b}_4\bar{b}_3\bar{b}_2\bar{b}_1b_0, \bar{b}_{12}b_{11}b_{10}b_9b_8b_7\bar{b}_6\bar{b}_5\bar{b}_4\bar{b}_3\bar{b}_2\bar{b}_1\bar{b}_0)$
8192	$(b_{13}b_{12}b_{11}b_{10}b_9b_8b_7b_6b_5b_4b_3b_2b_1b_0, \bar{b}_{13}b_{12}b_{11}b_{10}b_9b_8b_7b_6b_5b_4b_3b_2b_1\bar{b}_0, b_{13}b_{12}b_{11}b_{10}b_9b_8b_7\bar{b}_6\bar{b}_5\bar{b}_4\bar{b}_3\bar{b}_2\bar{b}_1b_0, \bar{b}_{13}b_{12}b_{11}b_{10}b_9b_8b_7\bar{b}_6\bar{b}_5\bar{b}_4\bar{b}_3\bar{b}_2\bar{b}_1\bar{b}_0, b_{13}\bar{b}_{12}\bar{b}_{11}\bar{b}_{10}\bar{b}_9\bar{b}_8\bar{b}_7b_6b_5b_4b_3b_2b_1b_0, b_{13}\bar{b}_{12}\bar{b}_{11}\bar{b}_{10}\bar{b}_9\bar{b}_8\bar{b}_7\bar{b}_6\bar{b}_5\bar{b}_4\bar{b}_3\bar{b}_2\bar{b}_1b_0, \bar{b}_{13}\bar{b}_{12}\bar{b}_{11}\bar{b}_{10}\bar{b}_9\bar{b}_8\bar{b}_7b_6b_5b_4b_3b_2b_1\bar{b}_0, \bar{b}_{13}\bar{b}_{12}\bar{b}_{11}\bar{b}_{10}\bar{b}_9\bar{b}_8\bar{b}_7\bar{b}_6\bar{b}_5\bar{b}_4\bar{b}_3\bar{b}_2\bar{b}_1\bar{b}_0)$

Lemma 5 The maximum number of disjoint faults for $N \times N$ DON is $2 \times 2^{\lfloor \frac{\log_2 N - 1}{6} \rfloor}$, when $N \geq 8$, and the time efficiency of the approach is $\Theta(N^{5/6})$.

Proof: If the number of internal stages excluding the first stage and the last stage is less than 6, then only one disjoint fault exists. From *Lemma 5*, two outlet signals are disjoint if six successive different bits exist for these two outlet signals. So, four outlet signals are disjoint if twelve different bits exist in succession, and so on. Meanwhile, the number of internal stages is $\log_2 N - 1$ and the maximum number of more than six successive different bits is $\lfloor \frac{\log_2 N - 1}{6} \rfloor$. Thus, the maximum number of disjoint faults is $2 \times 2^{\lfloor \frac{\log_2 N - 1}{6} \rfloor}$, while the total number of tests required in the two phases diagnosis algorithm is $4N$, and the number of tests is $\frac{4N}{2 \times 2^{\lfloor \frac{\log_2 N - 1}{6} \rfloor}}$. The time efficiency of the upper bound approach when N is infinity is $\Theta\left(\frac{N}{2^{\frac{\log_2 N}{6}}}\right) = \Theta\left(\frac{N}{2^{\frac{1}{6}\log_2 N}}\right) = \Theta\left(\frac{N}{N^{\frac{1}{6}}}\right) = \Theta(N^{5/6})$. \square

6. CONCLUSIONS

Algorithms for diagnosing crosstalk for $N \times N$ dilated blocking photonic switching networks based on 2×2 directional coupler photonic switches are the major issue in emerging photonic network architectures. However, the crosstalk for optical MIN can only be measured in the inlet and outlet ports. Before diagnosis, the status of each switching element must be configured in the proper sequence. Meanwhile, the input signals are injected from the proper inlet ports and then the undesirable crosstalk output signals occur in specific outlet ports is measured. A native approach requires $4MN$ tests, and a two-phase diagnostic algorithm is proposed to reduce the test requirement to $4N$ tests. This paper proposes an algorithm to find the maximum number of disjoint faults for an $N \times N$ dilated blocking network and execute the fault location algorithm for these disjoint signals in parallel. The time efficiency of the fault diagnosis can be significantly improved, from $\Theta(N)$ to $\Theta(N^{5/6})$, as the size of the dilated blocking photonic switching network grows. An important feature of the proposed diagnostic algorithm can be programmed as the automatic testing generation in the current sophisticated VLSI technology. Other possible dilated photonic networks, such as re-arrangeable and non-blocking networks, are also interesting topics for future research. This investigation has significant practical value and can also be applied to integrated photonic networks.

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